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A Digital Nuclear Reactor Control System

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HE GREAT EXPANSION in the nuclear reactor field places increased phasis on the problem of reactor con-. The complexity of a nuclear re-or installation suggests the use of ital computation in the over-all power-nt control system. The chief ad-tages to be gained from the use of a ital system would be a sizable increase the flexibility of the programing of control system and an increase in the uracy of the computations.

The problem of digital control for nuclear ctors is approached in this paper in following manner. First, a pres-ized water-type reactor, and in par-ular the reactor now in operation at ippingport, Pennsylvania, is selected the one upon which the study is based. rtain variables which can be used to scribe the behavior of such a reactor e selected as essential to the design of a actor control system. These variables e either measured directly (e.g., neutron vel, coolant temperatures) or calculated om measurable variables (e.g., reactor riod, power level in the power plant). ext, several commands (e.g., fast or ow insertion of control rods, changes in ow rates) which can be used to control e behavior of the reactor are chosen. hen, an organization of digital computer ements for calculating the nonmeasur-ble reactor variables from the measurable nes and relating both measured and alculated quantities to the reactor con-rol commands, are developed. A func-ional design of the necessary computer ogic is carried out, and the required egister capacities, operation frequencies, nd scale factors are determined. The ffect of both control system frequency nd gain on the reactor stability is con-idered for a simplified system, and a eneralized transfer function for a stability nalysis of any digital reactor control ystem is presented.

Paper 58-1362, recommended by the AIEE Feed-back Control Systems Committee and approved by the AIEE Technical Operations Department for presentation at the AIEE Winter General Meeting, New York, N. Y., February 1–6, 1959. Manuscript submitted December 1, 1958; made available for printing April 27, 1960.

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Quantities Chosen to Describe Reactor Behavior[1–3]

The measurable quantities include neutron density [neutrons per cm (centimeters)[3]], coolant inlet and outlet temperatures, reactor fuel temperature, moderator temperature, control-rod temperature, shield temperature, pressures and flow rates of both primary and secondary coolants, primary coolant level, control-rod positions, and gamma-ray flux in such places as the control room or exit air duct.

The calculated quantities include the reactor period as given by:

$$\tau=\frac{1}{\dfrac{dn}{dt}\times\dfrac{1}{n}} \tag{1}$$

the reactor power output as expressed by the relation:

$$P=F(T_h-T_c) \tag{2}$$

the average coolant temperature defined as:

$$T_{\text{avg}}=\frac{T_h+T_c}{2} \tag{3}$$

and an error temperature given by the difference between the desired, or reference, average coolant temperature and the actual coolant temperature in the form:

$$\Delta T=T_{\text{ref}}-T_{\text{avg}} \tag{4}$$

See Table I for explanation of nomenclature.

The desirable signal to govern the movement of the control rods has been found to include a component proportional to the rate of change of the neutron level as well as to the actual level itself. It is given by:

$$S_1=n+\frac{K_1}{\tau} \tag{5}$$

where K_1 is a constant chosen to include a desirable fraction of the neutron level derivative component in the composite signal S_1.

Another type of control-rod actuating signal which is sometimes used contains a component proportional to the rate of change of the neutron level and another proportional to the error temperature.[4] It affords both derivative and integral control and is given by:

$$S_2=\Delta T+\frac{K_2}{\tau} \tag{6}$$

Reactor Control Commands

The commands chosen to control the reactor are as follows:

SCRAM: This is an emergency situation in which there is grave danger if the reactor is not shut down immediately. All rods are to be inserted into the reactor very fast in order to reduce the reactivity as quickly as possible.

REVERSE: The reactor is shut down, but more slowly than in a SCRAM. This situation is caused by a difficulty which requires the reactor to be shut down, but which is not as urgent as the one requiring a SCRAM.

CUTBACK: The power level in the reactor is reduced to a given low level, but the reactor is not completely shut down. The difficulty can be corrected while the reactor is still operating at very low power.

REGULATOR RODS IN: This command is caused by a power level slightly greater than

Table I. Quantities Indicative of Reactor Behavior

	Variable	Symbol	Units
Measured reactor variables	Neutron level	n	neutrons per cm^3
	Primary coolant flow rate	F	gallons per minute
	Primary coolant inlet temperature	T_c	degrees F
	Primary coolant outlet temperature	T_h	degrees F
	Fuel temperature	T_f	degrees F
	Primary coolant level	L	% maximum level
	Primary coolant pressure	p_c	pounds per square inch
	Secondary coolant pressure	p_s	pounds per square inch
	Gamma radiation flux	ϕ	mr* per hour
	Regular rod position	X	cm
	Shim rod position	X_s	cm
Calculated reactor variables	Reciprocal period	$1/\tau$	seconds^{-1}
	Average coolant temperature	T_{avg}	degrees F
	Reactor power	P	kw
	Proportional-plus-derivative control signal	S_1	
	Error temperature	ΔT	degrees F
	Integral-plus-derivative control signal	S_2	

*Milliroentgens.

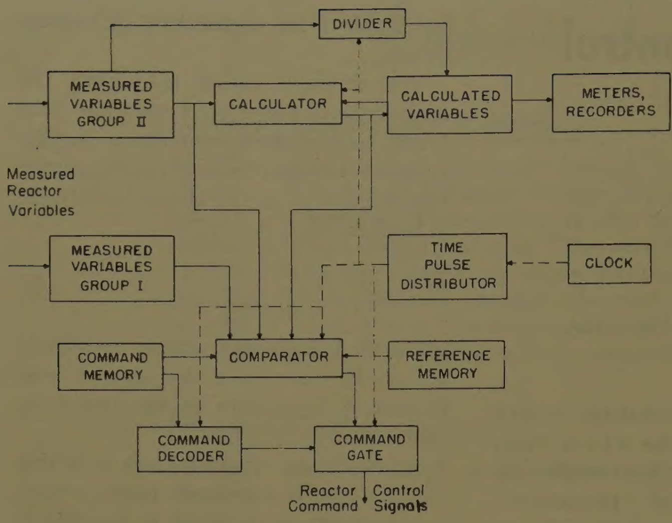


Fig. 1. Block diagram for over-all computer

the demand reference level. The rods are moved in an inward direction until the power level has returned to below the tolerated upper limit for the given power demand.

REGULATOR RODS OUT: Here the power level is slightly less than the given command, and the rods are moved in an outward direction until the power level has returned to above the tolerated lower limit for the given demand level.

EXCHANGE OUT: When a regulator rod has moved to its outermost position, its worth in reactivity is exchanged with that of the shim rods by moving the regulator rod from its outermost to its innermost position and at the same time moving the shim rods outward far enough to furnish the reactor with an amount of reactivity equal to that given up in the inward motion of the regulator rod. This command is necessary to accommodate the gradual outward movement of the regulator rod due to fuel depletion and poison buildup in the reactor.

EXCHANGE IN: The regulator rod is moved from its innermost position to its outermost one, while the shim rods are moved in by an amount necessary to make the net reactivity change zero.

INCREASE COOLANT FLOW RATE: The flow of the primary coolant is to be increased, for example by increasing pump speed.

DECREASE COOLANT FLOW RATE: The flow rate of the primary coolant is to be decreased.

In addition to these commands, the following subcommands are included:

INCREASE FLOW RATE (F): According to this subcommand coolant flow-rate conditions merit an increase in the flow rate, but before it is permissible to give a definite command signal to increase the coolant flow rate, certain temperature conditions must be fulfilled.

INCREASE FLOW RATE (T): Temperature conditions merit an increase in coolant flow rate, but before a definite command signal may be given, the coolant flow-rate conditions must also merit an increase in the flow rate.

The simultaneous occurrence of the subcommands INCREASE FLOW RATE (F) and INCREASE FLOW RATE (T) cause the full command INCREASE COOLANT FLOW RATE to be given. When the subcommands are not given coincidently they have no effect.

The following two subcommands are also available: DECREASE FLOW RATE (F) and DECREASE FLOW RATE (T). The simultaneous occurrence of the subcommands causes the DECREASE COOLANT FLOW RATE command to be given; their individual occurrence has no effect.

Relations Between Reactor Variables and Control Commands

The reactor variables initiate the control commands when the variable quantities become either greater or smaller than given references. To illustrate with some specific examples, a SCRAM command is given when the neutron level is greater than 150% of full power, a REVERSE command is given when the neutron level becomes greater than 125% of full power; and a REVERSE command is also made when the coolant flow rate becomes less than 70% normal. A complete listing of the relations between the reactor variables and the control commands is given in Appendix I, which gives the conditions on the magnitudes of the variables which initiate the various commands. In the nomenclature used, the symbols designating reactor variables are those given in Table I, and the additional subscripts are defined as follows: m refers to the absolute maximum, or SCRAM level; r the REVERSE level; c the CUTBACK level; h the higher of two variable levels which cause the same command; o refers to the level which initiates the EXCHANGE OUT command; i the level which initiates the EXCHANGE IN command, and "ref" to the reference power-demand signal from the turbo-generator. Numbered subscripts refer to various trip levels used in initiating the subcommands; while δ is a small fraction used in establishing a zone, or region of permitted variable values. For example, p_{cr1} refers to the lower REVERSE level for the coolant pressure variable.

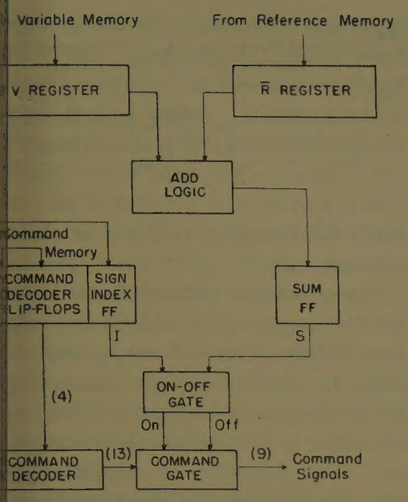
General Layout of Control Computer

General Description

A detailed block diagram of the over-all computer is shown in Fig. 1. The solid lines indicate the flow of processed information, while the dotted lines show the flow of the timing pulses necessary to operate the computer. The input to the computer, which consists of the measured reactor variables given in Table I, is sent directly to measured variable storage. The variables are categorized into two groups. Group I includes those variables which are not used in computing the calculated variables, whereas group II consists of those variables which are used in the calculations. The variables in group II are the neutron level, the primary coolant flow rate, the reactor inlet coolant temperature, and the reactor outlet coolant temperature. The remaining variables are included in group I.

The calculations are performed in two computing elements: a calculator which performs addition, subtraction, and multiplication; and a divider which performs division. The need for the two separate computing elements arises from the fact that the necessary division in the computation of the reactor period takes longer than the total time required to perform all other calculations. Also, the calculator, which is presently quite simple, would have to be substantially enlarged if it were to perform division in addition to the other arithmetic operations. The calculator and divider feed the calculated variable storage registers, the signals from which are sent to various indicating and recording devices.

The reactor variables (both measured and calculated) are sent to the comparator where they are compared with the reference levels. The comparisons are carried on at the same time the calculations are being performed in the calculator and the divider. Each reference level has an associated coded command signal which is read partly into the comparator and partly into the command decoder every time a comparison is made. The result of the comparison is gated with the decoded command signal to provide computer output signals which either initiate or terminate the reactor control com-



2. Comparator and its associated elements

nd signals. Timing pulses for all rations are furnished by a 64-kc clock its associated time-pulse distributor. operations are carried out in parallel n.

CULATOR

The operations of addition, subtraction, l multiplication are performed in the culator, which functions as an accumulator and which consists of a sumduct register, an addend-multiplicand ister, and a multiplier register. In the cess of adding two numbers, one num- is read into the sum register while the er number is placed in the addend ister. The logical expressions for the n and carry for each digit position are med when the numbers are inserted in ir respective registers, and the actual dition operation is performed upon the plication of an "add" pulse. The sum stored in the sum-product register and ay be read out upon the application of "read-out" pulse. Subtraction is permed by complement addition. Multiication consists of a series of additions of e contents of the multiplicand register those of the product register, and shifts the product and multiplier register gits according to whether the multiplier gits are ones or zeros.

IVIDER

Division is performed in the divider by series of additions and subtractions govned by the following rules.[5] First, the ivisor is subtracted from the dividend. the remainder is positive, a one is laced in the most significant quotient osition and the next operation is set up as subtraction. If the remainder is negave, a zero is placed in the quotient and ne next operation is set up as an addion. The divisor is then shifted and the process repeated, with the "answer" digit being placed in the next most significant quotient position.

Comparator and Associated Elements

The function of the comparator is to compare the reactor variables with various reference levels, and to initiate control command signals when the variables exceed (or in some cases become less than) the reference levels. The comparison is made by subtracting the reference level from the variable level. In the actual computation the complement of the reference level is added to the true value of the variable level.

As is shown in Fig. 2, the variable is placed in the V register and complement of the reference level in the $\bar{R}$ register. The addition is performed by means of add logic, which sends the sign of the remainder to the sum flip-flop. Each control command is given a specific code. If the control command signal is to be given when the variable exceeds the reference level, a sign index of 0 is assigned to the control command. If the signal is to be turned on when the variable becomes less than a particular reference value, a sign index of 1 is assigned. A list of all possible commands, their binary code, and sign index number is given in Table II.

These command codes and sign indexes are stored in the command memory. When a particular comparison is to be made, the command code and sign index associated with the reference level used in the comparison are read into the command decoder and sign index flip-flops. The command signal is turned on if the digits in the sign index and sum flip-flops are both 1 or both 0. The command signal is turned off if one flip-flop contains a 1 and the other a 0. This gating action is performed by the on-off gate. The command decoder is a diode matrix which causes one of its 13 output lines to be activated in response to inputs from the four command decoder flip-flops. The command gate functions to turn the command signals on and off at the proper times according to signals from the on-off gate and from the command decoder.

Timing-Pulse Generation

The timing pulses necessary for operation of the computer are furnished by a clock and its associated time-pulse distributor, the latter consisting of a six-stage flip-flop binary counter and an associated diode matrix. Output pulses, designed $P1$ through $P64$, are furnished and are routed to different places in the

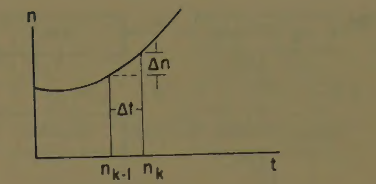


Fig. 3. Sampling for period calculation

computer. Since the clock frequency is 64 kc, a complete cycle of timing pulses lasts 1 ms (millisecond).

Calculation of Reactor Variables

Reactor Period

In the determination of the reactor period, due to ease of computation, the reciprocal period $1/\tau$ rather than the period itself is calculated. The digital computation is performed according to the approximate relation:

$$\frac{1}{\tau} \approx \frac{\Delta n}{n \Delta t} \approx \frac{n_k - n_{k-1}}{n_{k-1}} \times \frac{1}{\Delta t} \tag{7}$$

A hypothetical waveform of neutron level as a function of time is shown in Fig. 3. Samples of the neutron level are made every millisecond. The $(k-1)$th sample is subtracted from the kth sample, the difference is divided by the $(k-1)$th sample. The $(k-1)$th sample, rather than some mean value between the $(k-1)$th and kth is used in the denominator in order to ensure that any errors in period for an increasing neutron level will always be on the "safe" side. In other words, for an increasing neutron level, Δn will always be divided by the smaller of the two neutron levels, thus making $1/\tau$ greater than it would be if n_k or some mean value of n were used in the denominator.

The actual sequence of operations for performing the calculation of reactor period is as follows. On timing pulse $P64$ the neutron level is sampled, and a number proportional to the neutron level is read into the n_k storage register com-

Table II. Command Codes and Sign Indexes

Command	Code	Sign Index
INCREASE F	0001	1
DECREASE F(F)	0010	0
INCREASE F(F)	0011	1
REVERSE (variable reference)	0100	0
	0100	1
INCREASE F(T)	0101	0
DECREASE F(T)	0110	1
RODS IN	0111	0
RODS OUT	1000	1
DECREASE F	1001	0
SCRAM	1010	0
EXCHANGE IN	1011	1
EXCHANGE OUT	1100	0
CUTBACK (variable reference)	1101	0
	1101	1

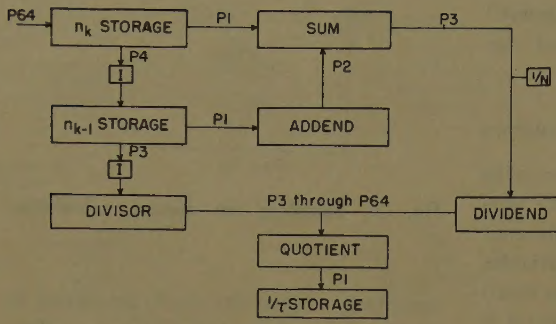
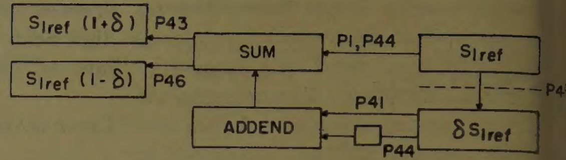


Fig. 4 (left). Sequence for period calculation

Fig. 5 (right). Sequence for reference control-signal calculation



posed of a parallel array of flip-flops (see Fig. 4). This operation may take the form of sending the output signal from an ionization chamber through an analog-to-digital converter. The analog-to-digital conversion problem is not discussed here but would have to be considered before a physical system could be realized.

On the following timing pulse, $P1$, the present neutron level n_k is read into the sum register of the calculator and the complement of the previously sampled neutron level, $\bar{n}_{k-1}$, is read out of the $\bar{n}_{k-1}$ storage register and into the calculator addend register. $P2$ is an add pulse which causes the contents of the addend register to be added to the contents of the sum register, the result being stored in the sum register. In this case the result is $n_k - n_{k-1}$. This quantity is read into the dividend register in the divider on $P3$. If $n_k - n_{k-1}$ is positive, $P3$ also reads a 0 into the sign position of the quotient register since n_{k-1} is always positive; hence the sign of the quotient depends only on the sign of $n_k - n_{k-1}$. If $n_k - n_{k-1}$ is negative, its complement is stored in the sum register and must be inverted before being read into the dividend register. The I/N block in Fig. 4 means the digits are to be inverted (complemented) as they are read into the dividend register if the sign is negative. $P3$ also reads n_{k-1}, the complement of $\bar{n}_{k-1}$, into the divisor register.

$P4$ reads the complement of n_k into the $\bar{n}_{k-1}$ register where the present n_k becomes the n_{k-1} for the next computation. The n_k register receives the next n_k sample on $P64$. During pulses $P3$ through $P64$ the division which forms

$$\frac{n_k - n_{k-1}}{n_{k-1}}$$

is carried out, and on the following $P1$ the quotient is read into the $1/\tau$ flip-flop storage register.

Additional Computations Performed by Calculator

The additional reactor control variables which must be calculated are the average coolant temperature, the coolant temperature difference between the inlet and outlet to the reactor, the reactor power, and the proportional-plus-derivative control-rod actuating signal.

The calculation of the proportional-plus-derivative control signal proceeds as follows. On $P4$ the present neutron level is read out of the $\bar{n}_k$ register into the sum register. At the same time the reciprocal period is read from the $1/\tau$ storage register to the addend register. Information is stored in the $1/\tau$ register in the form of the sign digit followed by the magnitude of the number in ordinary binary code (not complemented if negative). Therefore if the reciprocal period is negative, the digits must be complemented before being read into the addend register. $P5$ adds the two quantities, after which $P6$ reads the sum $n + K_1/\tau = S_1$ from the sum register to the S_1 register. In the read-out operation the digits are complemented if the sign is negative.

In the computation of the average coolant temperature, T_h is read into the sum register and T_c is sent to the addend register on $P7$. $P8$ adds the two quantities and stores twice the average temperature in the sum register. $P9$ reads out the average temperature to the T_{avg} storage register, the factor of 2 being eliminated by ignoring the least significant bit in the read-out process.

The calculation of the coolant temperature difference involves reading in T_h to the sum register and the complement of T_c to the addend register on $P10$, adding these quantities on $P11$, and reading $(T_h - T_c)$ out of the sum register on $P12$. The computation of reactor power is slightly more involved. $P12$ also reads $(T_h - T_c)$ from the sum register to the multiplier register and puts the coolant flow rate F into the multiplicand register. The following odd-numbered pulses are "add attempts," i.e., the least significant digit in the multiplier register is tested; if this digit is a 1, an "add" pulse is applied to the calculator, while if a 0, no pulses occur. The even-numbered pulses shift the contents of the multiplier and product registers. The multiplication is completed on $P39$; hence $P40$ reads the reactor power out of the sum product register.

The calculator must perform two additional computations, namely the calculation of the tolerated upper and lower limits for the reference proportional-plus-derivative control signal, which varies according to the load on the turbogenerator. The sequence of computations is illustrated in Fig. 5 and proceeds as follows. On $P40$ the signal S_{1ref} is read into the δS_{1ref} register. For the present computations δ has been chosen as 1/16. The multiplication of S_{1ref} by δ is accomplished by dividing by $1/\delta$. Since $1/\delta = 16$, all that is required for this operation is to ignore the four least significant bits in the transfer of information from the S_{1ref} register to the δS_{1ref} register. The δS_{1ref} register is made four bits shorter than the S_{1ref} register and the transfer process in effect shifts S_{1ref} to the right by four bits. $P41$ reads S_{1ref} into the sum register and δS_{1ref} into the addend register. $P42$ performs the addition, and $P43$ reads S_{1ref} $(1 + \delta)$ out of the sum register. $P44$ reads the complement of δS_{1ref} to the addend register and S_{1ref} to the sum register. After the addition is performed on $P45$ the result S_{1ref} $(1-\delta)$ is read out of the sum register on $P46$. A complete pulse timing sequence for the comparator operation is given in Appendix II.

Timing Sequence for Comparator

The principles of comparator operation have already been discussed. The actual

Table III. Comparator Command Timing Sequence

Command	Pulses on Which Command Can Occur
INCREASE F	P41
DECREASE F(F)	P42
INCREASE F(F)	P43
REVERSE (variable > reference)	P2, 5, 8, 12, 14, 16, 18, 22, 26, 28, 29, 33
REVERSE (variable < reference)	P20, 24, 35, 39, 4
INCREASE F(T)	P31
DECREASE F(T)	P32
RODS IN	P10
RODS OUT	P11
DECREASE F	P44
SCRAM	P1, 4, 7
RODS EXCHANGE IN	P38
RODS EXCHANGE OUT	P37
CUTBACK (variable > reference)	P3, 6, 9, 13, 15, 17, 19, 23, 27, 30, 3
CUTBACK (variable < reference)	P2, 1, 25, 36, 40, 46

ing sequence for comparator operation ow given. Appendix III shows what rence levels are read into the $\overline{R}$ register what timing pulses. Table III lists timing pulses which read the various mand codes into the command de- er. By comparing Appendixes II and it may be seen that the quantities which cause scramming of the reactor, nely excess neutron level or reactor iod, are tested as soon as possible after sampling of n and the completion of computation of $1/\tau$. Also, note that control signal S_1 is computed on $P6$ l sent to the comparator on pulses $P7$ ough $P11$.

Fig. 6 shows a block diagram with the iable storage registers and calculating isters as well as the timing pulses as- iated with each register.

agnitudes of Control Quantities

Table IV gives the magnitudes of each erence level and shows the criteria used their determination. Much of the data these criteria applies to the Shipping- rt reactor.[4]

In determining the magnitude of the ictor reciprocal period, the following ocedure was used. Since the variables e sampled every millisecond, the recip- cal period is:

$$/\tau)=\frac{\Delta n}{n}\times\frac{1}{\Delta t}=\frac{\Delta n}{n}\times\frac{1}{10^3} \qquad (8)$$

$$=10^{-3}\times\frac{1}{\tau} \qquad (9)$$

or a reciprocal period of 0.2 second^{-1}, e fractional reference level must be 10^{-3} $0.2=0.0002$, since $\Delta n/n$ is what is tually calculated. In the calculation of e proportional-plus-derivative control gnal $S_1=n+K_1/\tau$, K_1 is chosen so that a -second period is equally weighted gainst a full-power level. Using this iterion, K_1 turns out to be $2^{41}=2^{30}\times2^{11}$. nce the reciprocal period is a fraction nd has its decimal point at the left of e 30-bit register) while the neutron level a whole number (having its decimal oint at its right), the proper scaling is ccomplished by merely shifting the $1/\tau$ umber 11 bits to the left as it is read into e calculator for addition to the neutron vel. The capacity of each variable torage register is shown by the numbers n parentheses in Fig. 6.

erivative-Plus-Integral Control

As was previously mentioned, instead f employing a proportional-plus-deriva-

Fig. 6. Interrelations of storage and calculating registers

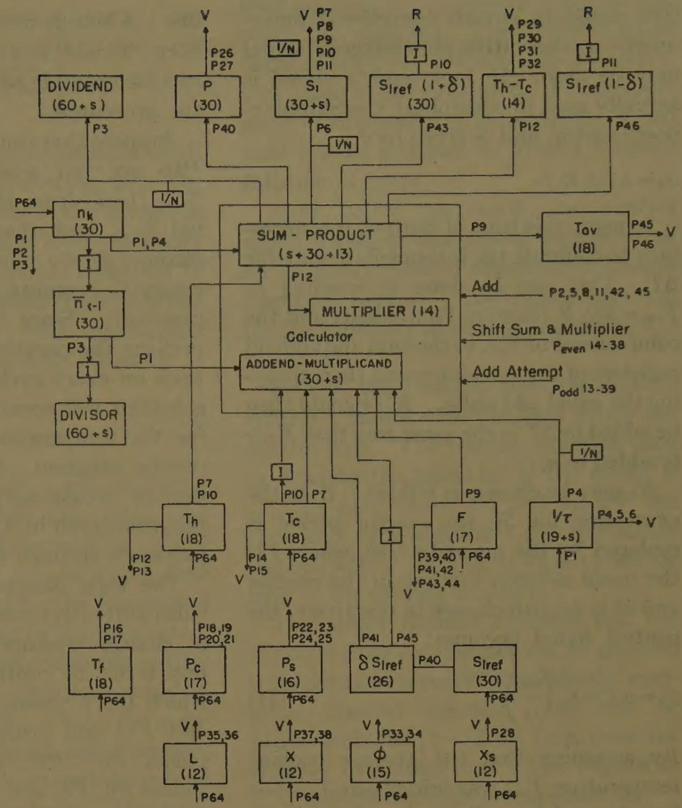


Table IV. Magnitude of Reference Levels

Reference Level	Criteria[1–4]	Magnitude of Level	Actual No. on Computer
n_m	150% full power	1.5×10^8	1.5×10^8
n_r	125% full power	1.25×10^8	1.25×10^8
n_c	112.5% full power	1.125×10^8	1.125×10^8
$(1/\tau)_m$	$\tau=5$ seconds	0.0002 second^{-1}	0.0002
$(1/\tau)_r$	$\tau=10$ seconds	0.0001 second^{-1}	0.0001
$(1/\tau)_c$	$\tau=15$ seconds	0.000067 second	0.000067
S_{1m}	150% full power	1.5×10^8	1.5×10^8
S_{1r}	125% full power	1.25×10^8	1.25×10^8
S_{1c}	112.5% full power	1.125×10^8	1.125×10^8
T_{hr}	T_h maximum = 636 F	625 F	62,500
T_{hc}		600 F	60,000
T_{fr}	T_f maximum = 636 F	625 F	62,500
T_{fc}		600 F	60,000
T_{cr}	(T_h-T_c) at full power = 35 F; $T_{hr}=625$ F; $T_{hc}=600$ F	590 F	59,000
T_{cc}		565 F	56,500
$(T_h-T_c)_r$	143% full power	50 F	5,000
$(T_h-T_c)_c$	126% full power	43 F	4,300
$(T_h-T_c)_1$	108.5% full power	38 F	3,800
$(T_h-T_c)_2$	50% full power	17.5 F	1,750
T_{avgr}	$k=0.01$	475 F	47,500
T_{avgc}	$k=0.005$	500 F	50,000
F_1	Flow Rate 115% Normal	51,750 gallons per minute	25,875
F_2	Flow Rate 105% Normal	47,250 gallons per minute	23,625
F_3	Flow Rate 95% Normal	42,700 gallons per minute	21,350
F_4	Flow Rate 90% Normal	40,500 gallons per minute	20,250
F_c	Flow Rate 80% Normal	36,000 gallons per minute	18,000
F_r	Flow Rate 70% Normal	31,500 gallons per minute	15,750
p_{crh}	p_c maximum = 2,500 psi*	2,400 psi	24,000
p_{cch}	p_c normal = 2,000 psi	2,200 psi	22,000
p_{ccl}	p_c varies 1,850–2,150 psi	1,800 psi	18,000
p_{crl}		1,600 psi	16,000
p_{srh}	p_s maximum = 975 psi	950 psi	9,500
p_{sch}	p_s full power = 600 psi	925 psi	9,250
p_{scl}	p_s no load = 885 psi	525 psi	5,250
p_{srl}		450 psi	4,500
L_c		80%	800
L_r		70%	700
ϕ_r		5 mr†/hour	5,000
ϕ_c		2 mr/hour	2,000
P_r	150% full power	90,000 kw	118,125,000
P_c	125% full power	75,000 kw	98,437,500
X_i	1/4 full rod travel	46 cm	920
X_o, X_r	3/4 full rod travel	138 cm	2,760

* Pounds per square inch.

† Milliroentgens.

tive signal to govern control-rod movement, a derivative-plus-integral signal may be used. In fact, such a signal is actually used in controlling the Shippingport reactor, and is given by:[4]

$$S_2 = \Delta T + K_2/\tau \tag{10}$$

In using this type of control, it is necessary to compute the intermediate variable ΔT. This can be done by reading in $T_{ref} = 525$ F (degrees Fahrenheit) and the complement of T_{avg} to the sum and addend registers of the calculator and then applying the usual add pulse. K_2/τ could then be added to ΔT in the same way that K_1/τ is added to n.

K_2 may be chosen as follows. If, in the expression for S_2, the reactor period is replaced by the quantity $l/\delta k$, where l is the mean neutron lifetime in the reactor and δk is a small change in reactivity, the control signal becomes:

$$S_2 = \Delta T + K_2\left(\frac{\delta k}{l}\right) \tag{11}$$

By assuming that the average coolant temperature T_{avg} was initially at T_{ref} but has changed by a slight amount $\Delta T'$, the change can be expressed as:

$$\Delta T' = T_{avg} - T_{ref} = -\Delta T \tag{12}$$

The change in reactivity is related to the temperature change by

$$\delta k = (-\alpha)\Delta T' \tag{13}$$

where $(-\alpha)$ is the temperature coefficient of reactivity of the reactor, which in the case of the Shippingport reactor is inherently negative. Equation 11 can now be rewritten as:

$$S_2 = \Delta T + K_2 \frac{\alpha}{l}\,\Delta T \tag{14}$$

For both terms in equation 14 to carry equal weight $K_2\,\alpha/l$ must equal 1. For a temperature coefficient of -2×10^{-4} per F and a mean neutron lifetime of 10^{-3} seconds, K_2 must equal 5.

Breaking into Calculator Timing Sequence

In the design of a computer similar to the one just described, it may be desired to use the calculator to compute a very slowly varying quantity, and there will not be enough timing pulses available to program the calculation into the regular timing sequence. One way to perform the calculation and still meet the timing-pulse restriction is occasionally to break into the calculator timing sequence at a point where other slowly varying quantities are being computed and to perform the new calculation instead of the regular one. A simple method for doing this has been devised and will be discussed. A specific example will be used to illustrate the procedure.

Suppose that timing pulses $P40$ through $P46$ are not available for calculating $S_{1ref}\,(1\pm\delta)$ from the power demand signal S_{1ref}, and that the power demand changes slowly enough so that it is necessary to compute $S_{1ref}\,(1\pm\delta)$ only occasionally. Since it is not necessary to perform the regular temperature calculations on every cycle, the $S_{1ref}\,(1\pm\delta)$ calculations will occasionally be substituted for the temperature calculations in the regular program. More specifically, it is desired occasionally to substitute the program given in Appendix IV for that of pulses $P6$ through $P12$ of Appendix II.

The logic designed to carry out this substitution functions as follows. When a "change sequence" pulse occurs, a flip-flop is set to control certain AND gates which block timing pulses $P7$, $P9$, $P10$, and $P12$ and other gates which permit pulses $P6'$, $P9'$, and $P12'$ to occur. Pulses $P6$, $P8$, and $P11$ are still allowed, and the timing sequence given in Appendix IV results. $P12'$ returns the flip-flop to its original state.

Decreasing Permissible Reactor Periods During Low-Power Operation

The proportional-plus-derivative control signal S_1 is given by equation 10 as equal to $n + K_1/\tau$. K_1 is chosen so that a 22-second period is equally weighted against a full (100%) power neutron level. The control signal is obtained, however, according to the load on the turbogenerator. Therefore, for a power demand of less than 100% the signal S_1 will be decreased from its full-power value. When this happens, the tolerated limit on the reciprocal period will be automatically reduced so that the shortest allowable period will be greater than 22 seconds. For example, for a power demand of 50%, the period limit will be 44 seconds. This means that for small power demands the permissible rate of change in reactivity is decreased; hence it will take longer for the reactor to reach desired power levels.

A method has been devised for speeding up changes in power level during low-power demand. Essentially, it consists of merely modifying the calculated reactor period. Table V shows the reciprocal period demands which correspond to several given power demands in the sense of having the same controlling effect on the regulator rods, and by what fraction

Table V. Power-Demand Reciprocal-Peri… Correspondence

Power Demand,%	Corresponding $1/\tau$ Demand	Multiply T… $1/\tau$ by
100	0.04550	1.0
50	0.02275	0.5
10	0.00455	0.1

the true reciprocal periods must be mul… plied in order to tolerate a 22-secon… period for each power demand.

As a specific example, assume the pow… demand is 50% and the reactor is cu… rently operating on a 22-second perio… The calculated $1/\tau$ will be 0.0455, but… the computer described in this article… used, the control signal S_1 will tolerate… reciprocal period no greater than 0.0227… Hence, the control rods will be moved… until the reactor is forced to operate… nothing less than a 44-second perio… Now, suppose that the calculated $1/\tau$… multiplied by one half, the ratio of actu… to 100% power demand, to obtain a ne… reciprocal period $(1/\tau)' = (0.0455\times{}^1\!/_2)$… 0.02275. When this quantity is com… pared with the control signal the limit… just reached, and the reactor is st… allowed to operate on a 22-second perio…

Using this method, the calculation… would proceed as follows. The computa… tion of $1/\tau$ would be carried out in exact… the same manner as before, the resu… being sent to the meters, recorders, an… the V register of the comparator the sam… as before. However, an addition… quantity $(1/\tau)' = (1/\tau)$ times the powe… demand divided by the full-power lev… would be computed, and this quantit… would be employed in computing the con… trol signal S_1. In other words, equatio… 10 would be replaced by the relation

$$S_1 = n + K_1\left(\frac{1}{\tau}\right)' \tag{1…}$$

This ensures that the allowable reciproca… period is not reduced when the powe… demand is below that of full power.

Stability

An essential requirement for any con… trol system is that it be stable for a… conditions for which the system is to b… operated. In a nuclear reactor contro… system the stability requirement is eve… more important since the damage whic… could result from an unstable atomic pil… is considerable.

A complete stability analysis canno… possibly be presented here; however, … brief and simplified analysis which indi… cates that the designed digital reacto…

rol system is stable and which throws
light on the general problem of ility of sampled-data reactor control ems follows. In this analysis, a gen- block diagram for a nuclear reactor its associated sampled-data control em is set up and reduced. The trans- unctions which pertain to the specific em under consideration are formu- d, and several simplifications are made. stability is studied by means of root- s techniques, two root loci plots being uded to illustrate the effect of varying sampling frequency on the allowable r-all system gain.

generalized block diagram for a pled-data nuclear reactor control em is given in Fig. 7(A). $F(s)$ is the nsfer function for the nuclear reactor relates changes in reactivity δk to nges in neutron level δn. The output is sampled by an impulse modulator and the samples (impulses) are sent ough a digital computer $G(z)$. The ation $G(z)$ is used to indicate a purely crete (sampled-data) system, whose place transform consists of functions the form $\epsilon^{sT}=z$, where T is the time tween samples. The output of the nputer is sent through some continuous ments, represented by $H(s)$, in the dback loop before being used to furnish e reactor with reactivity of opposite larity to the original disturbance.

The block diagram of Fig. 7(A) can be duced to the successive forms shown by gs. 7(B)–(E). The starred quantity notes that only the samples $(HF)^*$ the purely continuous transfer function F are of interest. The transmission T om δk to δn may be written by inspec- on of Fig. 7(E) as:

$$=F-\frac{F}{H}(MG)\left[\frac{1}{1+(HF)^*G}\right](HF) \qquad (16)$$

If the input δk consists of one sample, a nit impulse, the transmission can be ought of as representing the unit im- lse response of the system. If one is terested only in the samples of the im- lse response, the pure z-domain trans- ission may be given as:

$$^*=F^*-\left(\frac{F}{H}\right)^*\left[\frac{G(HF)^*}{1+(HF)^*G}\right] \qquad (17)$$

ence the system is represented by an tirely discrete transfer function and root cus or similar frequency domain tech- ques may be employed. The next step is determine the specific functions F, G, , F^*, $\left(\frac{F}{H}\right)^*$, and $(HF)^*$.

The transfer function for a nuclear actor can be shown to be of the follow- g form:[1]

$$\frac{\delta n(s)}{\delta k(s)}=\frac{n_o}{l}\frac{1}{s\left[1+\sum_{i=1}^{6}\frac{\beta_i}{l(s+\lambda_i)}\right]} \qquad (18)$$

where n_o is the steady-state neutron level, l is the mean neutron lifetime, β_i is the fraction of total neutrons in the ith group of delayed neutrons, and λ_i is the decay constant for the ith group. Equation 18 can be simplified by considering one group of delayed neutrons with appropriate constant β, λ. Equation 18 becomes:

$$\frac{\delta n(s)}{\delta k(s)}=\frac{n_o(s+\lambda)}{ls(s+d)} \qquad (19)$$

In the following analysis λ will be chosen as 0.075 second^{-1}, while the value of d will be taken to be 50 seconds^{-1}. Temperature effects may also be taken into consideration, although for the purposes of this analysis these effects will be neglected.

The feedback control consisting of $G(z)$ and $H(s)$ in Fig. 7(A) actually is made up of several components. First, the computer sends out a sampled signal proportional to the neutron level and the rate of change of the neutron level. The computer transfer function will be denoted by $D(z)$.

In order to facilitate the analysis, the derivative component of $D(z)$ will be ignored and the computer will be assumed to furnish merely a delay of some sample time. In other words,

$$D(z)=\frac{1}{z} \qquad (20)$$

Next, the signals are considered to be quantized, i.e., for any neutron level greater than $n_o(1+\delta)$ the computer sends out a $+1$ signal meaning move the rods in, while for any neutron level less than $n_o(1-\delta)$ a -1 signal is generated to move the rods out. For neutron levels between $n_o(1+\delta)$ and $n_o(1-\delta)$ no corrective signal at all is initiated. For the purposes of this analysis, however, the quantizer will be assumed to have a transfer function of unity.

After being quantized, the control signal is held, or clamped, at a given value for an entire cycle of computer operation (sampling period). The transfer function for the clamping operation is given by $1/s\,[1-(1/z)]$.

Finally, the clamped signal is used to control a motor, which in turn drives the control rods. The transfer function used to describe the behavior of the motor is $(K_m/1+\tau_m s)$ where K_m and τ_m are the gain and time constants for the motor respectively. A value of $\tau_m=0.1$ will be used in the ensuing analysis.

The motor transfer function and the $1/s$ term from the clamper will be grouped together and termed $H(s)$, while the computer transfer function $D(z)$ will be paired with the $[1-(1/z)]$ term from the clamper to obtain $G(z)$. The quantities F, G, and H can thus be expressed as:

$$F(s)=\frac{n_o(s+\lambda)}{ls(s+d)}=\frac{K_1(s+0.075)}{s(s+50)} \qquad (21)$$

$$G(z)=\frac{1}{z}\left(1-\frac{1}{z}\right)=\frac{z-1}{z^2} \qquad (22)$$

$$H(s)=\frac{K_m}{s(\tau_m s+1)}=\frac{K_m/\tau_m}{s(s+10)} \qquad (23)$$

Inspection of equation 17 reveals that the stability problem can be analyzed by focusing attention on the poles of F^* and $\left(\frac{F}{H}\right)^*$ and the zeros of $1+(HF)^*G$. $F(s)$ can be expanded in a partial fraction expansion of the form:

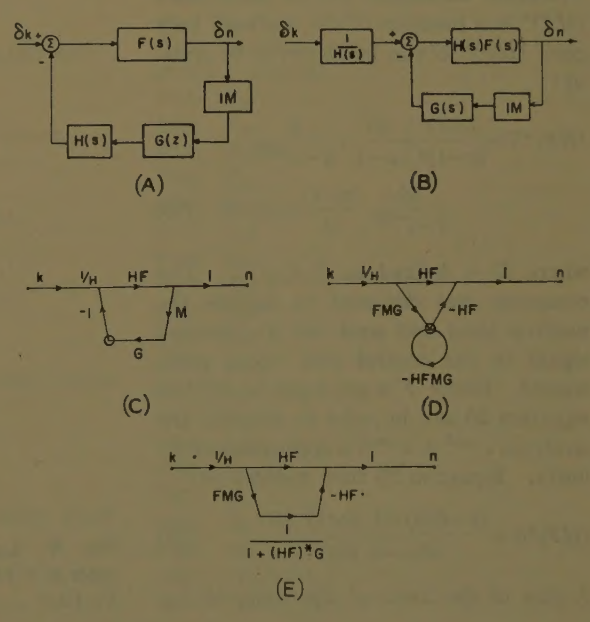


Fig. 7. Reduction of generalized reactor-control block diagram

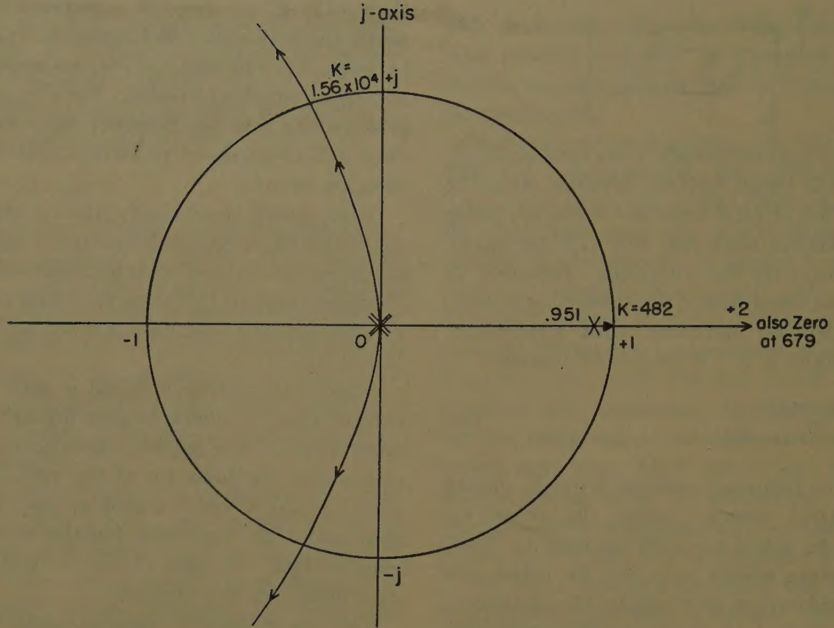


Fig. 8. Locus of roots of 1+(HF)*G for T=10⁻³ seconds

$$F(s)=\frac{a}{s}+\frac{b}{s+50} \tag{24}$$

The z-transform for $F(s)$ can then be written down by inspection as:

$$F^*(z)=\frac{a}{z-1}+\frac{b}{z-\epsilon^{-50T}} \tag{25}$$

where T is the sampling period.

$F^*(z)$ is seen to have poles at $z = 1$ and at $z = \epsilon^{-50T}$. For the transform $z = \epsilon^{sT}$ the unit circle corresponds to the j-axis of the s-plane, poles outside of the unit circle causing instability and poles inside of the unit circle giving rise to stable systems. Since the poles of $F^*(z)$ do not lie outside of the unit circle, $F^*(z)$ is seen to be stable. A similar argument holds for $\left(\frac{F}{H}\right)^*$.

In examining the zeros of $1+(HF)^*G$ root-locus techniques will be used, since $(HF)^*$ is a function of the feedback loop gain. $(HF)^*G$ can be shown to be given by:

$$(HF)^*G=\frac{1.5Tz}{(z-1)^2}+\frac{20}{z-1}+\frac{5}{z-\epsilon^{-50T}}-\frac{25}{z-\epsilon^{-10T}}\frac{(z-1)}{z^2}\times 10^{-4}K \tag{26}$$

where K is defined as K_1K_m/τ_m. The computer was designed to sample the neutron level and send out a command signal to the control rods every millisecond. Hence T is set equal to 10^{-3} in equation 26 and in order to simplify the analysis, $\epsilon^{-10T} = \epsilon^{-0.01}$ is approximated by unity. Equation 26 then reduces to:

$$(HF)^*G=\frac{(z-679)\times 1.5\times 17^{-7}K}{z^2(z-0.951)} \tag{27}$$

A plot of the locus of the roots of $1+(HF)^*G$ as given by equation 27 is shown in Fig. 8. The system is seen to be stable for all values of positive K less than 482.

In order to determine the effect of varying the sampling frequency, a sampling period of 10^{-5} seconds was hypothesized. A frequency greater than 1 kc was chosen, rather than one smaller, since for high sampling frequencies approximations can be made which greatly simplify the analysis, whereas for sampling frequencies less than 1 kc, the analysis becomes far more complicated. For a sampling period of 10^{-5} seconds both ϵ^{-10T} and ϵ^{-50T} can be approximated by unity, and equation 26 reduces to:

$$(HF)^*G=\frac{1.5\times 10^{-9}K}{z(z-1)} \tag{28}$$

Fig. 9 gives a plot of the locus of the roots of $1+(HF)^*G$ as given by equatic 28. The system is now seen to be stab for all positive values of K less than 6 $\times 10^8$, and for K less than 1.67×10^8, th loci remain entirely on the real axi Note that there is a substantial increase the allowable K when the sampling fr quency is increased from 1 kc to 100 kc.

For any sampling frequency great than 100 kc the approximations tha ϵ^{-10T} and ϵ^{-50T} are both unity still hol and in fact, become even more accurat The shape of the root loci is dentical t that shown in Fig. 9. However, th values of K which correspond to poin where the loci cross the unit circle var with the sampling frequency and a found to be:

$$K=\frac{6.67\times 10^8}{T} \tag{29}$$

Equation 29 shows that for high enoug sampling frequencies (small enoug sampling periods) the allowable feedback loop gain K increases and is directly pro portional to sampling frequency (in versely proportional to sampling period In the limit as the time between sample goes to zero, the allowable K is seen t approach infinity. This result is not sur prising, since the equivalent continuou reactor control system is stable for a positive K.

Conclusions

According to the preceding analysis, reactor control system consisting of sampler, delay, clamper, and a motor control rod-drive mechanism certainl appears to be stable. Moreover, for sampling frequency of 1 kc the allowabl feedback-loop gain seems tolerable. I the sampling frequency were increased t

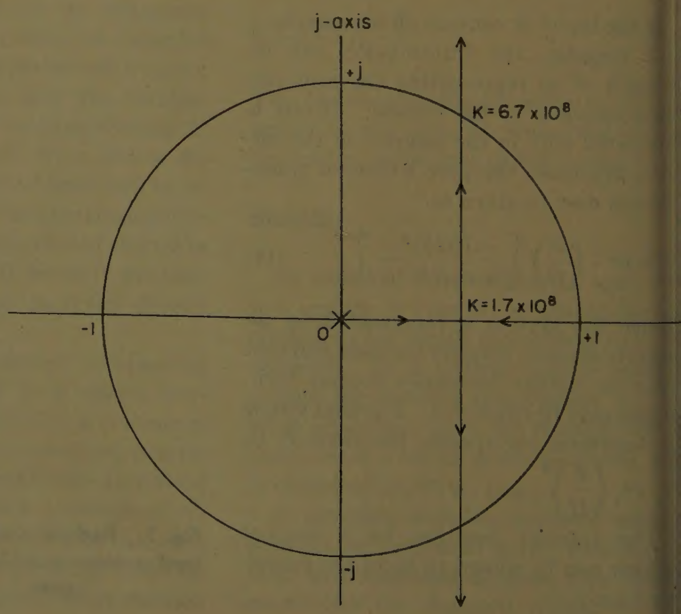


Fig. 9. Locus of roots of 1+(HF)*G T=10⁻⁵ seconds

kc, or greater, the allowable gain is eased substantially and, in fact, be-es directly proportional to the sam-g frequency. In the limit of infinite uency (a continuous system) the wable gain approaches infinity. It s appear, however, that excessively n sampling frequencies are not neces-y since sampling at the selected fre-ncy of 1 kc seems more than sufficient esult in a stable system.

ccuracy studies were not made for the cribed computations. However, the ure of digital computation does lend lf to high accuracies. In general, it y be expected that any reactor control ntities calculated by digital methods ld be computed to the same degree of uracy as the original data.

he designed reactor control computer ears considerably more flexible than sently used analog reactor control tems. The reference levels and scale tors can be changed with considerable e, simply by reading new numbers into reference memory. Also, it would possible to change the quantities to be culated, if the nature of the substituted mputations is such that a presently isting computer timing sequence could used. The described substitution eme adds to the flexibility afforded ce it permits additional quantities be calculated at variable time lags tween individual computations. One rious limitation in the flexibility of the signed computer does exist, however. is is due to the fact that the computer special-purpose in nature, and hence ssesses nowhere near the wide degree flexibility afforded by a general-pur-se digital computer, into which almost y sequence of operations can be pro-amed and new programs readily sub-ituted.

The designed computer should be as liable as any other digital computer of ual complexity. Since huge general-rpose computers much more involved an the designed computer have been ilt and operated successfully, a high liability would seem possible for the signed reactor control system. In the tailed design of the system high quality mponents (tubes, transistors, diodes, c.) would be used in order to minimize alfunction.

The design has not proceeded to the int where a meaningful cost estimate n be made. It may not be unreason-le to assume however, that the cost of digital system like the one described uld be higher than presently used alog systems. It would then have to determined whether any increase in cost is justified by increases in flexibility and accuracy.

Appendix I. Variable-Command Relations

The conditions for SCRAM are:

$n>n_m$
$(1/\tau)>(1/\tau)_m$
$S_1>S_{1m}$

The conditions for REVERSE are:

$n>n_r$
$(1/\tau)>(1/\tau)_r$
$S>S_{1r}$
$T_h>T_{hr}$
$T_c>T_{cr}$
$T_f>T_{fr}$
$(T_h-T_c)>(T_h-T_c)_r$
$p_c>p_{crh}$
$p_s>p_{srh}$
$\phi>\phi_r$
$p>p_r$
$X_s>X_r$
$L<L_r$
$F<F_r$
$p_c<p_{crl}$
$p_s<p_{srl}$
$T_{avg}<T_{avgr}$

The conditions for CUTBACK are:

$n>n_c$
$(1/\tau)>(1/\tau)_c$
$S_1>S_{1c}$
$T_h>T_{hc}$
$T_f>T_{fc}$
$T_c>T_{cc}$
$(T_h-T_c)>(T_h-T_c)_c$
$p_c>p_{cch}$
$p_s>p_{sch}$
$\phi>\phi_c$
$p>p_c$
$L<L_c$
$F<F_c$
$p_c<p_{ccl}$
$p_s<p_{scl}$
$T_{avg}<T_{avgc}$

The conditions for EXCHANGE OUT are:

$X>X_o$

The conditions for EXCHANGE IN are:

$X<X_i$

The conditions for RODS IN are:

$S_1>S_{1ref}(1+\delta)$

The conditions for RODS OUT are:

$S_1<S_{1ref}(1-\delta)$

The conditions for DECREASING FLOW RATE are:

$F>F_1$
$F>F_3$ and $(T_h-T_c)<(T_h-T_c)_2$

The conditions for INCREASING FLOW RATE are:

$F<F_4$
$F<F_2$ and $(T_h-T_c)>(T_h-T_c)_1$

Appendix II. Timing Sequence for Calculator Operation

$P64$: sample all measurable quantities.
$P1$: read n_k to sum, read $\bar{n}_{k-1}$ to addend.
$P2$: add.
$P3$: read $\bar{n}_k$ to divisor (complement), read sum to dividend.
$P4$: read n_k to sum, read n_k to $\bar{n}_{k-1}$ (complement), read $1/\tau$ to addend (complement digits if negative).
$P5$: add.
$P6$: read S_1 out of sum (complement digits if negative).
$P7$: read T_h to sum, read T_c to addend.
$P8$: add.
$P9$: read T_{avg} out of sum.
$P10$: read T_h to sum, read T_c to addend (complement).
$P11$: add.
$P12$: read (T_h-T_c) out of sum, read F to multiplicand, read (T_h-T_c) to multiplier.
Odd numbered pulses $P13$ through $P39$ are add attempts; even pulses are shift multiplier and product.
$P40$: read P out of sum, read S_{1ref} to δS_{1ref}.
$P41$: read S_{1ref} to sum, read δS_{1ref} to addend.
$P42$: add.
$P43$: read $S_{1ref}(1+\delta)$ out of sum.
$P44$: read S_{1ref} to sum, read δS_{1ref} to addend (complement).
$P45$: add.
$P46$: read $S_{1ref}(1-\delta)$ out of sum.

Appendix III. Comparator Reference-Level Timing Sequence

$P1$: n_m
$P2$: n_r
$P3$: n_c
$P4$: $(1/\tau)_m$
$P5$: $(1/\tau)_r$
$P6$: $(1/\tau)_c$
$P7$: S_{1m}
$P8$: S_{1r}
$P9$: S_{1c}
$P10$: $S_{1ref}(1+\delta)$
$P11$: $S_{1ref}(1-\delta)$
$P12$: T_{hr}
$P13$: T_{hc}
$P14$: T_{cr}
$P15$: T_{cc}
$P16$: T_{fr}
$P17$: T_{fk}
$P18$: p_{crh}
$P19$: p_{cch}
$P20$: p_{crl}
$P21$: p_{ccl}
$P22$: p_{srh}
$P23$: p_{sch}
$P24$: p_{srl}
$P25$: p_{scl}
$P26$: p_r
$P27$: p_c
$P28$: X_r
$P29$: $(T_h-T_c)_r$
$P30$: $(T_h-T_c)_c$
$P31$: $(T_h-T_c)_1$

$P32$: $(T_J-T_c)_2$
$P33$: ϕ_r
$P34$: ϕ_c
$P35$: L_r
$P36$: L_c
$P37$: X_0
$P38$: X_i
$P39$: F_r
$P40$: F_c
$P41$: F_4
$P42$: F_3
$P43$: F_2
$P44$: F_1
$P45$: T_{avgr}
$P46$: T_{avgc}

Appendix IV. Substituted Calculator Timing Sequence

$P6'$: read S_{1ref} to δS_{1ref}.
$P7'$: read S_{1ref} to sum, read δS_{1ref} to addend.
$P8'$: add.
$P9'$: read $S_{1ref}(1+\delta)$ out of sum.
$P10'$: read S_{1ref} to sum, read δS_{1ref} to addend (complement).
$P11'$: add.
$P12'$: read $S_{1ref}(1-\delta)$ out of sum, read F to multiplicand, read (T_h-T_c) to multiplier.

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Nonlinear Servomechanism of Limited Dynamic Range

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THE METHODS of analysis and synthesis of linear feedback control systems have now reached such a state of perfection that the design of a system to meet any set of performance specifications should be a fairly simple matter. Unfortunately no practical system can ever be completely linear, and therefore the theory developed for linear systems applies only approximately to practical systems.

A particular type of nonlinearity frequency encountered is dead band. A device having dead band will not respond to signals that are smaller than a certain threshold level. In mechanical systems, dead band is often caused by coulomb or dry friction. Another commonly encountered type of nonlinearity is saturation. Since no physically realizable component can deliver infinite output, all practical systems saturate. A device having both dead band and saturation may be thought of as having a limited dynamic range; that is, it responds in an approximately linear fashion only to signals large enough to exceed the dead-band threshold, but not so large as to cause saturation. The dynamic range may, in fact, be defined as the ratio of saturation level to dead-band level. For maximum performance of a feedback control system, the servomotor or other power element should have as large a dynamic range as possible. On the other hand, the cost of a motor generally increases with dynamic range, particularly if an effort is made to increase the dynamic range to its ultimate limit. Thus, to keep the cost of a system within reason a motor with limited dynamic range must be employed. Limited dynamic range of one or more elements of a feedback control system generally affects the relative stability of the system, and it may, in fact, cause actual instability. The static accuracy of such a system also has a definite upper limit. In addition, systems of this type generally exhibit a cut-off frequency. That is, for all frequencies above a given critical frequency the system fails to respond irrespective of the magnitude of the input signal.

A common unit having limited dynamic range is the combination of a saturating power amplifier and a d-c motor having stiction or static friction and coulomb friction. Such a system is investigated in this paper. The system block diagram is shown in Fig. 1.

Since the paper deals with a system having more than one nonlinear element the describing function method of analysis offers the most flexible approach for the solution of the problem. The procedure will be to combine the describing functions of the individual nonlinearities into one composite describing function. The composite describing function can be arranged such that the remaining linear function is the transfer ratio of the actual linear system. In general, the composite describing function will be both amplitude- and frequency-sensitive.

The saturating nonlinearity of the power amplifier is shown in Fig. 2(B) and has the describing function H_1 indicated by equation 1.

$$H_1=\frac{2}{\pi}\left[\sin^{-1}\frac{E_s}{E}+\frac{E_s}{E}\sqrt{1-\left(\frac{E_s}{E}\right)^2}\right] \quad (1)$$

for $E>E_s$.

Fig. 2(A) also gives a plot of H_1 and γH_1 versus γ for the saturating nonlinearity.

The stiction and coulomb friction nonlinearity is shown in Fig. 3. Tou[1,2] has evaluated a describing function relating effective torque to applied torque for stiction and coulomb friction and his results will be used for this nonlinearity.

The describing equations for stiction and coulomb friction are:

$$\dot{\theta}=0+ \qquad T_a=T_{s'} \quad (2)$$

$$\dot{\theta}>0 \qquad T_a=T_c+J\ddot{\theta} \quad (3)$$

$$\dot{\theta}=0- \qquad T_a=-T_{s'} \quad (4)$$

$$\dot{\theta}<0 \qquad T_a=-T_c+J\ddot{\theta} \quad (5)$$

T_c is a positive number and $T_{s'}$ is a variable which always exactly matches T_a until T_a exceeds some definite limit T_s. The effective torque can be defined as

$$T_e=T_a-T_f$$

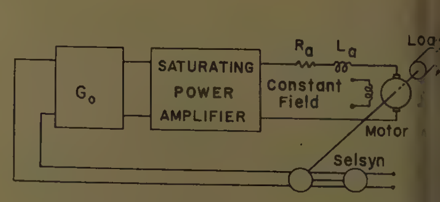

Fig. 1. System block diagram

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This work contains part of the results of a thesis submitted by R. L. Moruzzi in partial fulfillment of the requirements for the degree of the Doctor of Engineering at Yale University.

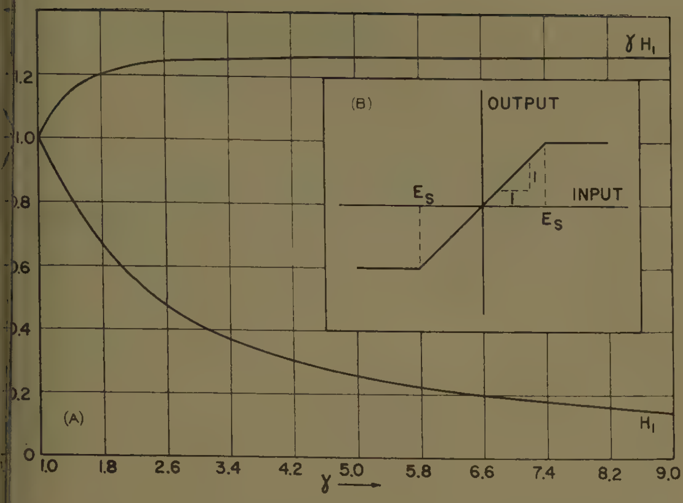


Fig. 2. (A) Describing function for saturation plot of H_1 and γH_1, and (B) saturation nonlinearity

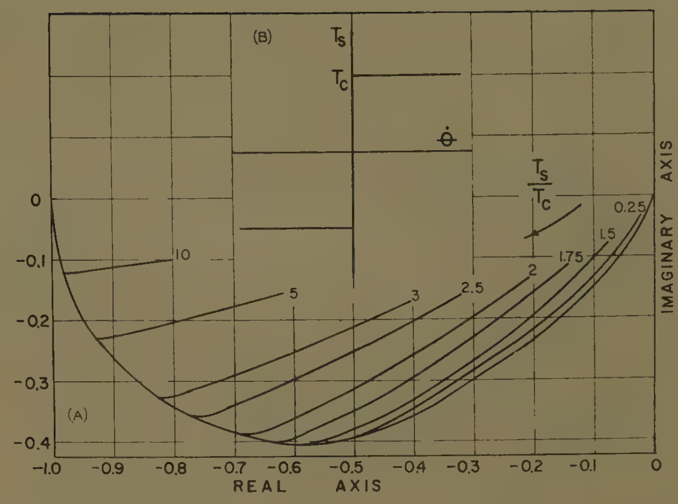


Fig. 3. (A) Describing function for friction, and (B) friction nonlinearity

The stiction and coulomb friction describing function is defined as follows:

$$\frac{T_e}{T_a} = H = F(\lambda)\,\underline{/\delta(\lambda)};\quad \lambda = T_c/T_a$$

where $F(\lambda)$ = the magnitude portion of the describing function and $\delta(\lambda)$ = the phase angle portion of the describing function. Fig. 3 gives a plot of $-H$ for T_s/T_c between 1 and 10.

A set of differential equations can be written which describes the operation to the selected system. The following equations are given in their Laplace transform form with $s = j\omega$ and all initial conditions set to zero. In addition, the describing functions H_1 and H are used for the respective saturation and friction nonlinearities.

$$E_0(j\omega)/E_s(j\omega) = KH_1 \tag{6}$$

$$E_0(j\omega) - V_m(j\omega) = R_a I_a(j\omega) + L_a \omega j I_a(j\omega) \tag{7}$$

$$K_t I_a(j\omega) = T_a'(j\omega) \tag{8}$$

$$T_a'(j\omega) - T_b(j\omega) = T_a(j\omega) \tag{9}$$

$$T_a(j\omega) \quad H = T_e(j\omega) \tag{10}$$

$$T_e(j\omega) = Jj\omega\dot{\theta}(j\omega) \tag{11}$$

$$V_m(j\omega) = K_v\dot{\theta}(j\omega) \tag{12}$$

$$T_b = B\dot{\theta}(j\omega) \tag{13}$$

Equations 6 through 13 may be represented by the block diagram shown in Fig. 4.

Evaluation of a system containing a motor unit represented by Fig. 4 is usually accomplished by breaking the loop at the $T_a(j\omega)$ point.[1] In this manner, the stiction and coulomb friction describing function H can be isolated. Breaking the loop at $T_a(j\omega)$, however, makes it awkward to investigate various forms of series equalization, feedback equalization, or a combination of the two. Also, the presence of a second nonlinearity further complicates the procedure. To overcome this difficulty, the block diagram of Fig. 4 may be reduced to the form of Fig. 5. Rearranging the block diagram and simplifying the results is permissible since the analysis will ultimately be conducted for a point-by-point evaluation of fixed amplitude and frequency, thus rendering the system linear at the given point of investigation.

In Fig. 5, the block $K\ H_1$ represents the saturating amplifier, $H_0'(j\omega)$ contains the friction nonlinearity, and $G(j\omega)$ in the linear transfer function of the d-c motor. $H_0'(j\omega)$ and $G(j\omega)$ can be derived from equations 6 through 13 and may be expressed in the form:

$$H_0'(j\omega) = H\left(\frac{1+\alpha T_J j\omega}{H+\alpha T_J j\omega}\right)\times\left[\frac{1+(1/\alpha)(T_r j\omega+1)(\alpha T_J j\omega+1)}{H\dfrac{1+\alpha T_J j\omega}{H+\alpha T_J j\omega}+(1/\alpha)(T_r j\omega+1)(\alpha T_J j\omega+1)}\right] \tag{14}$$

$$G(s) = \frac{\alpha K_v}{(\alpha T_J j\omega+1)(T_r j\omega+1)+K_v K_t} \tag{15}$$

where

$$T_J = \frac{JR_a}{K_v K_t}$$

$$T_r = L_a/R_a \tag{16}$$

$$\alpha = K_v K_t/BR_a$$

The composite describing function then becomes

$$H_0 = H_1 H_0'(j\omega) \tag{17}$$

Although by equation 14 $H_0'(j\omega)$ is a complicated function of both amplitude and frequency, it can be simplified in many practical cases. For instance, if α is very large and T_r is small compared to T_J, $H_0'(j\omega)$ can be simplified to

$$H_0'(j\omega) \approx H\frac{1+T_J j\omega}{H+T_J j\omega} \tag{18}$$

It can be shown that for $\alpha \geq 10$ and $T_r \leq 0.1\ T_J$, this approximate expression for $H_0'(j\omega)$ results in describing functions that are practically identical with those obtained from the exact expression. It should also be noted that coulomb friction is considered separately in this analysis, and that B, therefore represents only the linear component of friction. In most

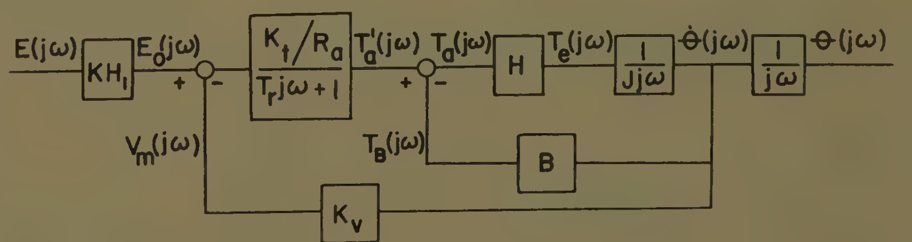


Fig. 4. Block diagram of saturating power amplifier and d-c motor having stiction and coulomb friction

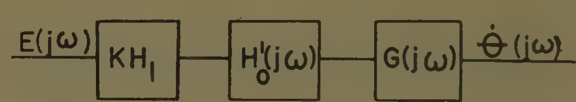


Fig. 5 (right). Modified block diagram

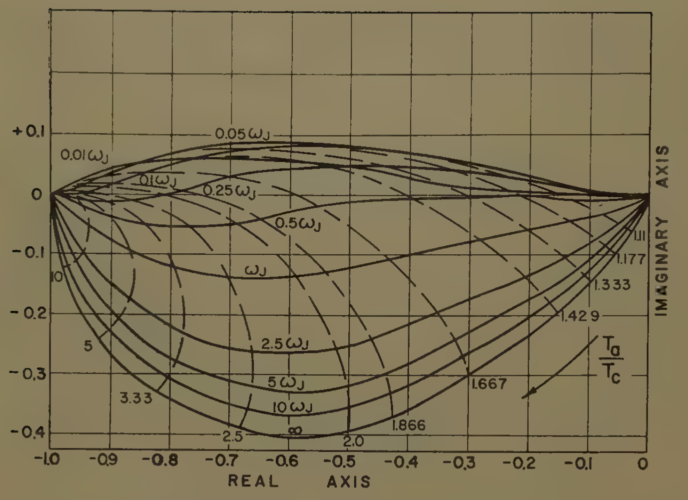


Fig. 6. Plot of H_0' for $\alpha \to \infty$ and $T_s/T_c=1$

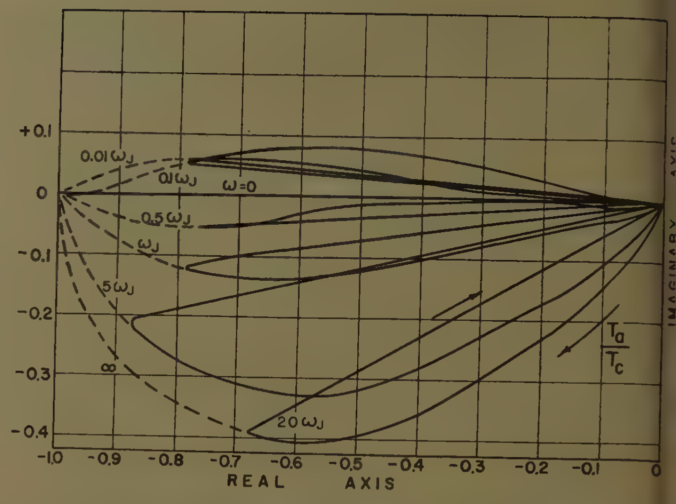


Fig. 7. Composite saturation and friction describing function H_0 for $\psi=5$ and $T_s/T_c=1$

motors, one would expect B to be very small, and, therefore, α to be very large. Thus equation 18 may be expected to hold in most practical situations.

A plot of equation 18 for the case of sliding friction only ($T_s/T_c=1$) is shown in Fig. 6. For $T_s/T_c>1$, part of the $-H_0'(j\omega)$ locus in the vicinity of the origin disappears. This can be deduced qualitatively by inspection of Fig. 3(A) and equation 18. Otherwise the loci for $T_s/T_c>1$ are similar in form to those shown in Fig. 6.

Effect of Saturation on H_0'

Having evaluated $H_0'(j\omega)$ one may now combine H_1 and $H_0'(j\omega)$ to form $H_0(j\omega)=H_1H_0'(j\omega)$, where H_1 is an amplitude reduction term having no phase shift. For a given operating point, the phase angle of $H_0(j\omega)$ is the same as that of $H_0'(j\omega)$. The output of the saturating power amplifier may be expressed by:

$$E_0/E=H_1$$

$$E_0=EH_1=E_s[E/E_s]H_1=E_s\gamma H_1 \quad (19)$$

A plot of H_1 and γH_1 versus γ is given in Fig. 2(A).

To establish the operating points of γ or T_a/T_c on $H_0(j\omega)$, make use of the following development.

Referring to Fig. 4,

$$T_a'(j\omega)=T_a(j\omega)[HB/Jj\omega]+T_a(j\omega)=T_a(j\omega)[HB/Jj\omega+1] \quad (20)$$

$$E_0(j\omega)=\frac{\dfrac{T_a'(j\omega)}{K_t/R_a}}{T_rj\omega+1}+\frac{T_a(j\omega)HK_v}{Jj\omega} \quad (21)$$

Substituting equation 20 into 21 and letting

$$T=\alpha T_J;\ T_J=JR_a/K_tK_v \quad (22)$$

$$E_0(j\omega)=T_a(j\omega)H(R_a/K_t)[(1/H+1/\alpha\times T_Jj\omega)(T_rj\omega+1)+1/T_Jj\omega] \quad (23)$$

Let $\psi=E_sK_t/R_aT_c$ = dynamic range; and reducing equation 22

$$\frac{E_0(j\omega)}{E_s}=\frac{T_a(j\omega)}{T_c}\frac{H}{\psi}\left[\frac{T_r}{\alpha T_J}+\frac{1+\alpha}{\alpha T_Jj\omega}+\frac{1}{H}(T_rj\omega+1)\right]=\gamma H_1 \quad (23)$$

The maximum value of the output of the saturating amplifier is $(4/\pi)$ E_s (i.e. the fundamental component of a rectangular wave). Thus, H_1 ranges from 1 to $4/\pi$ for operation in the saturating region. In this region of operation, a given T_a/T_c establishes a value of H, and for different frequencies, various values of $H_0'(j\omega)$ are established from equation 18, or plot of equation 18. Thus, for consistent values of T_a/T_c, H, and ω, it is possible to determine $H_0'(j\omega)$ and evaulate equation 23. Knowing γH_1 from equation 23, one may then establish the separate values of γ and H_1 from Fig. 2, thereby allowing the calculation of $H_0(j\omega)=H_1H_0'(j\omega)$. Note that $H_0(j\omega)$ may be specified in terms of γ or its equivalent T_a/T_c. Fig. 7 illustrates the effect of a dynamic range of five ($\psi=5$) on the $-H_0(j\omega)$ locus for coulomb friction.

The effect of decreasing the dynamic range is to eliminate part of the $-H_0(j\omega)$ locus in the vicinity of the -1 point.

Cut-off Frequency

The conditions for cut-off are $T_a/T_c=T_s/T_c$; $H=0$ and $E_0(j\omega)/E_s=4/\pi$. The

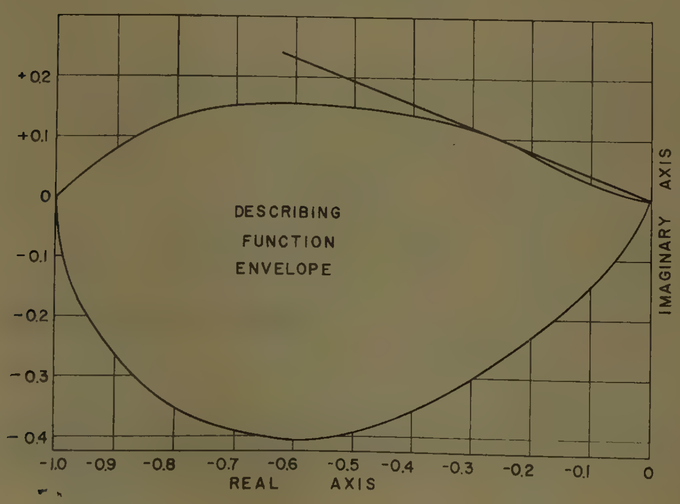


Fig. 8 (left). Describing function envelope

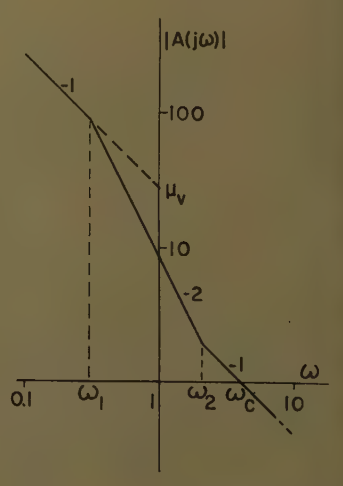
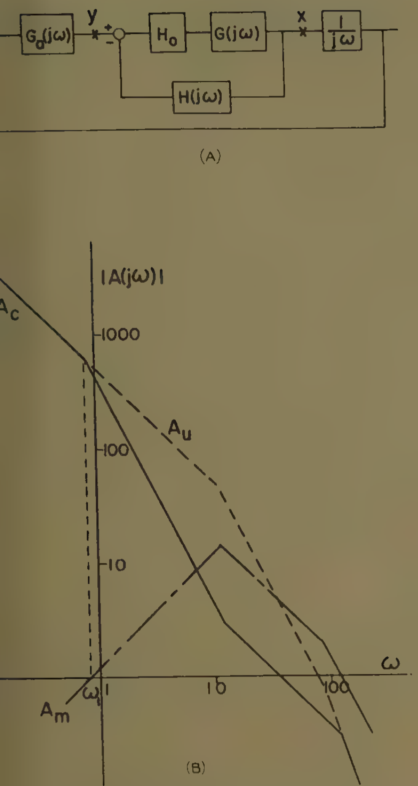


Fig. 9 (right). Bode diagram of a system with a simple equalizing network



g. 10. System with minor loop equalization

—Block diagram

—Bode diagram (values of A_u and A_c given in Fig. 13)

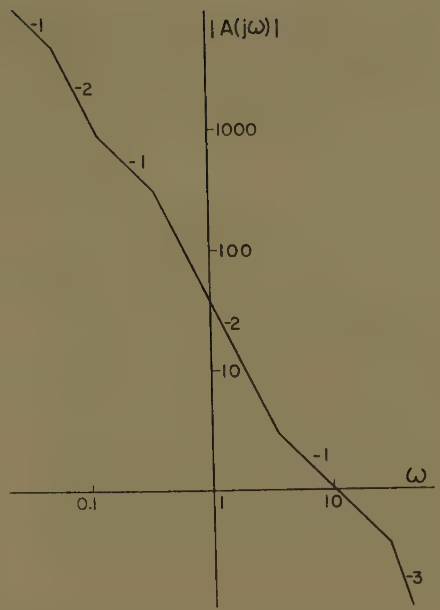


Fig. 11. Bode diagram with stepped series equalizer

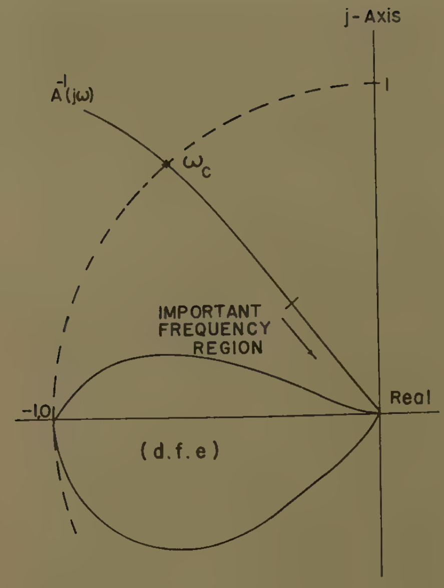


Fig. 12. Nyquist diagram of a typical system

rst two conditions satisfy the requirement that the system has no output, while ne last fulfills the condition that the ystem fails to respond irrespective of the ize of the input signal. Substituting he preceding cut-off conditions into quation 23 and setting $j\omega = j\omega_0$ where ω_0 s the cut-off frequency, one obtains:

$$/\pi = (T_s/T_c)(1/\psi)\,|(T_r j\omega_0 + 1)| \tag{24}$$

Assume that at cut-off, $T_r\omega_0 >> 1$. Also, et $\omega_r = 1/T_r$. Then,

$$/\pi \approx (T_s/T_c)(1/\psi)(\omega_0/\omega_r) \tag{25}$$

$$\omega_0 \approx (4/\pi)(T_c/T_s)\psi\omega_r \tag{26}$$

Conclusions

The nonlinear portion of a servomechanism of limited dynamic range may be arranged such that the remaining linear function is the transfer ratio of the actual linear system. The simplified form of equation 18 may be employed for the case of a saturating power amplifier and d-c motor having static and coulomb friction with $\alpha \geq 10$ and $T_r \leq 0.1\,T_J$. The describing function envelope, Fig. 8, or boundary curve which encompasses all describing functions for T_s/T_c between 1 and 10 may be used as a rather conservative, but simple stability criterion. The following results are developed in detail in Appendix I.

1. Stability. The system is stable if for monotonically increasing $|A^{-1}(j\omega)|$ the phase angle for all frequencies for which $|A^{-1}(j\omega)| \leq 1$ is less than 160 degrees.

2. Effect of Series Equalizer. A series equalizer which has a Bode diagram of the form of Fig. 9 has a maximum permissible length of -2 slope of $\omega_2/\omega_1 \leq 32.8$.

3. Minor Loop Equalization. Insofar as minor stability is concerned, the criterion for the nonlinear system is the same as for the linear system (Fig. 10).

4. Static Error. In general, for a system having a Bode diagram of Fig. 9, the velocity error constant μ_v can be expressed as $\mu_v = \omega_c(\omega_2/\omega_1)$. Hence, the minimum value of static error is Error $= \dot{\theta}_{max}/33\omega_c\psi$. The velocity error constant μ_v can be increased beyond the value of $33\omega_c$ if a more complex series network is used (Fig. 11), or if a combination of minor loop and series equalizing network is used. Fig. 12 illustrates a Nyquist diagram of a typical system.

5. Relative Stability. The typical behavior of both ω_m and $|M_m(j\omega)|$ is shown in Fig. 13.

6. System Cut-off Frequency. Insofar as stability is concerned, a system of limited dynamic range may be designed by conventional methods applicable to linear systems without any regard having to be paid to the cut-off frequency.

Appendix I

Analysis of Performance

Having established the $-H_0(j\omega)$ locus for the combined nonlinearities of stiction, friction, and saturation, it is now possible to investigate the performance of a servomechanism employing an output element of limited dynamic range.

Stability

Although quantitative statements concerning relative stability and system frequency response require point-by-point computations and can therefore be made only on the basis of specific values of $A(j\omega)$ and $-H_0(j\omega)$, it is possible on the basis of

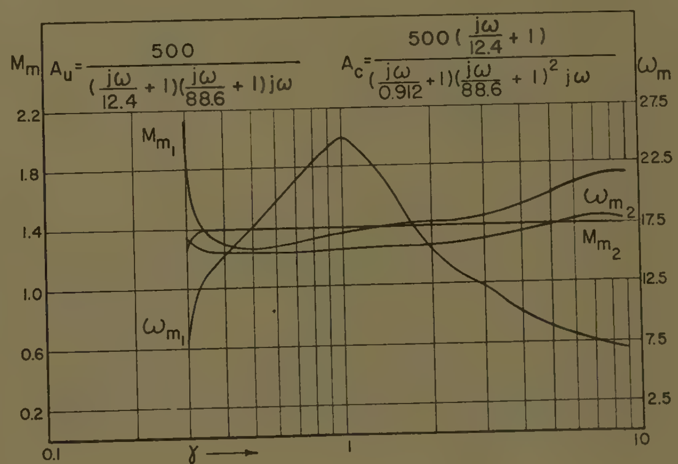


Fig. 13. Plot of $|M_{max}|$ and ω_m for a system using series equalization (M_{m_1} and ω_{m_1}) and for a system using minor loop equalization (M_{m_2} and ω_{m_2})

the general shape of the describing functions and of $A(j\omega)$ to make a number of qualitative deductions concerning the frequency response and relative stability.

First, the stability and the effect of various stabilizing networks are considered. As has been pointed out before, the describing functions for different values of T_s/T_c are all quite similar in form. It is, therefore, possible to construct a boundary curve surrounding all the describing functions. Fig. 8 shows such a boundary curve which encompasses all describing functions for T_s/T_c between 1 and 10. The describing functions for $T_s/T_c>10$ differ by only a negligible amount for those for which $T_s/T_c=10$, so that the boundary of Fig. 8 essentially includes all describing functions. This boundary curve is referred to as the describing function envelope (dfe), and it may be used as a rather conservative, but simple stability criterion.

In conventional describing-function theory, a system is stable only if the Nyquist diagram of the linear portion does not intersect the describing function. In the present system, there is an infinity of describing functions but they are all inside the dfe. Hence a system is certainly stable if the Nyquist diagram does not enter the inside of the dfe.

The criterion is conservative because intersection of the Nyquist diagram with the envelope does not necessarily mean that the system will be unstable. This is due both to the frequency dependence of the describing functions and the fact that for specific values of T_s/T_c, the describing functions for all frequencies may be far away from the envelope. The criterion is, however, desirable not only because of its simplicity, but also because in most practical cases the exact value of T_s/T_c is unknown and may even change during operation.

As is shown in Fig. 8, a tangent drawn from the origin to the dfe makes an angle of 160 degrees with the positive real axis. The Nyquist diagram, therefore, does not enter the dfe if the phase angle of the inverse loop gain function never exceeds 160 degrees. Thus, the simplified stability criterion may be phrased as follows: The system under study is stable if for monotonically increasing $|A^{-1}(j\omega)|$ the phase angle for all frequencies for which $|A^{-1}(j\omega)|\leq 1$ is less than 160 degrees.

In applying this criterion it should be noted that for a given T_s/T_c, the describing functions in the second quadrant are those for the low frequencies, while those for the higher frequencies are in the third quadrant. This can be seen in Figs. 6 and 7. Hence a system in which the crossover frequency is relatively high is generally stable even if the Nyquist diagram enters the dfe.

The Effect of a Series Equalizing Networks

The use of a simple series stabilizing network results in a Bode diagram of the form shown in Fig. 9. The -2 slope for frequencies below crossover may produce a phase lag exceeding 160 degrees, thus making an intersection with the dfe possible. The maximum length of the -2 slope for which the greatest phase lag does not exceed 160 degrees is easily computed by the standard techniques.[4] Assume, for the sake of simplicity, that the portion of the Bode diagram to the right of crossover continues along a -1 slope. Then for a maximum phase lag of 160 degrees at a frequency between ω_1 and ω_2, the result is:

$$\frac{\omega_2}{\omega_1}\leq 32.8 \qquad (27)$$

Minor Loop Equalization

Fig. 10(A) shows the system employing minor loop equalization and Fig. 10(B) is the representative Bode diagram.

Fig. 10(B) defines:

$A_u(j\omega)=$ linear uncorrected major loop gain $=G_0(j\omega)G(j\omega)(1/j\omega)$

$A_c(j\omega)=$ linear corrected system loop gain $=A_u(j\omega)/1+A_m(j\omega)$

$A_m(j\omega)=$ linear minor loop gain $=G(j\omega)H(j\omega)$

The transfer function of the part of the loop bridged by the minor loop is

$$x(j\omega)/y(j\omega)=H_0G(j\omega)/[1+H_0G(j\omega)H(j\omega)]$$
$$=H_0G(j\omega)/[1+H_0A_m(j\omega)] \qquad (28)$$

For $|H_0A_m(j\omega)|>1, x(j\omega)/y(j\omega)\approx 1/H(j\omega)$, and

$$A_c(j\omega)\approx\frac{G_0(j\omega)}{j\omega H(j\omega)} \qquad (29)$$

Minor loop stability is determined from equation 30.

$$1+H_0A_m(j\omega)=0-H_0=A_m{}^{-1}(j\omega) \qquad (30)$$

In general, the phase angle of $A_m{}^{-1}(j\omega)$ at low frequencies starts with -90 degrees, goes to 0 degrees, $+90$ degrees and approaches 180 degrees of phase angle. In respect to the $-H_0$ locus (dfe), it is only the 180-degree phase angle region that presents possible difficulty. Fortunately, this region represents the high-frequency portion of the Nyquist diagram. As has been previously pointed out, the describing functions for high frequencies are in the third quadrant. Hence, the minor loop will generally be stable even though the Nyquist diagram of the minor loop does enter into the dfe. In general, it is safe to say that insofar as minor-loop stability is concerned, the criterion for the nonlinear system is the same as for the linear system and the $-H_0$ term will not cause any difficulty. It may, in fact, improve the stability of the minor loop.

Next one needs to investigate the condition where the minor loop is not in control or $|(A_m(j\omega)|<1$. In this region it is necessary that the phase lag, due to the uncorrected loop gain $A_u(j\omega)$ below ω_1, in Fig. 10(B), be less than 160 degrees, to prevent possible intersection with the dfe.

This emphasizes the condition that best equalization is accomplished by the use of a minor loop. The amount of minor loop equalization is restricted by the linear stability requirements of the minor loop, and the condition that $A_u(j\omega)$ must introduce less than 160 degrees of phase lag below ω_1. If required, a series equalizer may be used in conjunction with a minor loop provided the preceding requirements are fulfilled.

Static Error

A simple position-control system of the type considered here exhibits a position error when a fixed torque is applied to output of the motor. If there is frict such a torque is produced by the frict The maximum error is then given by

$$\text{Error}=\frac{T_sR_a}{\mu_vK_tK_v} \qquad ($$

where T_s is the maximum static frict torque and μ_v is the velocity error const of the system. (For a derivation equation 31, see chapter 9 of reference This expression may be converted into form in which the dynamic range ψ appe explicitly. To this end it is noted th $E_{min}=T_sR_a/K_t$ is the minimum volta applied to the motor armature that w cause rotation. Also, if $\dot{\theta}_{mav}$ is the ma mum output velocity, then $E_{max}=K_v\dot{\theta}$ is the maximum armature voltage. T static error may, therefore, be written:

$$\text{Error}=\frac{-\dot{\theta}_{max}}{\mu_v}\frac{E_{min}}{E_{max}}=\frac{\dot{\theta}_{max}}{\mu_v\psi} \qquad (3$$

where $\psi=E_{max}/E_{min}$ is the dynamic ran defined previously. It is clear, therefo that for a given motor for which bo $\dot{\theta}_{max}$ and ψ are fixed, the static err can be reduced only by increasing In general, for a system having the Bo diagram of Fig. 9, μ_v can be expressed terms of the crossover frequency and t break frequencies by:

$$\mu_v=\omega_c\frac{\omega_2}{\omega_1} \qquad (3$$

Also, for a system with friction, it w shown previously in this paper that t maximum value of ω_2/ω_1 for which t system is certain to be stable is 33. Henc that the minimum value of the static error

$$\text{Error}=\frac{\dot{\theta}_{max}}{33\omega_c} \qquad (3$$

for most systems using d-c motors, $\omega_c<$ radians per second. Hence, a practic lower limit for the static error is on th order of $\dot{\theta}/2000\psi$.

The velocity error constant μ_v can b increased beyond the value of $33\omega_c$ if more complex series network is use Fig. 11 shows the Bode diagram of a syster with a series equalizer designed to produc a stepped -2 slope in $|A(j\omega)|$. By th means it is obviously possible to increas the gain indefinitely without exceeding th critical 160-degree phase lag. Howeve aside from the fact that a network t produce this Bode diagram might be re atively complicated, it also might resul in a rather poor transient response wit very long settling time.

Another way to improve static accurac is to use minor loop stabilization, or combination of a minor loop and a simpl series equalizing network. The improve ment cannot, however, be indefinitel continued without encountering the diffi culty of minor loop stabilization and larg time constant required by the serie equalizer.

Relative Stability

On the basis of the general shape of the describing functions and of $A(j\omega)$, it is possible to make a number of qualitative statements concerning the over-all system

uency response (M) and relative
bility. Relative stability may be defined erms of M_{max}, and becomes poorer as $_{ax}$ increases.

The system, considered as a linear system, uld be designed with adequate phase rgin. Also, good design usually requires t the phase margin is a maximum near crossover frequency. Therefore, the ve for $A^{-1}(j\omega)$ will intersect the unit le on an approximately radial line and will otherwise have the form shown in . 12.

The general shape of the describing ctions is shown in Fig. 7. It should be ed that the describing functions for low quencies are in the second quadrant ile the ones for the higher frequencies in the third quadrant. Therefore, e low-frequency describing functions are sest to the $A^{-1}(j\omega)$ curve. Also, it should noted that as γ increases from zero, the gnitude of $-H_0$ first increases and then creases again. Thus, for very small or ry large γ, the operating point of the scribing functions is near the origin, ile for γ near unity, the operating point near $-1+j0$.

The over-all frequency response for the rvosystem is given by the well-known rmula:

$$M(j\omega)=\frac{A(j\omega)H_0(\omega,\gamma)}{1+A(j\omega)H_0(\omega,\gamma)}$$

$$=\frac{H_0(\omega,\gamma)}{A^{-1}(j\omega)+H_0(\omega,\gamma)}$$

Both the numerator and denominator of his expression can be obtained from the Nyquist diagram and describing functions s shown in Fig. 12. The resonant frequency of the system, defined as the frequency for which $|M(j\omega)|=|M_{max}(j\omega)|$ is pproximately the frequency for which the enominator is a minimum; i.e., the frequency where the separation between $A^{-1}(j\omega)$ and $H_0(\omega,\gamma)$ is a minimum. It hould be clear, therefore, that since $|H_0(\omega,\gamma)|\ll 1$ for γ very small or very arge the resonant frequency will be a oint on $A^{-1}(j\omega)$ near the origin; i.e., a elatively low frequency. On the other hand, for $\gamma\approx 1$ so that $|H_0(\omega,\gamma)|\approx 1$, the esonant frequency will be near the crossover frequency of $A^{-1}(j\omega)$, where the cross-over frequency ω_c is defined by $|A(j\omega_c)|=1$. Thus, one expects the resonant frequency to be low for small input amplitudes, to increase to a value near ω_c for inputs such that $\gamma\approx 1$, and then to decrease again.

For low frequencies, corresponding points on $A^{-1}(j\omega)$ and on the describing function loci tend to be close together, since the low-frequency describing functions are mostly in the second quadrant. The low resonant frequencies associated with very large or very small values of γ will also tend to produce relatively large values of $M_{max}(j\omega)$. Hence, one expects $M_{max}(j\omega)$ to be large for small γ, to decrease to a minimum near $\gamma=1$, and then to increase again.

The typical behavior of both ω_m and $|M_{max}(j\omega)|$ is shown in Fig. 13, which shows curves computed for a particular system.

Also shown in Fig. 13, for comparison, is the range of $|M_{max}(j\omega)|$ and ω_m for the system stabilized by a minor loop rather than by a series network. It is clear from the figure that the variations of $M_{max}(j\omega)$ and ω_m as a function of input amplitude are much less when a minor loop is used. This is due to the fact that for the frequency range in which the minor-loop gain is greater than unity, the transfer function from x to y, Fig. 10(A), is approximately equal to the reciprocal of the transfer function of the feedback path, which consists of passive, linear elements. The nonlinearities of the system are therefore approximately masked by the minor loop, and the over-all system behaves approximately like a linear system; i.e., it exhibits constant $M_{max}(j\omega)$ and ω_m. For very small input amplitudes, the reduction in gain caused by the nonlinear friction will eventually also reduce the minor-loop gain to the point where the effectiveness of the minor loop is reduced. A similar effect is noted for very large inputs, where saturation reduces the gain. With both extremes, the system again begins to exhibit a somewhat nonlinear response.

System Cut-off Frequency

The cut-off frequency is defined by equation 26 and Fig. 7 illustrates the effect of decreasing the dynamic range ψ on the $-H_0$ locus. Even with low values of ψ, it is possible to have a high cut-off frequency, since this frequency depends on ω_r.

An important question arising in connection with the cut-off frequency is whether this frequency must be considered in the design of the feedback loop? That is, if instead of the nonlinear cut-off that is considered here, a similar effect were produced by a linear filter with a sharp cut-off characteristic, would it be accompanied by a very large amount of phase lag at frequencies below the cut-off value? This phase lag would make it difficult to obtain an adequate phase margin at the cross-over frequency unless the cross-over frequency were well below the cut-off point. However, it will be noted that the nonlinear cut-off phenomenon is only superficially similar to the linear cut-off filter because there is no large phase lag associated with it. This can best be appreciated by inspection of the H_0 loci of Fig. 7. This figure shows that all of the loci are in the second and third quadrant, with higher frequency loci generally in the third quadrant. A feedback system designed on the usual linear basis with the usually acceptable phase of about 50 degrees will have a Nyquist diagram for the linear part of the loop as shown in Fig. 12. This diagram represents a stable system quite irrespective of whether the cross-over frequency is near the cut-off frequency or not. In fact, from the point of view of system stability there is no reason why the cross-over frequency, for the linear part of the system, cannot be higher than the cut-off frequency.

Thus one reaches the important practical result that as far as stability is concerned, a system of linear dynamic range may be designed by conventional methods applicable to linear systems without any regard having to be paid to the cut-off frequency.

On the other hand, if the over-all performance of the system is considered, it is clear that the cut-off frequency must be taken into account. By definition, the system does not respond to frequencies above the cut-off value. Therefore, if the cut-off frequency is less than the cross-over frequency, the system pass band will be limited by the cut-off frequency. The M curve will have to be zero for frequencies above this value; hence the resonant frequency must be less than the cut-off frequency. Also because of the relationship between the frequency response and transient response, the transient response will reflect the cut-off frequency rather than the cross-over frequency. In particular, the rise time is greater for the system with the cut-off frequency than for the linear system.

If the cut-off frequency is higher than the cross-over frequency, its effect on system performance diminishes. This can be appreciated by reference to Fig. 12. As has been previously pointed out, the resonant frequency of the system for a given input amplitude is approximately the frequency where the separation between $A^{-1}(j\omega)$ and $H_0(\omega,\gamma)$ is a minimum. For a system having the Nyquist diagram of Fig. 12, the resonant frequency is always less than the cross-over frequency. Hence, if the cut-off frequency is high with respect to the cross-over frequency, its effect on the resonant frequency is negligible. By implication, its effect on other performance criteria will also be negligible.

A rather extensive analog-computer study has been made on the type of system discussed in this paper.[3] This study has confirmed the conclusions reached in this analysis.

Appendix II. Nomenclature

T_a=torque applied to the stiction friction unit
T_c=coulomb friction torque
T_s=static or stiction torque
T_f=total friction torque opposing motion
T_e=effective motor torque ($T_e=T_aH_1$)
H=static and coulomb friction describing function
H_1=saturation describing function
H_0'=modified stiction friction describing function
H_0=composite describing function (H_1H_0')
E=input to saturating amplifier
E_s=saturating value of input signal
E_0=output of saturating amplifier
$\gamma=E/E_s$
θ=motor angular position
$\dot\theta$=motor angular velocity
V_m=motor back electromotive force
R_a=motor armature resistance
I_a=motor armature current
L_a=motor armature inductance
$T_r=L_a/R_a$:$1/T_r=\omega_r$
K_t=motor torque constant
K_v=motor voltage constant
J=combined inertia of rotating elements referred to rotor
B=coefficient of viscous friction for motor
$T=J/B=\alpha T_J$
$T_J=R_aJ/K_vK_t$
$\alpha=K_tK_v/R_aB$
ψ=dynamic range
μ_v=system static gain
ω_c=cross-over frequency
ω_0=cut-off frequency
A=linear loop gain
A_μ=linear uncorrected loop gain
A_c=linear corrected loop gain

A_m = linear minor loop gain
M_m = maximum value of harmonic response
ω_m = frequency at which M_m occurs
S = Laplace operator $= j\omega$ for harmonic analysis
S = Laplace operator
G_0 = general transfer function

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Discussion

J. E. Gibson and **E. S. McVey** (Purdue University, Lafayette, Ind.): The authors are to be complimented for their analytical treatment of a difficult problem which is of theoretical and practical interest. However, there are a few points that should be called to the authors' attention. The authors are obviously aware that the expressions for the describing function are valid only under sinusoidal conditions. The use of s instead of $j\omega$ in these equations might make the unwary reader think that the inverse Laplace transform can be taken to obtain the time response. Along the same line, equation 28 should be called a "quasilinear" or "linearized" transfer function or should be qualified in some manner lest a reader treat it as a linear transfer function. The reader should be reminded that the equation for $M(j\omega)$ must be used with care although it is a well known equation. An excellent paper was written by Levinson[1] concerning the frequency response of nonlinear systems which bears out this statement.

Reference

1. Some Saturation Phenomena in Servomechanisms with Emphasis on the Tachometer Stabilized System, E. Levinson. *AIEE Transactions*, pt. II (*Applications and Industry*), vol. 7 Mar. 1953, pp. 1–9.

R. L. Moruzzi and **F. B. Tuteur**: We woul like to express our appreciation for th comments made by Messrs. Gibson an McVey which are quite pertinent since th unwary reader may not keep in mind tha for the harmonic response function th Laplace operator s is equal to $j\omega$, and tha it is not correct to attempt to obtain the sys tem time function by applying the invers Laplace transform to a quasilinear system of the form discussed in this paper. It i noted, however, that the paper as it ap pears here has been corrected to eliminate this possible difficulty.

A Modified Posicast Method of Control With Applications to Higher-Order Systems

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Synopsis: The theory of Posicast control,[1–3] by using a delayed step or steps, permits deadbeat step response from a very lightly damped system. The original development deals primarily with second-order systems, proposes a delay line technique for application of the second step, notes that use of this technique leaves the basic system underdamped, then proposes use of the delay line in a feedback path to damp load disturbances, and finally proposes the use of a tapped delay line for higher-order systems.

It has also been proposed[4] that a delayed step may be applied to a linear underdamped system of any order by using a switch operated on a constant time delay basis. The delay time and step magnitudes are determined experimentally. The resulting response is not deadbeat but retains most of the advantages of Posicast control.

The delay line techniques have disadvantages when applied to relatively slow control systems, and the timer proposal makes no provision to damp load disturbances. This paper proposes two basic modifications to the Posicast scheme. The first is that the switching operation which introduces the delayed step also introduce compensation circuits which overdamp the system without decreasing the static gain. These circuits do not affect the step response, but provide damping for load disturbances and noise without changing the stiffness of the system. The second is the application of the technique to higher-order systems with provision for handling the boundary condition problems at the switching instant by dissipating or counteracting the stored energy in the pertinent components. The basic theory, special techniques, including a phase space approach, and computer studies are detailed in the text, plus studies of an actual positioning servo with experimentally determined fourth-order transfer function.

Theory of Second-Order Systems

This theory has been adequately covered in the literature and is only summarized here. Given a second-order underdamped system with transfer function

$$\frac{\theta_c}{E}(s)=\frac{K}{s(s+p)} \qquad (1)$$

the system function is

$$\frac{\theta_c}{\theta_R}(s)=\frac{K}{s(s+p)+K}=\frac{K}{(s+\alpha-j\omega_c)(s+\alpha+j\omega_c)} \qquad (2)$$

and the transient response to a step input, $Au(t)$, is

$$\theta_c(t)=A+\frac{\omega_n A}{\omega_c}\epsilon^{-\alpha t}\sin(\omega_c t-\phi) \qquad (3)$$

where

$$\omega_n=\sqrt{\alpha^2+\omega_c^2}$$

The maximum overshoot occurs at $t=T_p=\pi/\omega_c$. At this instant, $\dot\theta_c=0$, and

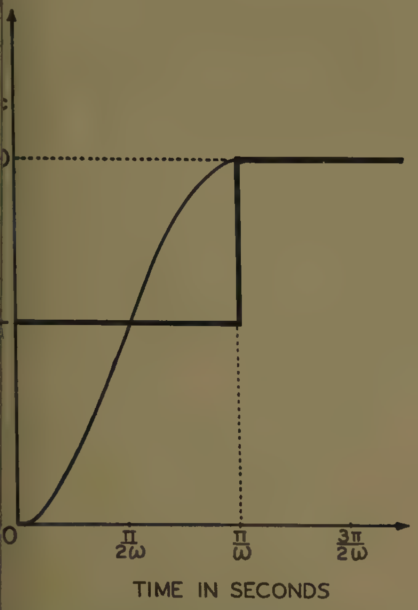
$$\theta_c(T_p)=A+\frac{\omega_n A}{\omega_c}\epsilon^{-\alpha\pi/\omega_c}\sin\phi \overset{\Delta}{=} aA \qquad (4)$$

From this it follows that if the magnitude of the step is set at $A=1/a$, then, at $t=T_p$

$$\theta_c(T_p)=1.0 \text{ (for } A=1/a) \qquad (5)$$

Paper 60-1022, recommended by the AIEE Feedback Control Systems Committee and approved by the AIEE Technical Operations Department for presentation at the AIEE Pacific General Meeting, San Diego, Calif., August 8–12, 1960. Manuscript submitted May 3, 1960; made available for printing June 17, 1960.

H. C. So (student) and G. J. Thaler are with the U. S. Naval Postgraduate School, Monterey, Calif.



. 1. **Typical transient response of second-order system with Posicast control**

It is apparent that at $t=T_p$, $\theta_c=1$ and $=0$, if a step $\theta_R=(1/a)u(t)$ is applied $t=0$. There still remains an error gnal $E=\theta_R-\theta_c=(1/a)-1$, but if this removed by changing θ_R from $1/a$ to) at instant $t=T_p$, then the system is standstill at the desired position, ere is no error signal to actuate the stem, and (for the second-order system) stored energy to disturb the equilib-um. The transient response is there-re as shown in Fig. 1.

The basic system is still underdamped d will oscillate if subjected to a load sturbance or noise. Since all boundary nditions are zero at the switching stant, compensation may be introduced ithout disturbing the system, and there no boundary value problem. Fig. A) shows a scheme for introducing a scade compensator, in which case vitching may be accomplished with a mer, and Fig. 2(B) shows a scheme for troducing tachometer feedback, in hich case the $\dot{\theta}_c$ signal is available and ay be used to actuate the switch when $=0$. Note that high static gain is adily maintained.

heory for Third-Order Systems

This discussion applies only to the se in which the transfer function poles e real. When complex poles exist, the asic theory is valid but the design of a scade compensator is necessarily differ-t. Consider a system with open-loop ansfer function

$$=\frac{K}{s(s+\alpha)(s+\beta)} \tag{6}$$

here K is large and the system is poorly

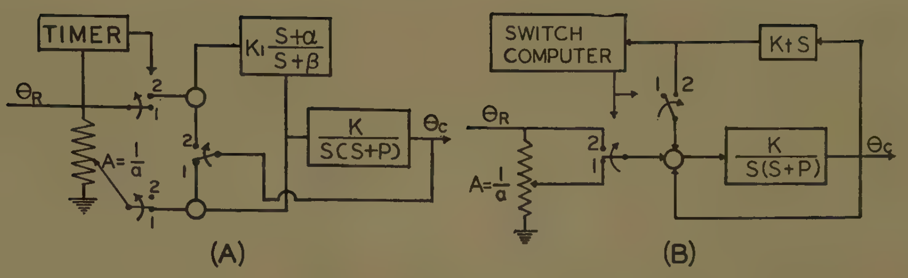


Fig. 2. **Cascade and feedback compensation schemes for second-order system with Posicast control. A—Cascade compensator. B—Feedback compensator**

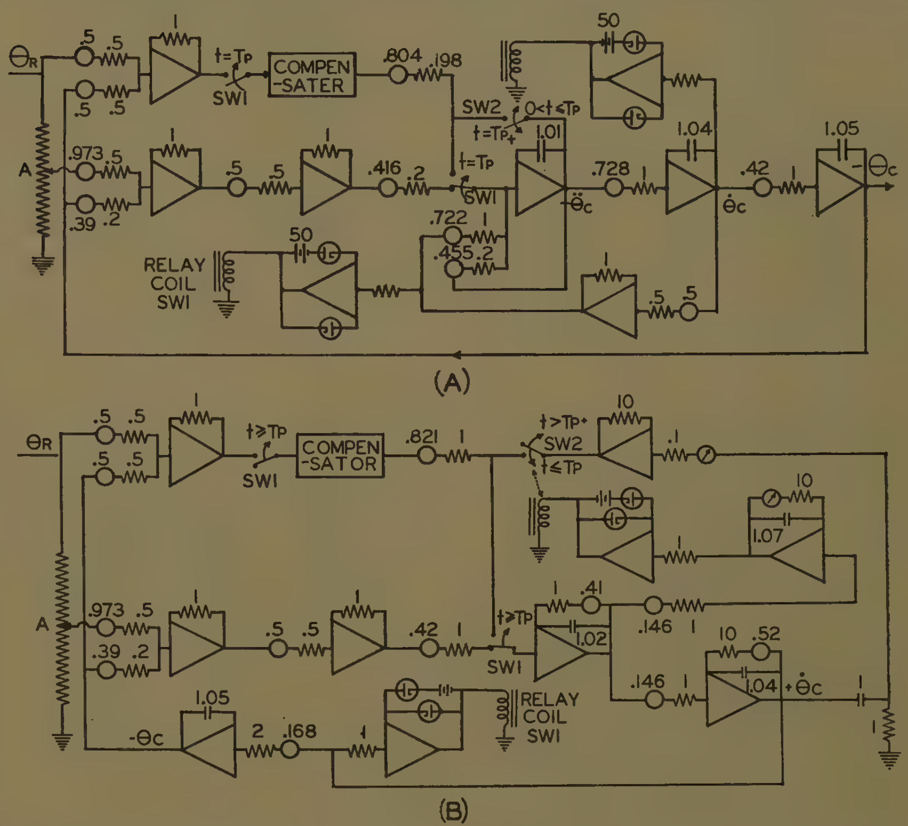


Fig. 3. **Circuit for third-order servo: $G=K/[s(s+\alpha)(s+\beta)]$. Series compensation: A—Acceleration boundary condition short-circuited out; B—Stored energy canceled by applied pulse**

damped or unstable. To apply Posicast control, the peak time and peak overshoot may be computed and the step sub-divided so that the system reaches corre-spondence at exactly the peak of the first overshoot. Assuming that K is set to place the complex roots on the imaginary axis, the characteristic equa-tion is

$$s^3+(\alpha+\beta)s^2+\alpha\beta s+K=0 \tag{7}$$

Applying Routh's criterion, for pure imaginary roots

$$K=\alpha\beta(\alpha+\beta) \tag{8}$$

and the three roots are at $-(\alpha+\beta)$ and $\mp j\sqrt{\alpha\beta}$. Solving the differential equa-tion shows that the peak time is approxi-mately

$$T_p=(\pi+\psi)/\omega \tag{9}$$

The first maximum overshoot is

$$M_{pt}=A(M+\gamma)/M \tag{10}$$

and the initial amplitude required of the step for Posicast control is

$$Au(t)=Mu(t)/(M+\gamma) \tag{11}$$

where

$\psi=\tan^{-1}\omega/\gamma$

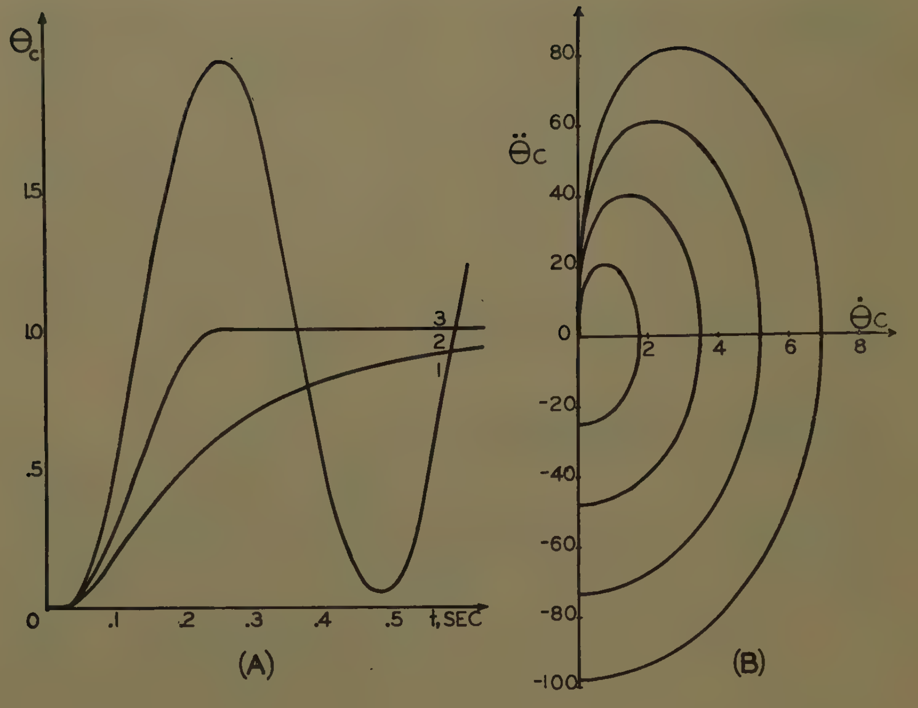
$\omega=(\alpha\beta)^{1/2}$

$M_{pt}=|\theta_c/\theta_R(t)|$ max

$M=[(\alpha+\beta)^2+\alpha\beta]^{1/2}$

$\gamma=\alpha+\beta$

If the error is reduced to zero at $t=T_p$ by making $\theta_R=1.0$, the output does not remain stationary because the stored energy due to the third pole does not permit a discontinuous change in accelera-



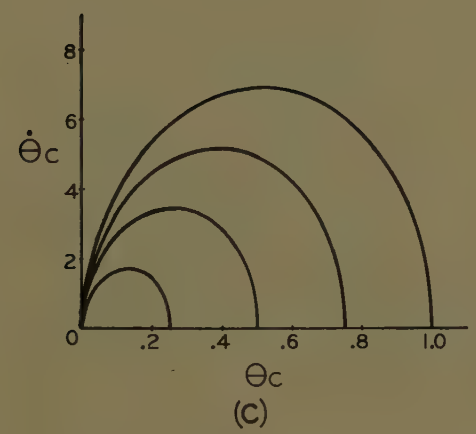


Fig. 4. Posicast control of third-order system using techniques of Fig. 3. A—Transient response curves: 1, underdamped; 2, over-damped; 3, with Posicast and over-damping. B—θ_c versus $\dot{\theta}_c$ phase plane. C—$\dot{\theta}_c$ versus $\ddot{\theta}_c$ phase plane

tion. Thus there exists a boundary value (or initial condition) problem which is inherent in the Posicast technique. Some solution is required to obtain a deadbeat response, even if compensation to an overdamped condition is not attempted. Three techniques are employed in this paper:

1. The stored energy is discharged directly.
2. A pulse of finite amplitude and duration is injected. This produces a transient which cancels that due to the boundary condition.
3. The boundary condition problem and the compensation are combined, and a phase space study of the compensated system shows that an eigenvector on the $\ddot{E}$ versus $\dot{E}$ plane may be used to obtain the desired deadbeat response.

The compensation problem is essentially an independent problem when methods 1 and 2 are used to solve the boundary condition problem. Either cascade or feedback compensation may be used, but feedback compensation seems preferable because it does not compromise the static gain. Further discussion follows with regard to specific cases.

Computer Studies, Third-Order System

Fig. 3(A) shows a computer circuit with which Posicast control is applied and the stored energy causing an acceleration at $t=T_p$ is short-circuited out by a switching device. Note that two relays are used, one marked *SW*1 to apply the Posicast principle, and the other marked *S*2*W*, to cancel the boundary condition. Both relays operate from the output velocity signal, which is readily obtained for most physical systems. *SW*1 operates at $t=T_p$, thus applying the Posicast principle. *SW*2 is closed until $t=T_p(+)$. This short-circuits out the indicated capacitor, reducing $\dddot{\theta}_c$ to zero, then opens. The slight delay is obtained b experimental adjustment of the rela amplifier. In a physical system simil switching arrangements may be used t short-circuit capacitors or open inducti storage elements. The design of t cascade compensator is accomplished i the usual fashion. If overdamping desired (but the static gain may b reduced) a single lead section with i zero used to cancel the smaller hardwar pole is usually satisfactory. The a tenuation of the filter is primarily r sponsible for the real roots obtained this fashion. If overdamping is require but gain reduction is not permissibl several cascaded sections may be nece sary. All of the usual compensati design techniques apply, and it shou be noted that there is no stored ener in the compensator when it is inserted the system, and that zero voltage applied to the compensator at the insta of insertion.

Fig. 3(B) shows a computer circu which simulates the energy stora elements in transfer function form, a which effectively cancels the stor energy by injecting a pulse at the switc ing instant. Both of the systems Fig. 3 were tested with step input The parameters (equation 6) were K 9,000, $\alpha=5$, and $\beta=40$, and the co pensator transfer function was $(s+5$ $(s+50)$. The results obtained with t two techniques for handling the bounda condition were identical; Fig. 4 shows single set of results.

To design feedback compensatio assuming that the transfer function the underdamped system is given b equation 6, and that a su table equivale transfer function for overdamped oper tion is known to be

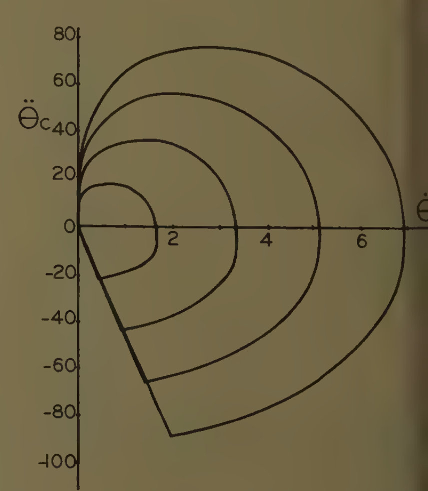


Fig. 5. Acceleration versus velocity jectories with damping connected and e signal cancelled at N = $-\omega_D$

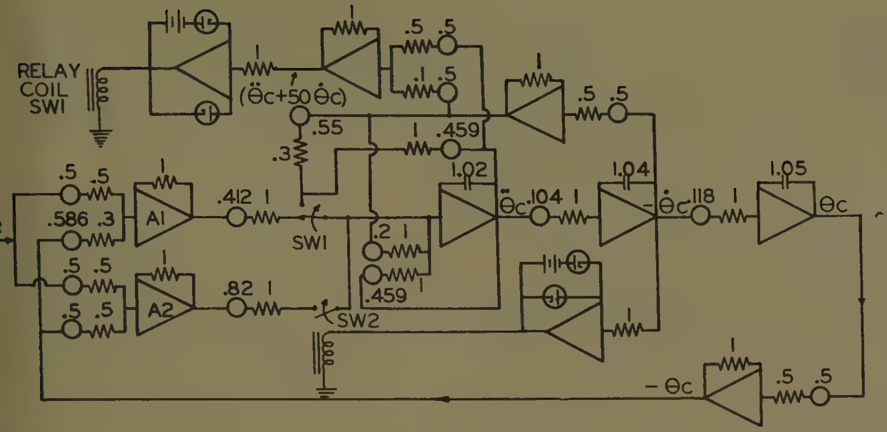


Fig. 6. Computer diagram for third-order system with use of phase space technique

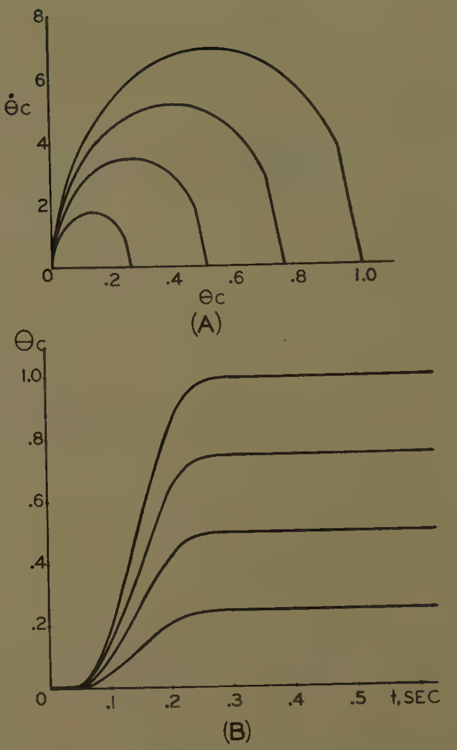


Fig. 7. A—Phase plane family obtained with phase space technique. B—Step response: θ_c-versus-t family

$$=\frac{\theta_c}{E}+\frac{K}{s(s+\beta)(s+\omega_D)} \tag{12}$$

is readily shown that a suitable feed-ck function converting equation 6 to uation 12 is

$$=(1/K)[(\omega_D-\alpha)s^2+\beta(\omega_D-\alpha)s] \tag{13}$$

To obtain overdamped operation it is cessary to connect this feedback path ien the second step of the Posicast ntrol is applied. It may be noted at the static gain is not altered when e feedback compensator is introduced, it the boundary condition problem mains. Though the methods of Fig. 3 e applicable, a new and interesting chnique has evolved from a phase ace study of the problem.

For the third-order system the differen-al equation in terms of the error variable of the form

$$+X\ddot{E}+Y\dot{E}+KE=0 \tag{14}$$

he numerical values of X and Y are ifferent for the underdamped and over-amped cases. The phase space defined y equation 14 is the E, $\dot{E}$, $\ddot{E}$ space. component of the slope of the phase rajectory is given by

$$\frac{d\ddot{E}}{d\dot{E}}=-\frac{X\ddot{E}+Y\dot{E}+KE}{\ddot{E}} \tag{15}$$

By choosing a constant value $N=$ $\ddot{E}/d\dot{E}$, a plane is defined in the phase pace. By further imposing the condi-ion $E\equiv0$ the phase space is collapsed o the $\ddot{E}$ versus $\dot{E}$ plane, and

$$N=-\frac{X\ddot{E}+Y\dot{E}}{\ddot{E}} \tag{16}$$

lefines a line in the $\ddot{E}$ versus $\dot{E}$ plane, which is a straight line through the rigin with the property that at any oint on this line the slope of the $\ddot{E}$ ersus $\dot{E}$ trajectory has the designated alue N. When the second-order system lefined by equation 16 has real roots, then two eigenvectors are defined in the $\ddot{E}$ versus $\dot{E}$ plane. Manipulating equation 16 yields

$$\frac{\ddot{E}}{\dot{E}}=\frac{-Y}{N+X}\overset{\Delta}{=}N \tag{17}$$

Thus the values of N which define the eigenvectors are

$$N=\frac{-X\mp(X^2-4Y)^{1/2}}{2} \tag{18}$$

For the underdamped system this evaluates to $N=-\alpha$, $-\beta$, and for the overdamped system $N=-\beta$, $-\omega_D$.

The physical interpretation of these results is simple. If the underdamped system is set into motion by any forcing function (such as a step), and at an instant when $\ddot{E}/\dot{E}=N=-\alpha$(or $-\beta$) the error signal E is forced to zero (such as by opening the error channel), then the system will drift to standstill with $\ddot{E}$ and $\dot{E}$ going to zero simultaneously, and at every instant the state point is on the straight line $\ddot{E}/\dot{E}=N$. In like manner, if the error signal is forced to zero and the feedback circuit connected simultaneously when $\ddot{E}/\dot{E}=N=-\beta$ (or ω_D), the overdamped system behaves the same way. Fig. 5 shows acceleration versus velocity trajectories obtained in this fashion, and Fig. 6 the feedback-compensated system.

To combine this concept with Posicast control, the following procedure applies: the initial step magnitude is reduced to a value slightly less than that which would be used with Posicast control; the damping circuits are connected and the error signal forced to zero when the ratio of $\ddot{E}$ to $\dot{E}$ is at the predetermined value; when the system stops, the error channel is reconnected, and the full input signal (second step) is used, the system is in correspondence with the feedback damping circuits connected as desired. A reduction in the amplitude of the initial step results from a change in the dynamics of the system when the theory of the $\ddot{E}$ versus $\dot{E}$ plane is applied.

To obtain deadbeat response it is necessary that E, $\dot{E}$ and $\ddot{E}$ reach zero simultaneously. By following the eigenvector in the $\ddot{E}$ versus $\dot{E}$ plane it is guaranteed that $\ddot{E}$ and $\dot{E}$ reach zero simultaneously and quickly, but reduction of the error signal to zero in this manipulation reduces the system to open-loop operation for a short time, during which the physical error does not seek a null. If the initial step required by Posicast theory is applied, switching to the eigenvector in the $\ddot{E}$ versus $\dot{E}$ plane permits a slight overshoot in position due to the change in the velocity function. Therefore a smaller initial step is required. Illustrative computations are given in the Appendix.

The operation of the system of Fig. 6 is now explainable. For a step displacement input θ_R is fed through A_1, which reduces the step magnitude, and through $SW1$. The underdamped system re-

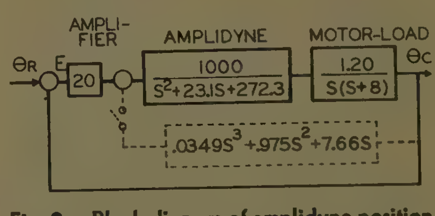


Fig. 8. Block diagram of amplidyne positioning servo with feedback compensation

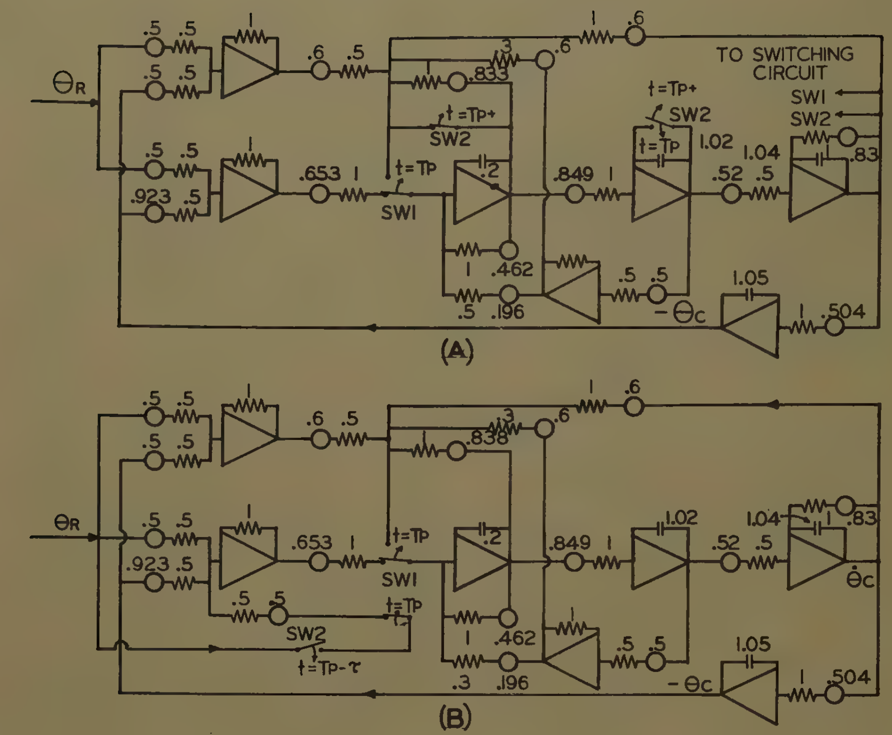


Fig. 9. **Computer circuits for posicast control of fourth-order system. A—With use of short-circuiting switches to simulate opening inductive circuit. B—With use of pulse input to counteract stored energy**

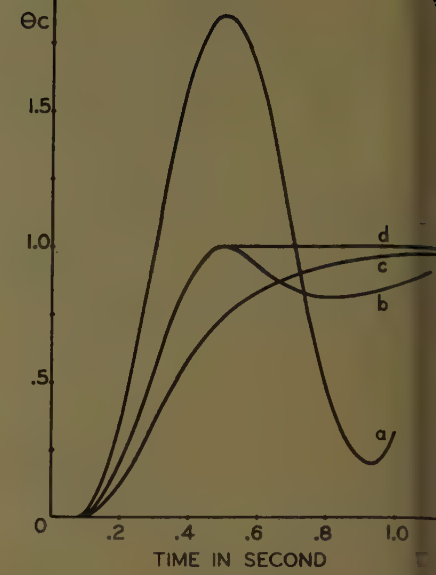


Fig. 10. **Result of computer studies. A—Underdamped. B—Posicast with overdamp ing circuit but without canceling store energy. C—Overdamped. D—Final resul with either short circuit or pulse**

sponds rapidly. When $\ddot{\theta}+50\dot{\theta}=0$, the switch $SW1$ operates, opening the error channel and applying the feedback damping loops. The open-loop arrangement brings $\ddot{\theta}_c$ and $\dot{\theta}_c$ to zero simultaneously, and for proper adjustment of the Posicast step $\theta_c=\theta_R$ when $\dot{\theta}_c=0$. At this instant, $SW2$ recloses the error channel and locks the system. Fig. 7(A) shows a family of $\dot{\theta}_c$ versus θ_c phase trajectories obtained with the system of Fig. 6. It is readily seen that they are essentially the same as those of Fig. 4(B). Fig. 7(B) shows a family of step-response curves, indicating that the performance is satisfactory over a range of step sizes.

Theory for a Fourth-Order System

As an additional extension of the method, it was decided to try application to a physical system: a positioning servo utilizing an amplifier, amplidyne, and 1/4-horsepower shunt motor. Transfer functions were determined experimentally; see Fig. 8. With the gains indicated, the system is stable but badly underdamped with system function

$$\frac{\theta_c}{\theta_R}=\frac{24{,}050}{(s+0.3\pm j8)(s+15.25\pm j11.95)} \qquad (19)$$

With feedback compensation as indicated, the system may be overdamped with real roots at -4.08, -6.1, -26.8, and -36.0.

Application of the Posicast principle requires that all stored energy be removed when the system reaches zero velocity and the second step is applied. In the amplidyne system, with output at zero velocity, energy storage is expected in the armature inductance, in the amplidyne quadrature field, and in th amplidyne control field. To dissipat this energy two schemes are attractive feeding a pulse into the amplifier, an open-circuit switching. It was decide to try both, with the use of an analo computer study to guide the technique To provide positive control of the pulse

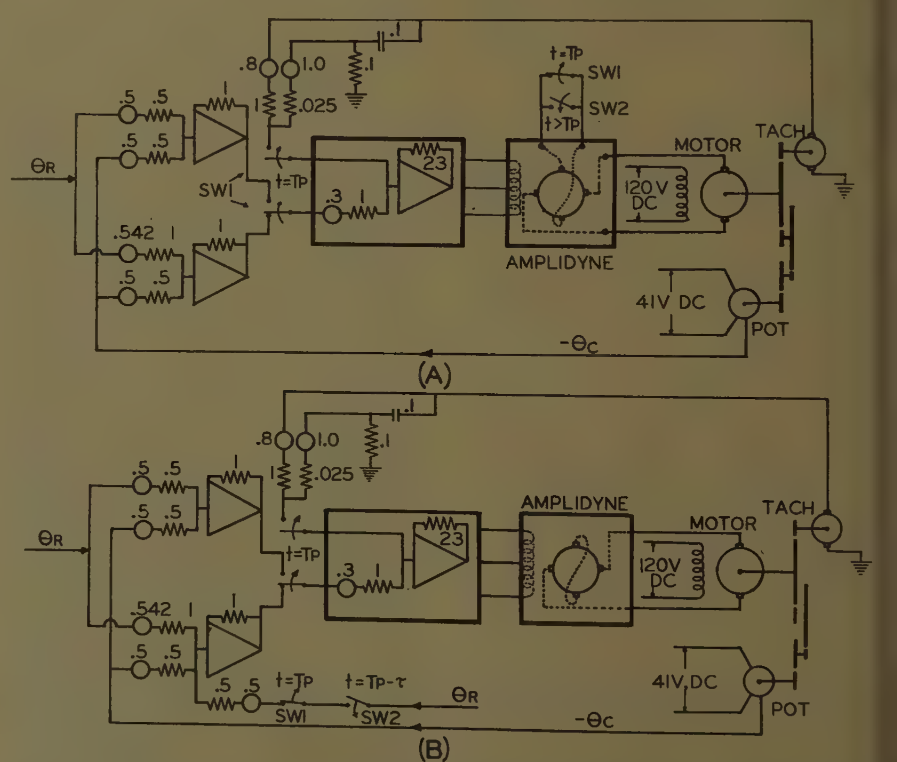
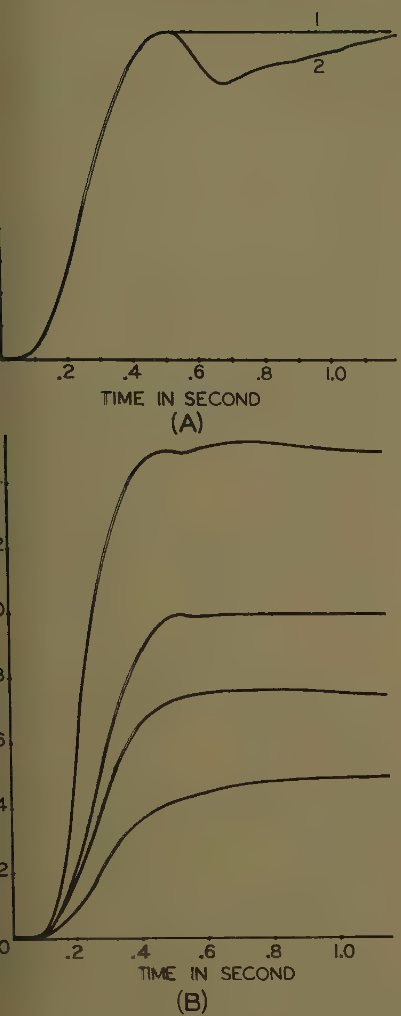


Fig. 11. **Circuit diagrams for modified Posicast control of positioning servo. A—With open-circuit switching to dissipate stored energy. B—With pulse injection to cancel transient due to stored energy**



g. 12. Step response of servo with modified sicast control. A—With use of circuit Fig. 11(A). 1—Posicast, with switching quadrature field. 2—Posicast without ncellation of stored energy. B—With use of circuit of Fig. 11(B)

et keep its amplitude proportional to ne command, this signal was obtained om θ_R. When open-circuit switching used, all three energy storage units ould be opened; however, it did not eem desirable to open the armature rcuit of the motor where the energy orage would be quite small. The uadrature field of the amplidyne was vailable for opening and reclosing with elays, and the control field is opened in ne process of changing the step input.

omputer and Physical Systems Studies

Fig. 9 shows the computer circuits mulating these conditions; typical re- ults are given in Fig. 10. Only one set f curves is shown because identical esults were obtainable with both chemes. Adjustment of the pulse is ecessary.

Fig. 11 shows the circuit diagrams for the positioning servo with open-circuit switching technique and pulse injection technique. Note that in each case only velocity and acceleration feedback were used, since noise prevented the addition of a $\ddot{E}_c$ channel. As a result the compensated system had two real and two complex roots instead of four real roots. However, the real roots were dominant, and the compensated system had an overdamped step response.

Fig. 12(A) shows the typical transient response obtained when the quadrature field was opened, and compares it with that obtained with Posicast control but without opening the quadrature field. Note that the characteristic dip obtained when the energy is not cancelled was predicted by the computer study of Fig. 10, but in this case small amplitude oscillations due to the complex roots are apparent. Similar results were obtained for various magnitudes of step.

Fig. 12(B) shows a family of step responses using the pulse injection technique with carefully adjusted pulse width (τ). Note that the responses are all essentially deadbeat as desired. For large input steps there is a slight oscillation, and it is presumed that saturation in the magnetic circuits has prevented proportional energy storage, so that the pulse width as adjusted for normal steps is too wide for larger steps.

Adjustment of the pulse width τ is fairly critical. Fig. 13 shows the effect of variations in pulse width on the step response. Note that the pulse must be applied somewhat before the second Posicast step to compensate for the time lags in the system.

Conclusions

This paper has proposed two modifications for the application of Posicast control, which remove some objectionable characteristics while retaining all of the desirable features and which also permit ready extension of the Posicast principle to higher-order systems. The first modification is that of designing damping circuits connected simultaneously with the application of the second part of the divided step. The purpose of this modification is to permit very light damping with consequent short rise time for command signals, while providing a stiff, heavily damped system to oppose noise and load disturbances. It has been shown that such compensation can be designed for either cascade or feedback connections, and is easily and satisfactorily inserted in the system.

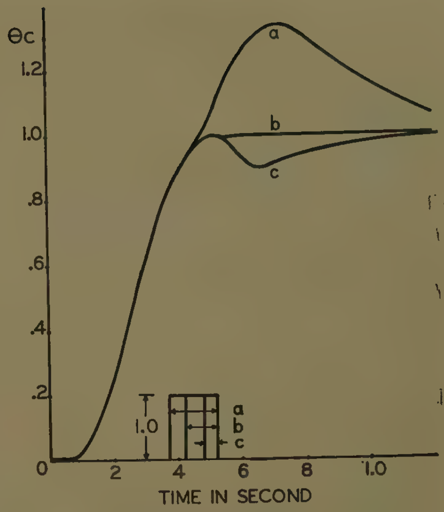


Fig. 13. Effect of pulse width τ on transient response

Cascade compensators are more difficult to design if the static gain is to be maintained at the level used to obtain the desired rise time, but is not difficult to obtain if this restriction may be relaxed. Feedback compensation is relatively easy to design, and the static gain is automatically retained. For higher-order systems the noise level in the higher derivatives may introduce a practical difficulty, but such difficulties can be overcome, as has been shown.

The second modification is the suggestion that simple techniques may be used to dissipate stored energy, thus permitting ready application of the Posicast principle to higher-order systems. Three specific techniques have been suggested. The first of these is direct dissipation of the stored energy by open- or short-circuit switching. This has been proved satisfactory by application to a physical servo system.

The second technique is cancellation of the transient due to the stored energy by injection of a pulse, and has also been proved satisfactory. The authors feel that dissipation of energy by switching is better than the pulse technique when both are applicable.

The third technique, studied in conjunction with a simulated third-order system, evolves a method for forcing E, $\dot{E}$ and $\ddot{E}$ to go to zero simultaneously. This results from a phase space study and provides a circuit which forces the system to follow a selected eigenvector in the $\dot{E}$ versus $\ddot{E}$ plane. This technique worked satisfactorily in simulation and suggests numerous possibilities for future development.

Finally, it has been shown that both modifications may be applied simultaneously with satisfactory results.

Appendix

The following are illustrative computations for adjustment of Posicast step magnitude:

Consider an underdamped system described by equation 6 with $\alpha=5$, $\beta=40$, $K=9{,}000$, and an overdamped system described by equation 12 with $\beta=40$, $\omega_D=50$ and $K=9{,}000$. The chosen eigenvector on the $\ddot{E}$ versus $\dot{E}$ plane for damping purposes is

$$\frac{\ddot{E}}{\dot{E}}=N=-\omega_D=-50 \qquad (20)$$

from which

$$\ddot{E}+50\dot{E}=0 \qquad (21)$$

The solution of the differential equation for the output motion of the underdamped system gives

$$\theta_c=\frac{M}{M+\gamma}-\frac{\gamma}{M+\gamma}\cos(\omega t-\psi)-\frac{\omega^2\epsilon^{-\gamma t}}{M(M+\gamma)} \qquad (22)$$

$$\dot{\theta}_c=\frac{\gamma\omega}{M+\gamma}\sin(\omega t-\psi)+\frac{\gamma\omega^2\epsilon^{-\gamma t}}{M(M+\gamma)} \qquad (23)$$

$$\ddot{\theta}_b=\frac{\gamma\omega^2}{M+\gamma}\cos(\omega t-\psi)-\frac{\gamma^2\omega^2\epsilon^{-\gamma t}}{M(M+\gamma)} \qquad (24)$$

where the roots of the equation are γ; $0\mp j\omega$, and the other parameters are defined following equation 11. At the instant of switching $t\cong(\pi+\psi)/\omega$ and the exponential terms may be neglected.

$$\ddot{\theta}_c+50\dot{\theta}_c=0=\frac{\gamma\omega^2}{M+\gamma}\cos(\omega t-\psi)+\frac{50\gamma\omega}{M+\gamma}\sin(\omega t-\psi) \qquad (25)$$

which reduces to

$$\tan(\omega t-\psi)=-\frac{\omega}{50}=-0.283 \qquad (26)$$

from which a first approximation is $\omega t-\psi=2.87$ radians$=164.2$ degrees. Substituting this into the equation for θ_c yields

$$\theta_c=\frac{M}{M+\gamma}-\frac{\gamma}{M+\gamma}\cos 164.2°=0.981 \qquad (27)$$

which is the value of the output displacement when the damping loop is closed and the error signal nullified; $\dot{\theta}_c$ and $\ddot{\theta}_c$ are readily computed in like manner, so the initial conditions for the damping mode are $\theta_{ci}=0.981$; $\dot{\theta}_{ci}=1.88$ and $\ddot{\theta}_{ci}=-\omega_D\dot{\theta}_{ci}=-94.0$.

Using these initial conditions in conjunction with equation 12 yields

$$s(s+\beta)(s+\omega_D)\theta_e=KE+(s+\beta)(s+\omega_D)\theta_{ci}+(s+\beta+\omega_D)\dot{\theta}_{ci}+\ddot{\theta}_{ci} \qquad (28)$$

and noting that E is zero by definition,

$$\theta_c=\frac{(s+\beta)(s+\omega_D)\theta_{ci}+(s+\beta+\omega_D)\dot{\theta}_{ic}+\ddot{\theta}_{ci}}{s(s+\beta)(s+\omega_D)} \qquad (29)$$

Then

$$\theta_c(t)=\theta_{ci}+\frac{\beta+\omega_D}{\beta\omega_D}\dot{\theta}_{ci}+\frac{\ddot{\theta}_{ci}}{\beta\omega_D}+\sum \qquad (30)$$

where Σ designates exponential terms which are negligible.

Noting that in this damping mode $\ddot{\theta}_c+\omega_{Dc}\dot{\theta}=0$, it follows that

$$\theta_c(t)=\theta_{ci}+\frac{\beta+\omega_D}{\beta\omega_D}\dot{\theta}_{ci}-\frac{\omega_D\dot{\theta}_{ci}}{\beta\omega_D}=\theta_{ci}+\frac{\dot{\theta}_{ci}}{\omega_D} \qquad (3$$

Inserting numbers gives

$$\theta_c(t)=0.981+\frac{1.88}{50}=1.0186$$

This indicates about 2.0% overshoot. is desired that $\theta_c(t)=1.0$, or

$$\theta_{ci}+\frac{\dot{\theta}_{ci}}{\omega_D}=1.0 \qquad (3$$

The magnitude of step used was $M/(M\ \gamma)=0.512$. Let the new magnitude be so that

$$\frac{A}{1.0}=\frac{0.512}{1.0186}$$

from which $A=0.502$ as a first approximation.

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esign and Development of a 600 F Pulse Preamplifier for Nuclear Instrumentation

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ESIGN considerations and evaluations are described for a high-temrature pulse preamplifier which can nplify low-level fission-chamber pulses ar the source. The preamplifier can thstand the combined environmental nditions of:

Equipment temperatures of 542 F egrees Fahrenheit).

Radiation environment equivalent to:
(a). 5.5×10^{11} gamma photons/cm (centimeter)^2/second
(b). 0.78×10^{10} neutrons/cm^2/second

The desirability of amplifying low-evel-fission counter pulses near their ource makes it mandatory that elecronic amplifier components and design pproaches be drastically revised. Temeratures in the range of 600 F and higher an be expected along with high levels of eutron and gamma radiation. Some f the high points established for the rogram to be discussed are given in the Appendix. It should be noted that the umidity requirement is still present ince the equipment must operate in common environments as well as high emperatures. It is also of importance to note that there are other added requirements accompanying the nuclear radiation and high-temperature environments. For example, this equipment is expected to operate under considerably more severe vibration and sonic-noise conditions than was normal in the past.

Design Considerations

There are four possible approaches to the high-temperature problem: 1. insulate the amplifier; 2. refrigerate the amplifier; 3. use high-temperature components and maintain a constant high-temperature environment; and 4. use components and techniques tolerant to the entire spectrum of anticipated temperatures.

The best approach, of course, will depend on the requirements of the specific application and the state of the art of components required. Possibly even combinations of these four approaches will be beneficial. The fourth approach, the equipment-tolerant approach, was the one selected by the Aircraft Nuclear Propulsion Department of the General Electric Company, for the pulse preamplifier discussed in this paper.

When one realizes that an amplifier temperature of 600 F is well above the melting point of tin and the useful life of most organic materials, it becomes obvious that drastic changes in packaging philosophy are required. The problem of electrical insulation becomes most acute, particularly in hookup wire and capacitors, where resistance to humidity at low temperatures is still a requirement. Materials suitable for use at these high temperatures, such as the micas and ceramics, are greatly affected by humidity.

Components and materials, from electron tubes and capacitors to paints, required extensive investigation and evaluation as did new manufacturing techniques.

Special studies were necessary in such areas as: 1. oxidation protection, 2. deviations of electrical characteristics, 3. thermal conductivity, 4. structural materials, 5. electric connections, and 6. spring materials.

At the beginning of this program, it was found that there were very few "on-the-shelf" electronic components suitable for these environments. Before proceeding with equipment design, it was necessary to use considerable engineering effort to investigate materials and design techniques. This was then followed by extensive evaluation testing to establish suitability for application and environments. With the number of entirely new materials and components required in the construction of high temperature and nuclear radiation-resistant amplifiers, there is a considerable variety of new manufacturing and design problems presented.

Electric Connections

One of the most difficult design problems encountered was to establish a mechanical layout which permitted accessibility for making a reliable electric connection while still maintaining miniaturization. Very early in the design stage it is necessary to establish the exact method to be used in making electric connections. Actually, it can be said that the high-temperature amplifier has to be designed around the electric connection. The need for analyzing and comparing all conceivable methods and processes of making electric connections became readily apparent.

In comparing the different methods of making connections, the following factors were considered prime requirements for an ideal electric connection: 1. reliability, 2. low and consistent resistance, 3. low mass and size, 4. high strength, 5. easy repairability, 6. obtainable with a minimum effort, 7. equally obtainable in the field, and 8. ability to withstand extremes of temperature, corrosive conditions, vibrations, and shock.

The following are some of the possibilities that were considered for meeting these objectives:

1. Arc fusing.
2. Brazing and high-temperature solders.
3. Resistance welding.
4. Wire wrap.
5. Crimping.
6. Nut and bolt.
7. Wire nuts.
8. Metalizing.
9. Taper pins.
10. Kold welding.
11. Ultrasonic welding.
12. Conductive cements.
13. Torch welding.
14. Thermit welding.

A general conclusion that can be drawn from the investigation is that there seems to be no single process or method of making high-temperature connections which can take the place of soldering. Limitations of materials and accessibility forces one to resort to a combination of methods in any given application. The designer of high-temperature electronic equipment must carefully analyze each method relative to a specific application

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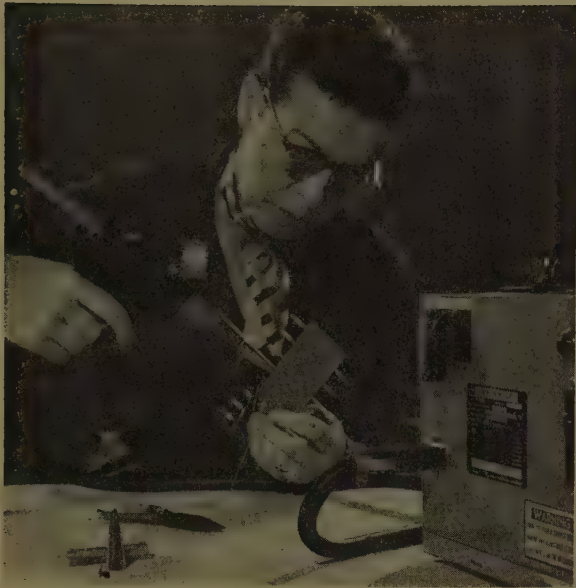

Fig. 1 (left). Arc fusing

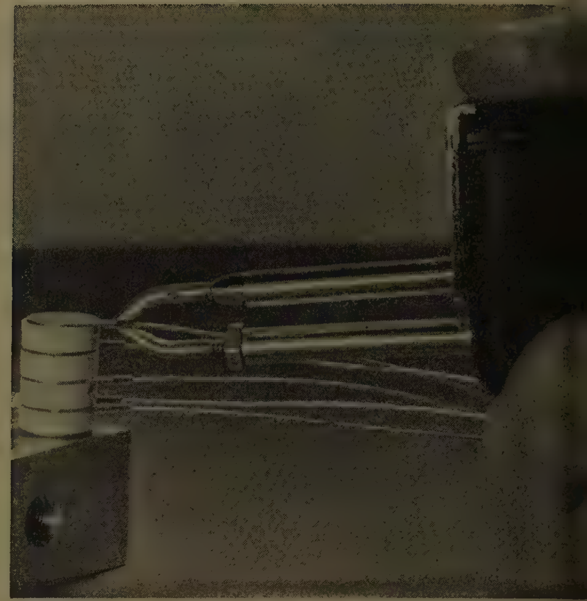

Fig. 2 (right). Typical application for resistance welder

to determine the most applicable method or methods.

In this particular preamplifier, arc-fusing was found to be a very satisfactory method for making electric connections. Resistance welding and brazing were also used advantageously on some components prior to assembly in the amplifier. Figs. 1 and 2 show typical equipment used for making arc-fused and resistance-welded connections.

The photomicrograph, Fig. 3, shows a cross section of a typical arc-fused connection, and was made in conjunction with a study of the quality of various other electric connections.

Structural Material and Configuration

The selection of the structural material and configuration requires consideration in order to realize adequate heat transfer and vibration characteristics. Some thought was given to the possibility of selecting a material for the amplifier which would help attenuate the level of nuclear radiation and thereby reduce the effect on amplifier components. However, calculations demonstrate very clearly why it is not generally practical from a weight and size standpoint to attempt to shield an amplifier. For example, when using common shielding materials in the radiation environment of this application, it takes at least a 22-inch-diameter enclosure around a $3^1/_2$-inch-diameter object to reduce the intensity of nuclear radiation by only a factor of 10. This is a relatively small percentage reduction of the total radiation level when intensities in the order of 10^{11} are being considered. It was clear, therefore, that electronic components and materials must be obtained which are inherently affected less by nuclear radiation. Consequently the choice of structural materials was primarily a problem of heat transfer and high-temperature strength. Unfortunately, these two factors are not usually found in the same material. Fig. 4 compares the relative high-temperature tensile strengths of some of the possibilities considered. As can be seen, the commonly used aluminum alloys are out of the picture in most applications for temperatures much above 500 F. Some of the possibilities considered were powdered aluminum, titanium, and the stainless steels. Although these generally were satisfactory from a strength standpoint, none was appropriate for use in the 590 F amplifier application. These materials were either unsatisfactory from a heat-transfer standpoint or because of poor weldability. For example, if stainless steels had been used, the heat rise within the amplifier would have been far too excessive.

Fig. 3 (left). Typical arc-fused connection

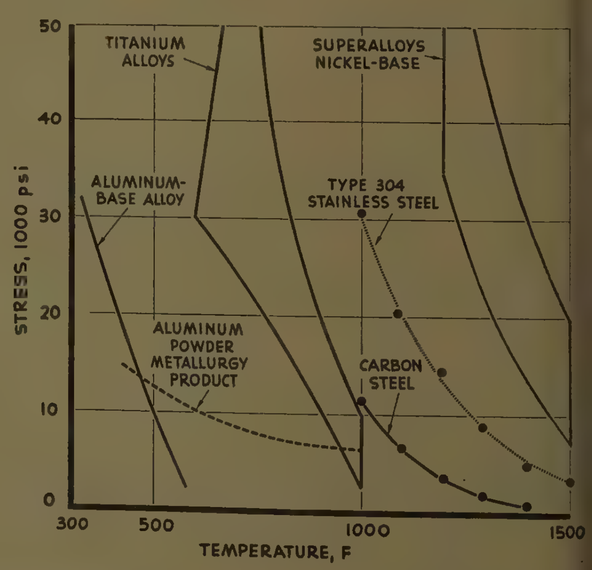

Fig. 4 (right). Stress versus temperature curves of structural metals for rupture in 1,000 hours

th 1/8-inch-thick stainless steel and y 40 watts total input, temperature would have been in the order of 200 300 C (degrees centigrade) contrasted h a design objective of 50 C.

Fortunately, stainless clad copper meets objectives without having excessive ight as would be the case with stainless el alone. The clad material combines od heat transfer, high-temperature ength, weldability, and good drawing aracteristics; however, when using it ecial attention must be given to welding brazing fixtures, otherwise considerable tortion will result.

The following are typical of other probns encountered as a result of the highnperature and nuclear-radiation reirements:

High-temperature corrosion.

Prevention of seizure of socket and conctor contacts.

Establishment of appropriate mounting ethods for relatively more brittle and agile components (ceramics and glass).

Special plating techniques and facilities quired.

Obtainment of test equipment and onitoring devices capable of operation der these severe environments.

Obtainment of small coaxial cables with w-capacitance per-unit length.

Establishment of hermetically sealed nnectors with provisions for making hrough" coaxial connections.

Development of new techniques for ssembling coaxial and interconnecting bling.

Preparation of new specifications, manucturing instructions, and operating inructions for all phases of the design, prourement, manufacture, and evaluation of e equipment.

ircuitry Considerations

Since several textbooks and considerble research have been devoted to the cience of pulse amplification, only a rief treatment will be made of these techiques, primarily to establish the basis or dealing with the high-temperature spects.

. Accept and faithfully amplify input ulses having wide ranges of amplitudes and urations.

. Amplify high average repetition rates vithout large resolution losses.

. Provide linearity of output up to a high value of pulse voltage.

. Maintain a high degree of stability of ulse amplitude.

. Prevent spurious additions or losses of ulses.

In addition, though not as important, or pulse preamplifiers:

1. Coupling time constants should be satisfactorily compromised between fast count rates and desire to faithfully reproduce shape and avoid pulse-height clipping or excessive underswings.

2. Ratio of low- to high-frequency half-power points should be no greater than 0.01 for the best signal-to-noise ratio.

3. Rise time of the amplifier should be equal to the collection time of the pulse source, unless the collection time of the source is under study.

4. All tubes should be operating at biases well outside the contact potential region.

5. Low-impedance techniques should be adhered to, especially in grid circuits.

6. The supply-voltage decoupling networks should possess time constants as high as is compatible with application size limitations.

7. Maximum consideration should be given to the shielding of heater leads from the unfiltered circuits of the final stage.

Several basic relationships necessary to specify such characteristics as preamplifier bandwidth and rise time, signal-to-noise ratio, gain bandwidth, gain per stage, pulse fall-off and low-frequency 3-db (decibel) point (f_1) are listed as: for preamplifier rise time:

$$Tr \cong \frac{1}{f_2}\sqrt{\frac{\ln 2}{2\pi}} = \frac{1}{3f_2} \qquad (1)$$

Signal-to-noise ratio:

$$S/N = \text{constant} \times \frac{1+\alpha}{f_1} \times \alpha^{\alpha/(1-\alpha)} \qquad (2)$$

Pulse fall-off:

$$f_1 = \frac{\Delta}{2\pi T_w \sqrt{N \ln 2}} \qquad (3)$$

Midband gain:

$$G_n = \frac{gm\phi(n)}{2\pi f_2 C} \qquad (4)$$

R_1 per stage:

$$R_1 = \frac{\phi(n)}{2\pi f_2 C} \qquad (5)$$

where

Tr = rise time of the pulse
f_1 = low-frequency half-power of 3-db point
f_2 = high-frequency half-power of 3-db point
α = ratio of f_1/f_2
Δ = fractional fall-off of pulse introduced by the amplifier
T_w = pulse width
N = number of stages or time constants
G_n = midband gain of each of n stages
$\phi(n)$ = a calculated fraction determined by the relationship: $\phi(n) = \sqrt{2^{1/n} - 1}$
n = number of stages
gm = tube transconductance
C = input capacitance per stage

Since detailed discussions of these equations are contained in reference 1, familiarity is assumed. Applying these relationships to this particular application, a preamplifier bandwidth up to 1.5 mc (megacycles) is established, a 3-stage preamplifier and cathode follower is selected, a theoretical plate-load resistor of 2.4 k (kilohms) is calculated, and a minimum transconductance per tube is calculated to be 4,100 μm (micromhos).

Of the primary components used in a high-temperature preamplifier, resistors were the most advanced for high-temperature use. Film-type resistors displayed a resistance temperature coefficient of −200 ppm (parts per million)/C (−6% resistance change to 600 F), and a 100-hour life test at 750 F indicated stabilities of better than 1%. Wire-wound resistors displayed a resistance temperature coefficient in the order of 0 to +100 ppm/C with excellent stability in high-temperature life tests. In size and electrical characteristics, resistors presented no problem.

Fortunately, a broadband amplifier does not require large-value capacitors. Considerable evaluation at high temperatures narrowed the choice to a particular capacitor displaying a dielectric quality of 1.25 megohm × microfarads (μf) at 600 F with a favorable size factor. A 1,000-$\mu\mu$f unit with the dielectric involved would display 1,250 megohms resistance at 600 F, sufficiently high to eliminate capacitor-leakage current as a problem. Decoupling capacitors of 0.01 μf would display resistances as high as 125 megohms at 600 F, so that a decade of further deterioration could be permitted. The temperature coefficient was only 70 ppm/C, of negligible importance in this application.

Owing to the boron content in the hard-glass family of tubes and the requirement to withstand nuclear irradiation, metal-ceramic rather than hard-glass tubes had to be utilized. For pulse use, tubes displaying high figures of merit in Gm/C and Gm/Ib are required, but availability in 1957 was confined to a very few tubes, and ultimate choice was a family of newly developed high-mu triodes; Gm = 4,800 μm, mu − 94 at ma (milliamperes). It is particularly significant that high-gain triodes were available in metal-ceramic construction, since the predominating noise of an amplifier is apt to be the shot effect and is worse in a pentode than in a triode.

Circuit techniques at high temperatures in the component tolerant approach are altered by: 1. compromise of noise, gain, and count rates of coupling circuits; 2. the danger of excessive grid emission of tubes; 3. closed-loop gain drift caused by component variations; and 4. the de-

Table I

Consideration	Make-Grid-Return Resistor
1. Less thermal noise	large
2. Maximum amplifier gain	large
3. Prevent grid-current overloading	small
4. Minimize capacitor leakage, current-bias change	small
5. Keep coupling-capacitor value low	large
6. Minimize grid-emission bias change	small

sirability of preventing excessive cathode temperatures.

Table I gives some of the considerations to be taken into account when selecting resistors for coupling circuits in high-temperature applications.

Another consideration common to all preamplifiers is the selection of a specific time constant compatible with expected pulse repetition rates.

It is perhaps surprising that the larger the grid-return resistor the less the amplifier thermal noise, for it is well known that the thermal rms voltage of a resistor is proportional to resistance and temperature in the following relationship:

$$e_{noise} = \sqrt{4KTbR\Delta f} \tag{6}$$

The following table represents typical noise voltages generated at the two temperatures of interest:

Resistance	e_{noise}, Microvolts 78 F	590 F
10 k	15.7	22
50 k	35	49
100 k	49.6	69.3
1 megohm	157	219
100 megohms	1,570	2,190

When the equivalent preamplifier input circuit is reduced to the form shown in Fig. 5, it is obvious that thermal-input noise is divided between the impedances

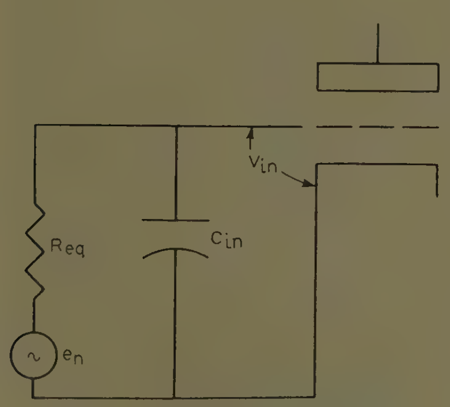

Fig. 5. Equivalent thermal-noise input circuit

$R\,eq$ and input capacitance C_{in} as shown in equation 7.

$$V_{in} = e_n \frac{-j/\omega C}{R - j/\omega C} \tag{7}$$

This expression is squared and integrated over the pass band to obtain thermal power:

$$Pn = \frac{KT}{RC^2\pi^2 f_1}$$

$$\left(\frac{1}{f_2} << \frac{1}{f_1}\right) \tag{8}$$

Rg should be made large to reduce thermal noise. Attempts to increase the bandwidth by dropping the low-frequency cutoff point f_1 will cause the thermal noise to become appreciable. If f_1 is kept at a reasonably high value, and the grid-return resistor is maintained at a large value, thermal noise need not be considered. If a differentiating circuit were used, the thermal-noise determining factor becomes f_2, the high-frequency cutoff point, but even then thermal noise is insignificant if R is large.

Since the voltage across the detector will shift by an amount nqR when the detector is exposed to radiation, the bias of the first stage will be shifted by a like amount if the detector electrode is connected directly to the first-stage grid element. Although this shift may be expected to be negligible, the effect could be disturbing where pulses from fission fragments are being counted in the presence of strong alpha-ray background. In such applications, it is desirable to choose an unusually low value of grid-return resistance for the first stage or decouple the detector from the first grid with a capacitor. In this application the latter method is used since an exceptionally low value of grid-return resistor will result in increased noise out of the amplifier.

Since there are, from the tube-plate element to a-c ground, two paths (one through the plate-load resistor and one through the coupling capacitor and grid-return resistor), the grid-return resistor is essentially shunting the plate-load resistor. Values of the plate resistor are apt to be in the order of 2 k to 10 k, and any value of grid-return resistor below about 50 k will cause serious drop-off in stage-by-stage gain.

Choice of the grid-return resistor is also affected by the characteristics of the application: the expected pulse-count rates and the permissible pulse fall-off. Higher pulse-repetition rates affect preamplifier design only in the choice of interstage coupling networks. If time constants are made very short (approaching the rise time of the incoming pulse) then maximum pulse-repetition rate can be handled, but pulse-height clipping will become a consideration. If time constants are made especially long to lengthen the duration of underswings and overshoots, the pulse will be stretched and a reduction in resolution will result (accurate counts of high repetition rates will not be possible). If choice of the coupling circuits is such that stretching occurs, then two pulses occurring very close together may overlap and be counted as one pulse.

The low-frequency response of the preamplifier will determine the fractional fall-off introduced or imposed upon the input pulse by the pre-amplifier coupling circuits. It is noteworthy that this fractional fall-off is in addition to the fall-off characteristic inherent in the preamplifier input pulse. The slope of the fall-off is determined by the time constants of the coupling circuits: the longer the time constant, the less rapid will be the drop-off. The magnitude and duration of the underswing is also affected by the coupling circuits: the shorter the time constant the greater will be the magnitude of underswing. Ordinarily, time constants several hundred times larger than pulse rise time may be used in the preamplifier since pulse magnitudes are small with respect to bias voltages and the danger of overload is therefore not serious unless repetition rate is high.

Appropriately covered in reference 2 are the advantages in selecting low grid-return resistors to prevent pulse overshoot and possible introduction of new pulses. An amplifier stage with a plate-load resistor of 1 k, a coupling capacitor of 0.01 μf, and a grid-return resistor of 1 megohm has the same time constant and gain as a stage with 20 k, 0.5 μf, and 1 k

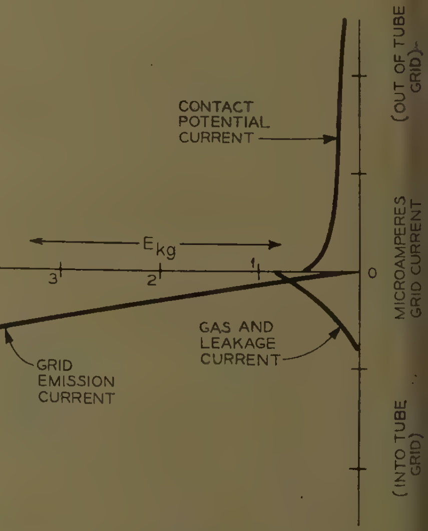

Fig. 6. Vacuum-tube grid currents

Fig. 7. High-temperature pulse-preamplifier assembly

spectively, but the second case could ndle a pulse many times larger prior to ocking.

The assumption is made that any tube ich shows as much as 2 μa (microam- res) of grid-emission current at an nbient operating temperature of 600 F ould be considered a failure. Based on is assumption then, various likely lues of grid-return resistor are tabu- ted with the magnitude of bias change sulting from the maximum expected ange in grid current throughout the erating ambient temperature excur- on. Fig. 6 shows the various grid rrents which can be encountered.

Obviously, the lowest possible value of id-return resistor should be utilized, mpatible with circuit gain and noise mitations. It would appear that 50– 00 k would be a good compromise for any applications.

Rg, Ohms	Volts *Eg*
10 k	0.02
47 k	0.09
82 k	0.16
00 k	0.20
50 k	0.30
20 k	0.45
70 k	0.90
1 megohm	2.0

Indications are that it may be desirable ither slightly to reduce heater voltage to prolong tube life, or (for large temperature xcursions) to schedule heater voltage ecause of the anticipated increase in cathode temperature as the result of the higher ambient temperatures. The design philosophy used to achieve a gain virtually independent of temperature excursions without the use of a heater power-derating device was to utilize temperature coefficients of components to provide compensation for the gain drift caused by changes in cathode temperature, as described in the following:

1. At room temperature, curves showing gain change caused by variations in value of circuit elements were compiled.

2. At room temperature, amplifier-gain changes caused by variations of heater voltage by an amount equivalent to the derating recommended by the tube manufacturer was determined.

3. Predicted amplifier drift caused by component temperature coefficient was determined from step 1 and compared to the drift caused by the heater power variation, step 2. The assumption is made that the component manufacturer's heater-derating curve represents constant cathode temperature.

4. Resistors with opposite temperature coefficient from those assumed for step 3 were used at key locations as determined by the curves of step 1 to either compensate or overcompensate as test results indicated.

Initial runs at 600 F indicated a voltage-gain drift of less than 4%.

Conclusions

The state of the art for high-temperature radiation-tolerant electronic equipment has advanced to the state where equipments are being manufactured and evaluated at 600 F in nuclear environments. Fig. 7 shows the result of one of these programs, the pulse preamplifier. In this application no significant size and weight penalties were caused by high-temperature requirements. A weight penalty can be expected where handling and severe vibration are requirements.

In order further to advance the state of the art of high-temperature electronic equipment: 1. major scientific breakthroughs are needed in dielectrics, and 2. survival-rate studies are needed as soon as components are in pilot production.

Appendix. Pulse Amplifier Environmental Specifications

The following breakdown gives some of the high points established for the discussion in the paper.

Temperature: −65 F to 542 F
Altitude: 70,000 feet
Humidity: 100% condensate including frost
Vibration: 5–2,000 cps, 40 *g*
Life: 1,000 hours total, 100 hours at 542 F
Sonic noise: 160 db, 150–6,000 cps
Salt spray: 50 hours
Impact: 24 impacts at 30 g, 10-millisecond duration
Radiation: 1,000 hours 10^9 rep, 1 meg-electron-volt gamma
10^8 rep, 1 meg-electron-volt neutrons

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The Principle of Equivalent Areas

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WHEN THE STATE of a linear system is changed by the application of a forcing function $r(t)$, the change occurring during a time interval may be determined approximately by considering only the initial conditions and the total impulse

$$I=\int_{t_1}^{t_2} r(t)dt$$

applied to the system during the interval. If the time interval is small enough, the approximation will be a good measure of the behavior of the system. This idea is exploited in this paper as the basis for the *principle of equivalent areas*, a principle establishing the condition which must be satisfied if two forcing functions which have different instantaneous characteristics are to be equivalent as far as their effects on a linear system are concerned. The principle may be briefly stated as follows:

Two input signals (forcing functions) to a linear continuous element are dynamically equivalent if their integrals evaluated over a set of corresponding equal time intervals are equal, and the time interval is suitably small. The quantities $r(t)$ and $r'(t)$ are considered equivalent if

$$\int_{(n-1)T}^{nT} r(t)dt=\int_{(n-1)T}^{nT} r'(t)dt$$

where T is a time interval based on the characteristics of $r(t)$, $r'(t)$, and the linear system, and n takes on all values sufficient to cover the total time duration of interest.

In many control system analyses the central problem is the representation of the signals in a form which is amenable to analysis. The principle of equivalent areas provides a means for transforming signals from one form into another dynamically equivalent form which may be more easily analyzed by mathematical or graphical means. This idea has been previously used in the analysis of pulse-duration sampled-data systems, and in connection with the analysis of nonautonomous nonlinear systems. The objective of this paper in introducing the principle of equivalent areas is to describe some of the factors which must be considered in its application. Graphical and mathematical derivations are given. The approach is in terms of the concepts of convolution, frequency spectra, and sampling theory.

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This paper is based on some of the results of research performed at Northwestern University in partial fulfillment of the requirements for a Ph.D. degree. The writer wishes to thank Prof. Richard W. Jones for his counsel during the course of the work and Prof. Gordon J. Murphy for reviewing the paper in its early stages.

Graphical Derivation

The problem may first be approached by referring to Figs. 1 and 2 and the convolution theorem[1] stated in equations 1 and 2. If

$$C(s)=H(s)R(s) \tag{1}$$

then

$$c(\tau)=\int_{-\infty}^{\tau} h(\tau-t)r(t)dt \tag{2}$$

The terms in equations 1 and 2 refer to Fig. 1, in which $r(t)$ represents the forcing function or input, and $h(t)$ the impulsive response characteristic of some linear system: $c(t)$ represents the state of that system or output. $R(s)$, $H(s)$ and $C(s)$ are the corresponding Laplace transforms. Fig. 2 graphically illustrates the process indicated in mathematical form by equation 2. In Fig. 2(A) the curve is the inverse Laplace transform of $H(s)$, which of course is the impulsive response of $H(s)$, plotted for convenience with the time axis reversed. In Fig. 2(B) the curve is a plot of the input r as a function of time. For illustration r is plotted as a sequence of step and ramp functions, but what we have to say is true for any arbitrary $r(t)$. The origins of the time co-ordinates of the curves in Fig. 2(A) and (B) are displaced by the amount τ. The value of the output c at the time τ is found by multiplying corresponding ordinates of the shifted curves to obtain the product curve in (C); then the curve in (C) is integrated with respect to time. The result of the integration is the area under the curve, and also the value of the output c at time $t=\tau$.

For the moment, suppose that $h(t)$ can be replaced by another curve $h_p(t)$, which is a staircase approximation to $h(t)$ as shown in Fig. 3. We can consider $h_p(t)$ as the output of a system which includes a sampler and a zero order hold circuit at the output as shown in Fig. 4. Later, the validity of this substitution and the determination of the sampling period T will be discussed. The quantity $h_p(t)$ can be expressed mathematically as follows:

$$h_p(t)=\sum_{n=0}^{\infty} h(nT)[u_{-1}(t-nT)-u_{-1}(t-nT-T)] \tag{3}$$

It has been tacitly assumed, as is customary, that $h(t)$ is zero for negative time. Also $h(t)$ is shown in the figures as approaching zero as t becomes large. The discussion, however, is general and does not only apply to systems having this special characteristic.

Fig. 5 is equivalent to Fig. 2, but redrawn with $h_p(t)$ replacing $h(t)$. As before, the area under the product curve is obtained in order to determine the value of the output c at the time $t=\tau$. In this case, however, the area is approximately equal to $c(\tau)$. It is clear that the smaller the sampling period, the better the approximation becomes. In the limit, when the sampling period approaches zero, Figs. 2 and 5 become identical. The procedure described by Fig. 5 is very similar to the approximate procedure often used for finding the time responses of a system, due to an arbitrary input, when the impulsive response of the system is known. This approximate procedure approximates the convolution integral by the following series:

$$C(mT)=\sum_{n=-\infty}^{m} h(mT-nT)r(nT)T \tag{4}$$

In using equation 4, one effectively replaces both $h(t)$ and $r(t)$ by staircase approximations.

Now refer to Fig. 6, which shows the same $h_p(t)$ as Fig. 5, but a different input function $r'(t)$. $r'(t)$ was constructed from $r(t)$ according to the principle of equivalent areas; i.e., for any n, the following relation is true:

$$\int_{nT-T}^{nT} r(t)dt=\int_{nT-T}^{nT} r'(t)dt \tag{5}$$

The exact shape of the curve within an interval T is not of importance in the construction of $r'(t)$; thus there is an

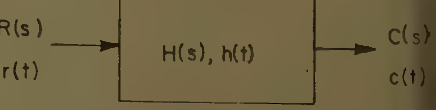
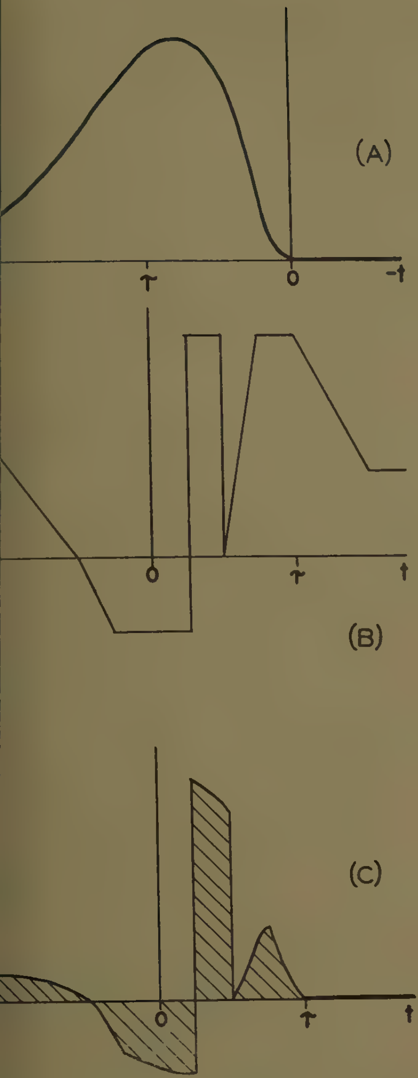


Fig. 1. Block diagram of a dynamic system



g. 2. **Graphical representation of convolution process**

A—h(t). B—r(t). C—h(τ−t)r(t)

finitely large number of curves which re equivalent according to the criterion xpressed by equation 5. When the roduct curve in Fig. 6 corresponding o $r'(t)$ is integrated we find that exactly

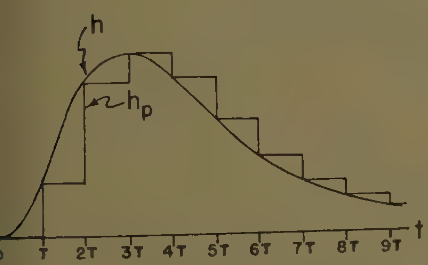


Fig. 3. Comparison of h(t) and h$_p$(t)

ig. 4 (right). **Block diagram of ystem with staircase type of impulsive response**

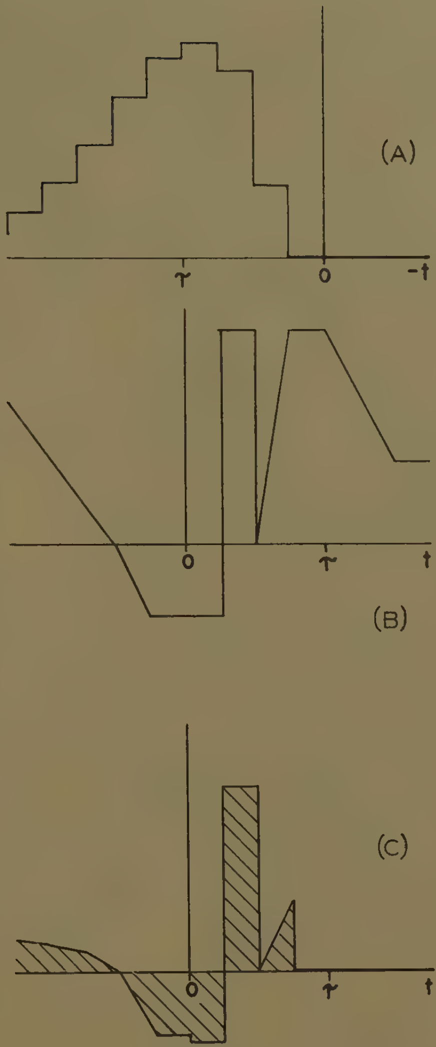
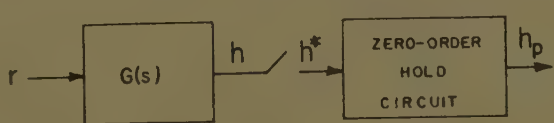


Fig. 5. Convolution with h(t) replaced by h$_p$(t)

A—h$_p$(t). B—r(t). C—h$_p$(τ−t)r(t)

the same area is obtained, and consequently exactly the same value for the output $c(\tau)$, as when the product curve corresponding to $r(t)$ in Fig. 5 was integrated. This may be verified by examining the contribution to the total integral of the integrals taken over the individual sampling periods. For example, in the interval $t=2T$ to $3T$, $r'(t)$ is constant in amplitude; $r(t)$ is represented by a triangular pulse of twice the amplitude. It is clear that the areas of the $r(t)$ and $r'(t)$ curves over the interval $t=2T$ to $3T$ are equal. Since $h_p(\tau-t)$ is constant at the value $h_p(t)$ over the interval under consideration, the product curves in Figs. 5 and 6 are given by $r(t)$ and $r'(t)$ times the same constant, $h_p(T)$. The integrals of the product curves over the same interval must then also be equal and have a value



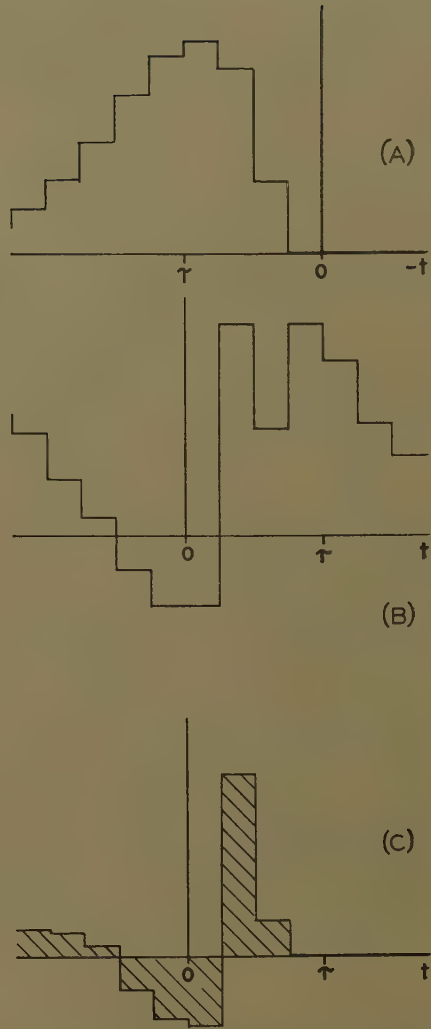


Fig. 6. Convolution with staircase approximation of impulsive response and equivalent input

A—h$_p$(t). B—r′(t). C—h$_p$(τ−t)r′(t)

$h_p(T)$ times the integral of $r(t)$ or $r'(t)$ over the interval. Since this holds true for each sampling interval, the total integrals of the product curves from $t=-\infty$ to τ are also equal. We conclude that the principle of equivalent areas does, in truth, hold.

Mathematical Derivation

In the foregoing a kind of graphical proof of the principle of equivalent areas has been attempted; a mathematical proof follows. Starting with the convolution integral, equation 2, and assuming that $h(t)$ can be replaced by $h_p(t)$, equation 6 is obtained.

$$c(\tau)=\int_{-\infty}^{\tau} h_p(\tau-t)r(t)dt \qquad (6)$$

Assume that τ is an integral number of sampling periods:

$\tau=mT \quad m=0, 1, 2, 3\ldots$

Then

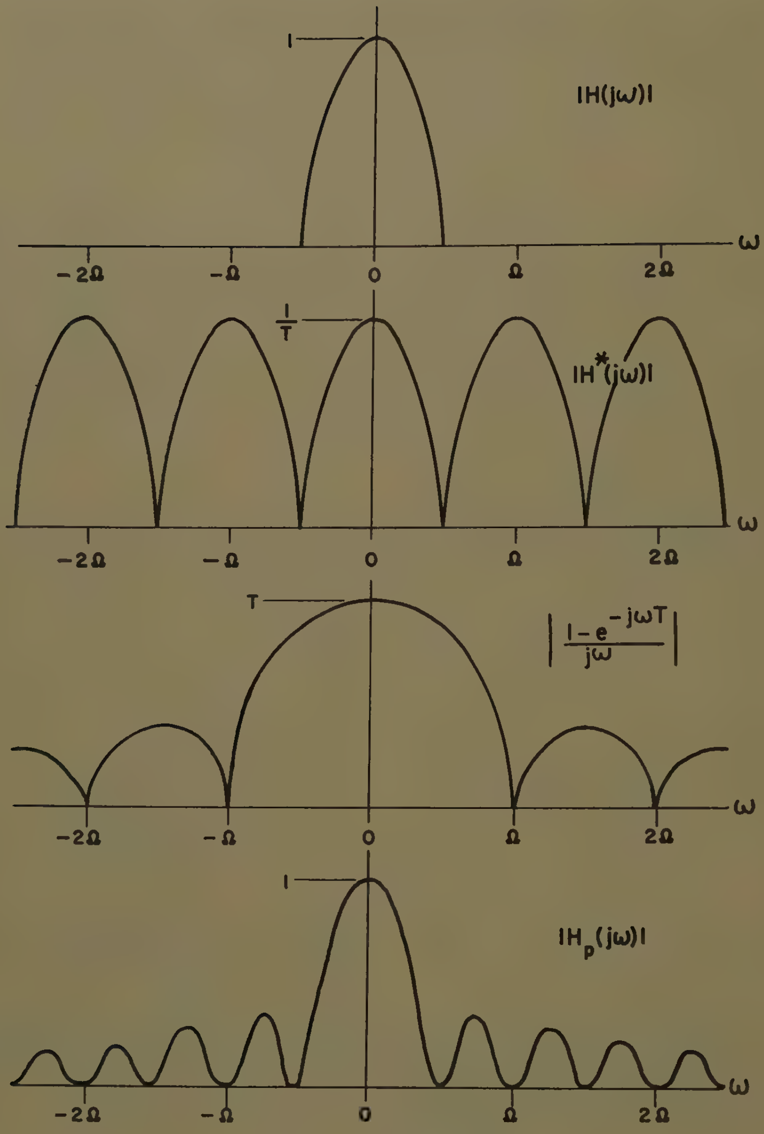


Fig. 7. Frequency spectra

$$c(mT)=\int_{-\infty}^{mT} h_p(mT-t)\, r(t)dt \tag{7}$$

Furthermore, the integral from $-\infty$ to mT can be broken down into a sum of integrals over the intervals $(n-1)T$ to nT, where n goes from $-\infty$ to m:

$$c(mT)=\sum_{n=-\infty}^{m}\left[\int_{(n-1)T}^{nT} h_p(mT-t)r(t)dt\right] \tag{8}$$

However, during the interval $(n-1)T$ to nT, $h_p(mT-t)$ is constant and equal to $h_p(mT-nT)$. Consequently the following is obtained:

$$c(mT)=\sum_{n=-\infty}^{m}\left[\int_{(n-1)T}^{nT} h_p(mT-nT)r(t)dt\right] \tag{9}$$

$$c(mT)=\sum_{n=-\infty}^{m} h_p(mT-nT)\int_{(n-1)T}^{nT} r(t)dt \tag{10}$$

Now, define $I(nT)$ by

$$I(nT)=\int_{(n-1)T}^{nT} r(t)dt \tag{11}$$

which yields the final result,

$$c(mT)=\sum_{n=-\infty}^{m} h_p(mT-nT)\, I(nT) \tag{12}$$

Equation 12 proves the principle of equivalent areas, subject of course to the validity of replacing $h(t)$ by $h_p(t)$, for the output of the system H is determined by the function h_p and I. The function h_p has been previously described; it takes on the value $h(nT)$ when $t=nT$. I is a function which has the value given by equation 11 when $t=nT$. In equation 12 $r(t)$ appears only through the subsidiary function I, and as soon as I has been calculated by integrating $r(t)$ over the suitable intervals of time, the information on the exact form of $r(t)$ during the intervals can be discarded. In probabilistic sense h_p and I are sufficien to determine the output $c(nT)$; i.e., th amplitude of the output at uniforml spaced intervals of time T seconds apart.

If the output at the sampling instant is considered in relation to the samplin theorem[2] it is noted that there are tw distinct cases which may occur. Th first concerns a dynamic element with continuous input $r(t)$ whose bandwidth i limited to one half the sampling frequency Note that in this case the outputs at th sampling instants completely define th output at all times, including the interva between sampling instants.

The second case concerns a system which has a continuous input whos bandwidth is limited to one half the sampling frequency, however this input i sampled, and the sampled signal is th actual input to the dynamic element of th system. In this case the sampling introduces high frequency components which were not present in the original signal The high frequency components are undesirable in the output and consequently practical systems act as a low-pass filte to attenuate these high frequencies. Th amount by which they are attenuate determines the amount of ripple in the output, and also the extent to which the output at the sampling instants define the output. The ripple which occurs i not accounted for; however, as the system characteristics are improved in a direction to reduce ripple, the definition of the output at all instants of time by the samples given at uniformly spaced intervals of time improves.

Replacement

The replacement of $h(t)$ by $h_p(t)$ must now be justified, and the conditions shown under which it is valid. It is instructive to consider this question in terms of the frequency spectra of $h(t)$ and $h_p(t)$. The Laplace transform of $h(t)$ is $H(s)$ and the Laplace transform of $h_p(t)$ is $H_p(s)$. From Fig. 4, $H_p(s)$ is related to $H(s)$ by equations 13 and 14:

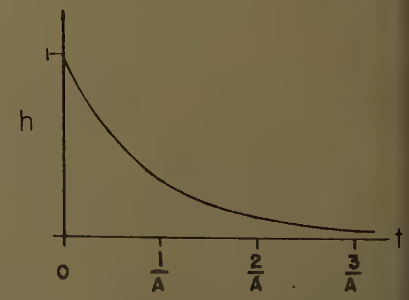
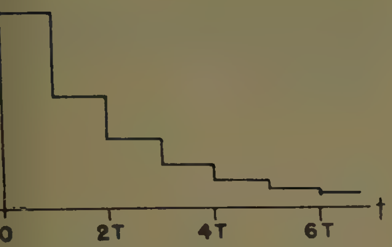


Fig. 8. Impulsive response of simple lag network



9. Staircase approximation to impulsive response of simple lag network

$$\cdot)=\frac{(1-\epsilon^{-sT}}{s}H^*(s) \quad (13)$$

$$;)=\frac{(1-\epsilon^{-sT})}{sT}\sum_{n=0}^{\infty}H\left(s\pm j\,\frac{n2\pi}{T}\right) \quad (14)$$

irst, it is evident from equation 14 that s) reduces to $H(s)$ when T approaches . The problem can therefore be ated as follows: how large can we ce T without introducing appreciable or? Fig. 7 illustrates the relations ween the magnitudes of $H(s)$, $H^*(s)$, ϵ^{-sT}/s, and $H_p(s)$, which are plotted a function of angular frequency. ω) is symmetrical about the zero fre- ncy axis. The sampled transform $(j\omega)$ is periodic and, if $H(j\omega)$ cuts off ow $\Omega/2=\pi/T$ radians per second, sists of the frequency spectrum of $j\omega$) attenuated by the factor $1/T$ and eated at intervals of $\pm n\Omega$. The nsform of the hold circuit, $1-\epsilon^{-j\omega t}/j\omega$, s as a filter on the sampled transform $(j\omega)$. $H_p(j\omega)$ is the product of $1-$ $^{\omega T}/j\omega$ and $H^*(j\omega)$.

Observe from Fig. 7 that when the npling frequency is twice the highest nificant frequency in $H(j\omega)$, $H_p(j\omega)$ proximates $H(j\omega)$ over the interval $\pm\Omega$ d that the higher frequencies in $H^*(j\omega)$ are attenuated by the filtering action of the hold circuit.

The empirical criterion thus advanced for the replacement of $h(t)$ by $h_p(t)$ to be valid is that the highest significant frequency in the spectrum $H(j\omega)$ does not exceed one half the sampling frequency $\Omega=2\pi/T$. For a low-pass element this is roughly equivalent to requiring that the sampling period T be less than one half the dominant time constant τ of the dynamic element. When this condition is satisfied, the spectrum of $H(j\omega)$ will be closely approximated by $H_p(j\omega)$, and on this basis the dynamics represented should be similar.

It should be noted that the cutoff of the frequency spectrum of $H(j\omega)$ will not ordinarily be as well defined as it appears in Fig. 7, but will usually have a long tail. This will complicate the situation somewhat; however, the above considerations will yield a tentative value of Ω or T which can be checked by deter-

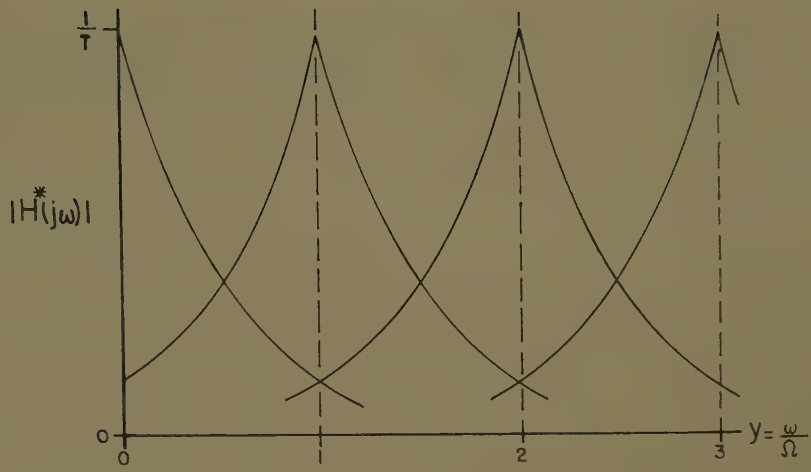
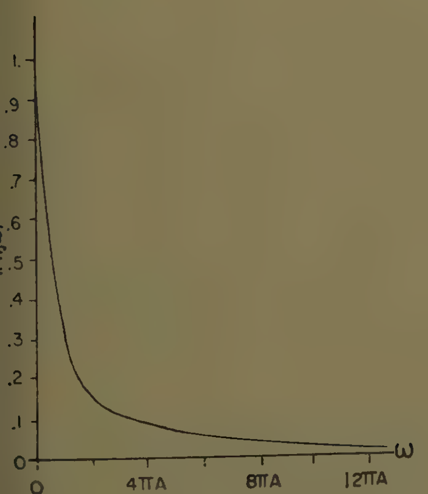


Fig. 11. Sampled amplitude frequency spectrum of simple lag network



g. 10. Amplitude frequency spectrum of simple lag network

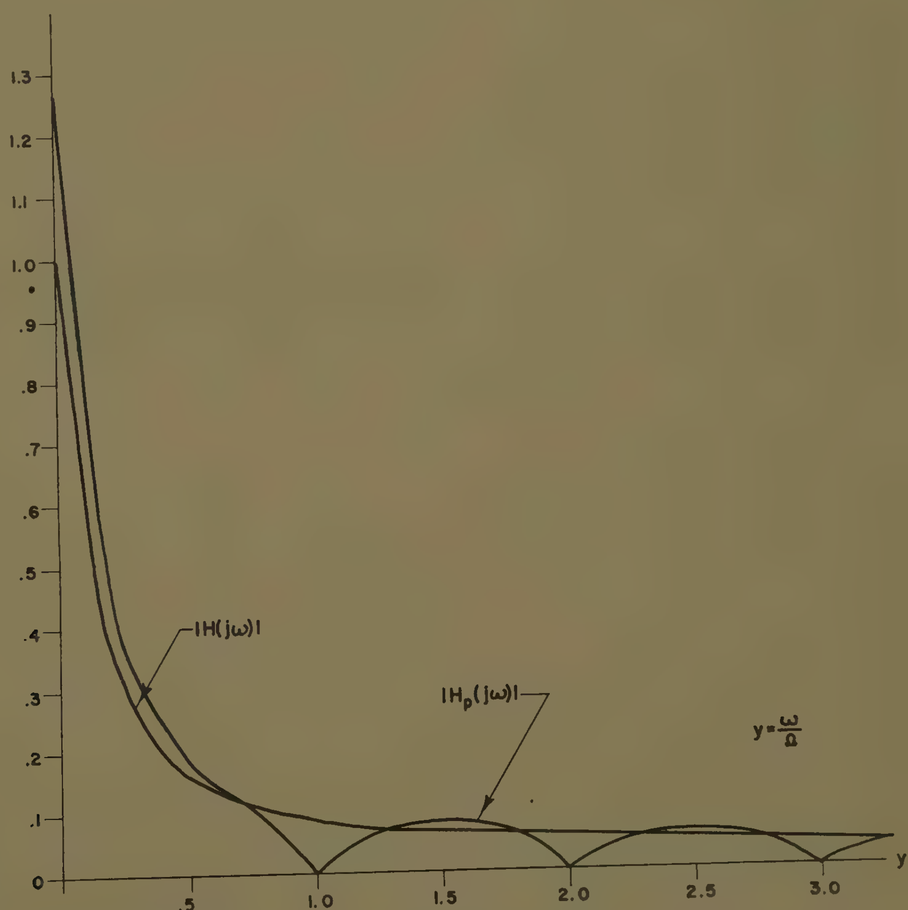


Fig. 12. Comparison of true and approximate frequency spectra of simple lag network

mining the agreement between the frequency spectra of $H(j\omega)$ and $H_p(j\omega)$ based on the tentative value. An example follows to illustrates this.

The illustration chosen is a simple lag network represented by the Laplace transfer function:

$$H(s)=\frac{A}{s+A} \tag{15}$$

The impulsive response $h(t)$ of this element is plotted in Fig. 8. From equation 15, the dominant time constant is $\tau=1/A$. A tentative value of the sampling interval equal to $T=1/2A$ is thus chosen. Fig. 9 shows $h_p(t)$ based on this value of T. The frequency spectrum $H(j\omega)$ is shown in Fig. 10, and the frequency spectrum of $H^*(j\omega)$ in Fig. 11. It should be noted that the superimposed curves most be added vectorially, since there is also a phase angle associated with each amplitude. Fig. 12 is a plot comparing the frequency spectra $H(j\omega)$ and $H_p(j\omega)$ when $T=1/2A$. The correspondence between the two curves is good, and on this basis we would say that the tentative value of $T=1/2A$ is satisfactory. The author has experimentally confirmed this; satisfactory agreement between calculated and experimental results was obtained for a system represented by equation 15. Also studied were higher-order systems which also gave good agreement between calculated and experimental results. We conclude that correspondence between $H(j\omega)$ and $H_p(j\omega)$ similar to that shown in Fig. 12 is acceptable for practical engineering calculations.

Errors

The error between the output calculated using the principle of equivalent areas and the true output is always a function of the system, the input to the system, and the value of T used. While it is possible to derive mathematical expressions which give the error as a function of these variables, it is not possible to evaluate them unless the exact form of the input is known. It appears to the writer that the considerations discussed provide the best practical guides to choosing the largest value of T which will yield satisfactory results. In the case of pulsed systems, the interval between pulses is usually the natural choice for T, since it must be small in relation to the significant time constants of the system, and the periods of the significant components of the input signal.

Principle of Equivalent Areas Restated

In conclusion, and in the light of the preceding discussion, the principle of equivalent areas is restated as follows:

The output of a linear system at the uniformly spaced sampling instants $t=nT$ to some input forcing function $r(t)$ is uniquely determined by the dynamic characteristics of the system (its impulsive response or Laplace transform transfer function) and a set of numbers I_n, based on the input, which represent the time integrals of the input evaluated over the individual sampling intervals; i.e.,

$$I_n=\int_{nT-T}^{nT} r(t)dt$$

which, given a plot of $r(t)$ versus time, is the area under the curve between $t=nT-T$ and nT. Two forcing functions which may have different instantaneous time characteristics, but which yield the same set of I_n values are said to be equivalent according to this principle, since they will produce the same ou puts at the sampling instants.

Thus, for equivalence forcing functio are required to satisfy

$$\int_{nT-T}^{nT} r(t)dt=\int_{nT-T}^{nT} r'(t)dt$$

for all n from $-\infty$ to m, where $c(mT)$ the output at the latest sampling insta in which we are interested. If initi conditions are given at $t=m_0T$ then w need only satisfy this relation for value of n from m_0 to m, since knowing the initi conditions is equivalent to knowing th inputs for $t<m_0T$, as far as the subseque behavior of the system is concerned.

The principle of equivalent areas is a approximation whose validity depen upon the characteristics of the dynam element and the input to that elemen The approximation depends upon th time interval T, and can be made as goo as desired by making T small. The be practical guide to choosing a value T which yields a satisfactory approx mation is to make $\Omega=2\pi/T$ at least twic the highest significant frequency of th input and the highest significant fre quency of the spectrum of the dynami system. A tentative value of T can b confirmed by comparing the frequenc spectrum of the continuous system $H(j\omega$ and the frequency spectrum of an appro imating system which has a staircase im pulsive response $H_p(j\omega)$, based on th tentative value of T. Fig. 12 is a example of an acceptable fit.

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Performance of Electrical Connectors at High Altitude

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HE PERFORMANCE of electrical connections at high altitudes has n reasonably well explored in recent s up to approximately 65,000 feet, ch has been sufficient for most elecal purposes in connection with aircraft. vever, the data are no longer adequate to the extreme altitude requirements present flight vehicles.

he problems of obtaining reliable innation in the altitude ranges in which are presently interested are much more icult to solve, as are also the problems trying to improve the existing pernance characteristics at these altitudes. n the range from sea level to 65,000 the pressure drops from 760 mm llimeters) Hg to approximately 43 mm . Observations of electrical breakvn sparkover and corona are not uny difficult to make and to distinguish from another in this range of pressures. wever, between 65,000 and 300,000 t the pressure drops from approxitely 42 mm Hg to approximately 5 crons Hg. For the geometry involved electrical connectors, this pressure nge encompasses a critical point in ctrical performance.

inimum Sparking Voltage

It has long been established that all ses have a minimum sparking voltage low which a spark cannot occur regards of density or spacing of the gap. is is in accordance with Paschen's v which, simply stated, says that the arkover voltage between electrodes is a nction of the density and the length of e gap separating the electrodes. As this oduct is decreased the sparkover voltage tween electrodes is decreased until the nimum sparking voltage is reached. rther decreases in the product result in creases in the sparkover voltage.

A widely quoted theoretical value for is minimum sparking voltage for air is 346 volts for uniform fields. Competent authorities state that 275 volts is an absolute minimum voltage below which sparkover cannot occur in air under any circumstances.[1]

With the type of terminations involved in electric connectors and the termination spacings applicable, this minimum sparking voltage is encountered in the range of 100,000 to 200,000 feet. The value of the minimum sparking voltage observed is approximately the same, independent of the contact spacing in the range of spacings applicable. Changes in contact spacing change the density at which the minimum sparking voltage is observed but do not significantly change its value.

Further decreases in density result in a rise in sparkover voltage which, if followed to the extreme, results in sparkover voltages many times greater than those found at sea level pressures. This phenomenon has resulted in the development of high-vacuum devices wherein dielectric strength up to 5,000 volts per mil is obtained.

This phenomenon also looks inviting to the designer of circuits for vehicles operating at altitudes where its effect can be utilized. The effects of intense radiation and other ambient conditions existing in space on these electrical performance characteristics are not generally known and are beyond the scope of this paper. Assuming, however, that good electrical performance can be obtained in the extreme low-density areas of outer space, it is still necessary to pass through the very critical area between 100,000 and 200,000 feet to attain this region. Telemetering circuits and other control circuits must function while passing through this critical range.

Test Criteria

At sea level pressures and up to approximately 65,000 feet, data on sparkover voltages and corona are easy to obtain and the criteria for breakdown are not difficult to establish; however, as the minimum sparking potential is approached, this criterion for sparkover becomes increasingly difficult to establish. It is difficult and possibly not practical to try to distinguish between corona and sparkover, as the characteristics of these phenomena are greatly changed from those at higher densities. Corona discharges at each electrode gradually appear and gradually grow larger as voltages are increased, until the glow encompasses both electrodes.

To set a criterion of the first incidence of corona, or for the extinction of corona, requires a definition of the corona detection device as the sensitivity of the detection device then becomes the controlling factor. A visual criterion may be established. Working in a dark room, discharges can be readily observed and a criterion of first visual appearance of discharge can be called breakdown. This appears to be a practical level at which to set the criterion but may not always be

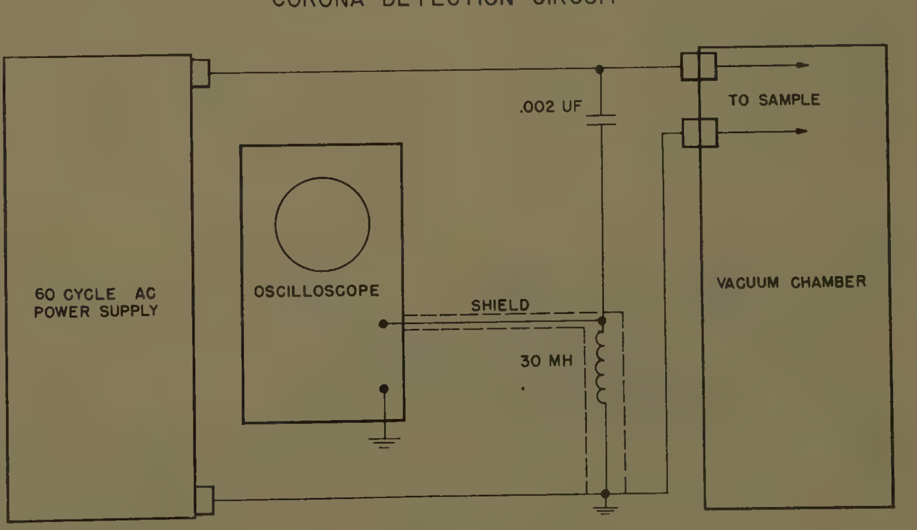

Fig. 1. Corona detection circuit

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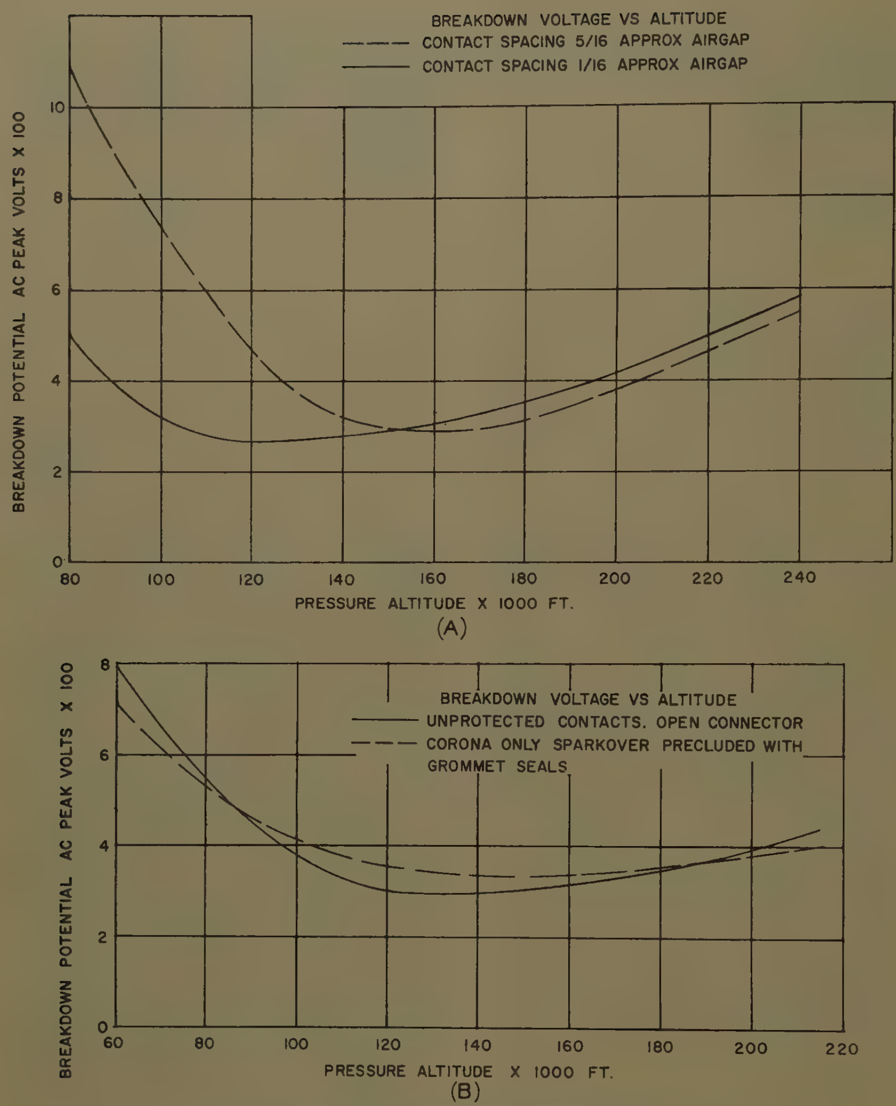

Fig. 2. Breakdown voltage versus altitude

practical for other reasons. A simple corona detection circuit with a sensitivity roughly corresponding to this visual level is perhaps a more practical tool. Such a circuit has been used by various groups and has provided useful information and good correlation among different laboratory groups. Fig. 1 illustrates such a circuit.

As the lower densities are reached and the minimum sparking potential region is passed, the characteristics of the discharge gradually change until corona and sparkover can again be separately recognized as more closely resembling the characteristics observed at sea level operation.

Fig. 2(A) illustrates data obtained on two electrical connectors of widely different contact spacing through this critical range. These samples represent approximately the maximum and minimum spacings practical in electrical connectors, and illustrates the fact that improved performance in this critical range cannot be solved by providing wider spacing between contacts. A visual observation in a dark room was the breakdown criterion used for these data.

Improving Performance

The obvious solution to the problem of improving performance in this region is to imbed the terminals in plotting compound or in resilient grommets to provide continuous insulation, thus preventing sparkover. This also requires a seal at the mating joint of the connector.

This approach is being used in many applications. However, most present designs are not sufficiently reliable for use in carrying voltages higher than those that can be carried on an open connector; rather they serve to provide moisture-proofing and freedom from contamination. A very high degree of reliability of sealing is required if voltages above those that can be demonstrated on open connectors are to be used.

If it is assumed that the approach just described, in improved form, will reliably prevent actual sparkover, the voltage rating of the connectors in many cases still cannot be increased because corona will continue to be a severe problem.

Fig. 2(B) illustrates this condition; it shows that on an imbedded connector corona can occur at a voltage approximately the same as that at which breakdown has occurred on the same connector with unprotected terminations. Visual observation in a dark room furnished the criterion for these data.

The point at which the wires enter the metal shell of the connector is a particular stress point; the conditions at this point will cause corona to occur there first. A similar condition, however, will exist any place along the wires where they are adjacent to a ground surface. Simple shielding of the cables will not materially improve this condition; to be effective the shield must be so installed that no air at reduced density can exist between the shield and the cable insulation.

A technique for successfully handling this condition has been worked out. It calls for a harness assembly with a gas-tight outer sheath sealed to all connectors. The shields are carried inside the gas-tight sheath and so terminated at the connector shells that no particular stress point

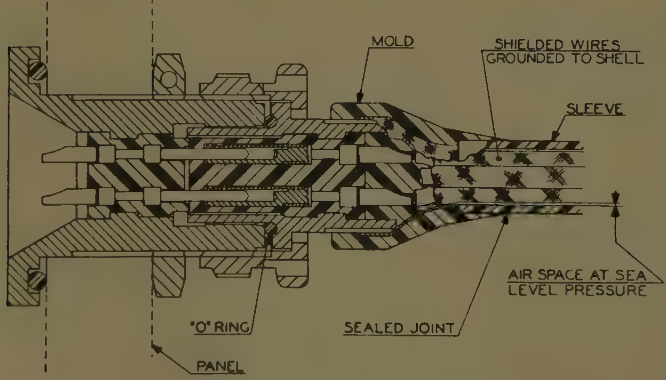

Fig. 3. Corona-resistant connector and cable assembly

ist. In plotting the contact terminations extreme care must be taken that no bubbles exist in the compound, as they ll be subject to corona. A special connector was designed to provide a reliable ain joint seal for the shell components. end interference seal of resilient dielecic insulators alone will not provide rona-free operation. All internal parts the connector must be maintained at ormal air density to control corona discharges.

Fig. 3 illustrates the design concept of his approach. This type of harness-and-onnector combination will provide corona ee operation of critical altitudes of voltges in excess of those that could normally e employed. Laboratory models of this ype have been found to be corona free hroughout this critical altitude range ith voltages as high as 1,000 volts rms -c. The corona detection circuit of Fig. was used for this work. Visual observation under these conditions is not satisfactory.

Due to the extreme care required in manufacturing, however, this level of performance is difficult to maintain reliably. With present techniques, voltages in excess of 800 volts rms should not be planned for this approach.

Conclusions

In the altitude range, of 100,000 to 200,000 feet, a critical pressure is reached at which the air gaps between wire terminations in electric connectors exhibit their poorest electrical breakdown characteristics. Electric circuits that must operate in this range in unpressurized areas can be expected to experience breakdown at voltages less than 300 volts peak unless special provisions are made.

Applying a normal safety factor, it would be well to limit operating voltage on unprotected connectors in these areas to approximately 150 volts peak.

Elimination of sparkover presents, theoretically, a relatively simple problem, but practically it is difficult to solve with sufficient reliability for good engineering practice.

Accomplishing the elimination of corona presents a more difficult problem than the elimination of sparkover but it is possible with special handling of circuits.

The effects of solar and cosmic radiation and other high-altitude ambient conditions on these electric circuit performance characteristics must be studied. Such studies, to be most useful, should be made on complete wiring systems installed as in service.

Reference

1. Fundamental Processes of Electrical Discharge in Gases (book), Leonard B. Loeb. John Wiley & Sons, Inc., New York, N. Y., 1939, chap. 10, p. 413.

Design and Development Problems of Radiation-Resistant High-Temperature-Tolerant Electronic Equipment Components

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NUCLEAR RADIATION and high temperature encountered near an operating atomic reactor reduce the lifetime of electronic control equipment. Conventional equipment acceptable for applications throughout industry cannot, in most cases, survive in the environment found in a nuclear reactor. Obviously then, to provide reliable reactor controls for long-term service, more reliable components that can withstand the influence of radiation and high temperatures need to be found.

The Aircraft Nuclear Propulsion Department of the General Electric Company (GE) is engaged in a research and development effort to produce a nuclear power system for aircraft propulsion. Space for the power plant within an air frame is necessarily limited; therefore much of the control equipment must be placed closer to the core than in any other reactor application.

Realizing that certain electronic equipment would be operating near the shield structure of their aircraft nuclear power plant, GE in 1954 initiated a study to develop components that would raise radiation and temperature barriers. By 1956, progress had been made: A 3-stage preamplifier was operated successfully in a high neutron flux exposed to a temperature of 1,000 degrees Fahrenheit. It was one of several pieces of equipment demonstrating feasibility.

Materials

The basic problem in developing this circuit was to find materials with the characteristics required for electronic equipment that could also withstand the reactor environment. The effects of high temperatures on materials, of course, have been determined over the years, but the effects of radiation have yet to be fully explored. Radiation damage varies with the kind, amount, and rate of radiation; the elemental composition of the material; the molecular composition of the material; and the volume of the material that is subjected to irradiation. The kind of radiation to which a material is subjected affects the extent of damage, but not all classes of materials are damaged by all of the energy transfer processes.

There are three interactions that cause damage; ionization, displacement, and transmutation. The effects of the interactions vary with the makeup of the materials. For instance, ionization reduces the effectiveness of certain resistors in a radiation field, but it does not appreciably affect the conductivity of wire. Displacement of atoms as the result of collision with radiating particles causes materials to become embrittled. Transmutation, which is an actual change in the nucleus of an atom introduces foreign matter that may change the engineering characteristics of the original material.

In the evaluation of materials for radiation tolerances and resistance to high temperatures, the following information was sought:

1. The temperature at which materials deform physically.

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Table I. Sensitivity of Engineering Properties to Radiation

Degree of Irradiation	Material	Result
10^{14}	Germanium transistor	loss of amplification
	Glass	coloring
10^{15}	Polytetrafluorethylene; Polymethyl methacrylate and cellulosics	loss of tensile strength
	Water and least stable organic liquids	gassing
10^{16}	Natural and butyl rubber	loss of elasticity
	Organic liquids	gassing of most stable ones
10^{17}	Butyl rubber	large change, softening
	Polyethylene; Mineral-filled phenolic polymer	loss of tensile strength
10^{18}	Natural rubber	large change, hardening
	Hydrocarbon oils	increase in viscosity
	Metals	most show appreciable increase in yield strength
	Carbon steel	reduction of notch-impact strength
10^{19}	Polystyrene	loss of tensile strength
	Ceramics	reduced thermal conductivity, density, crystallinity
10^{20}	All plastics	unusable as structural materials
	Carbon steels	severe loss of ductility, yield strength doubled
	Carbon steels	increased fracture-transition temperature
10^{21}	Aluminum alloys; Stainless steels	ductility reduced but not greatly impaired

2. The ability of materials to conduct electricity.
3. The temperature which oxidation becomes excessive.
4. The effect of prior treatment (heat treatment, mechanical treatment) on the materials.
5. The effect of nuclear radiation on chemical composition.
6. The effect of impurities.
8. The effect of surface imperfections on physical integrity.

Table I indicates the results of irradiating materials that are often used in electronic components. From the table it may be seen that the susceptibility of various materials to change as the result of irradiation varies widely. The changes noted in the table were in most cases at least 10%.

Design

After materials testing and evaluation, the preamplifier circuit shown in Fig. 1 was designed. Individual parts ultimately selected for the circuit were as follows:

1. GE ceramic single-triode tubes.
2. Platinum leads, spot-welded to the tube elements.
3. Standard 7-pin ceramic tube sockets secured with Saureisen cement.
4. Titanium–strontium resistors.
5. Phlogopite mica.
6. Alumina chassis.
7. Platinum connecting pin.
8. Single-twist gold-brazed connections.

However, before the final configuration and materials were successfully in test, material had to be substituted and fabrication techniques refined. New capacitors and vacuum-tubed resistors had to be developed. Component cases, lead confirmation, and mounting provisions had to be standardized. Better connection methods between component parts; suitable chassis and base materials; mechanical design and shock mounting provisions; and simplified assembly and testing techniques had to be worked out. Among the problems encountered was that concerning procedures for working with materials such as titanium, which had not yet been established. Welding of titanium to platinum proved trouble-some, merely because of limited exper-
ence with these materials.

The preamplifier shown in Fig. 2 had a high-impedance input to a high-gain am- plifier first stage, which was capacitiv coupled to the second-stage amplifie which in turn was capacitive-coupled to c cathode follower of low-impedance ou put.

Testing

Operating characteristics were estab lished in a high-temperature test prio to operation of the circuit in the reacto It was operated, to establish reliability, fo 115 hours at 500 degrees centigrade, a which temperature it had a gain of 1,00 and a flat frequency response from 3 t 80 kc.

When placed in the reactor, the pre amplifier operated satisfactorily for 5 hours. It exhibited the same operatin characteristics as those evident in the hig temperature test. While in the reacto preamplifier received a total radiation dos of approximately 1.5×10^{17} neutrons pe square centimeter. Operation was te minated when a tube short-circuited. Th short circuit was attributed to an electrica failure and was probably not a resu of radiation. When the short-circuite tube was replaced, the preamplifier circu was again operable.

The successful test of this circuit estab

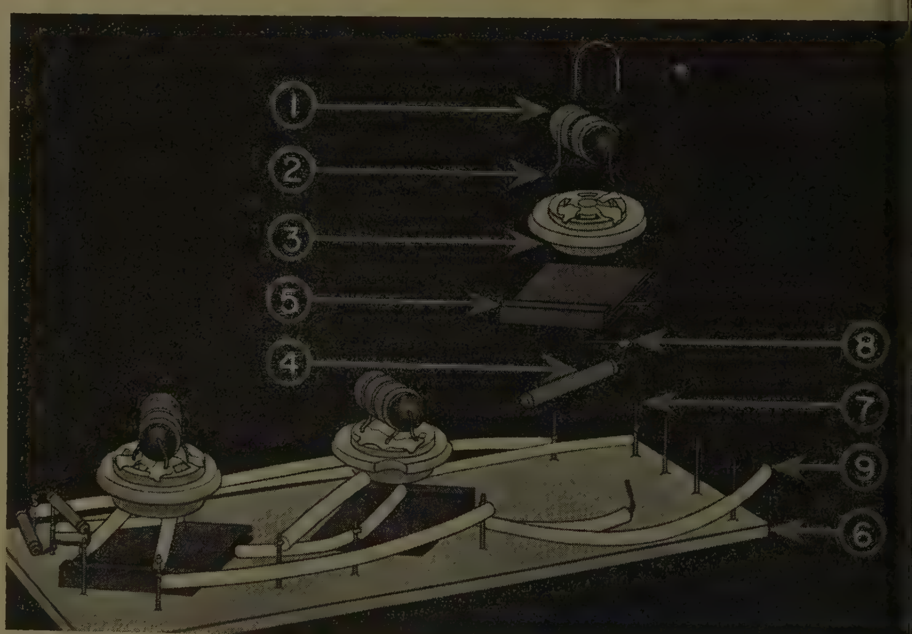

Fig. 1. Preamplifier blow-up circuit. 1—GE ceramic single-triode tubes. 2—Platinum lead spot-welded to tube elements. 3—Tube sockets standard 7-pin ceramic secured wit Saureisen. 4—Titanium–strontium composition resistors. 5—Capacitors made of phlogopit mica on inconel case and stainless steel separators (310SS). 6—Alumina ceramic chassis 7—Platinum connecting pin. 8—Single-twist connections brazed with gold. 9—Fiberglas insulation

ig. 2. High-tension radiation resistant circuit

ished the feasibility of designing electric omponents to operate in a high-tempera- ure radiation field. For the aircraft uclear propulsion application, it demon- trated that electronic components of this ature may be used advantageously with- ut space-consuming and power-consum- ng cooling equipment.

dditional Development Work

Although a success from the viewpoint f sustained operability in a reactor en- ironment, the testing of the preampli- ier circuit left problems that still require nvestigation. The use of expensive naterials such as platinum is impractical. Cheaper materials with equal or better roperties must be developed. Also it was felt that the component would be ruined if there were any vibration while the component is in the reactor.

Fig. 3. Shock mounting for circuit

To overcome the vibration problem, further research was conducted. The container shown in Fig. 3 proved quite satisfactory. The component base plate was made of a ceramic material, into which nickel tie points that extended through the base were cemented. The completed assembly was inserted into the container, which had locating tracks that provided a degree of shock mounting. However, most of the shock was absorbed by the powder that occupied the remaining space in the container. The top plate of the container was seated in a resilient gasket. The packed container underwent vibration and shock tests with no adverse effect on the assembly.

Conclusions

The need for high-temperature and radiation-tolerant components is expressed throughout the atomic energy industry. Nuclear detectors that can operate reliably at elevated temperatures are being developed as reactor technology advances and improvements in materials allow higher and higher operating temperatures.

Molecular engineering studies of the relationships between ionic, atomic, electronic, and physical-chemical factors must be fostered to obtain factual data, for further experimentation.

As is generally the case, dissemination of information in a new field can save time and money, and provide a new set of standards for applications not covered by the handbooks.

Neutral Grounding as Applied to Marine A-C Systems

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ELECTRIC POWER first became common on shipboard around the turn of the century. At that time it was solely for lighting, and the rating of the plants was, of course, very small. Since that time, and particularly within the last several decades, the growth of shipboard electric plants has been extremely rapid.

In the middle 1930's it was a rare passenger vessel which had more than 2,000 kw installed. Today there are passenger vessels being designed which will have a 10,000-kw installed capacity. In cargo ships, the growth has been even greater. The Liberty ships, designed as late as the early 1940's, have only 60 kw installed. Today there are few modern cargo ships built, either in the United States or abroad, without at least 1,200 kw installed. These increases have been occasioned by the greatly increased power requirements of the modern ship. For example, lighting levels are much higher, electronic aids to navigation are much more powerful and numerous, most modern ships are air-conditioned, galleys are almost entirely electric, electric cargo-handling equipment has almost completely replaced steam-driven winches, and machinery plants use far more motors for auxiliary drives than in the past.

Prior to World War II, commercial shipboard electric plants were almost universally d-c at 240/120 volts. In the United States, distribution systems were commonly 3-wire, with lighting at 115 volts and power equipment at 230 volts. Abroad, there were some 2-wire distribution systems, with both power and lighting at 230 volts. More commonly, however, foreign ships were built with 240/120-volt generators, power loads being supplied by 2-wire distribution at 230 volts, and lighting being supplied at 115 volts by 1-wire distribution with hull return. As the size of plants and the numbers of electric motors increased, a-c systems supplanted d-c systems, offering the advantages of less weight, less maintenance, greater ruggedness, and lower cost. Alternating-current marine plants adopted the 450/120-volt 60-cycle 3-phase system which had been selected by the U.S. Navy in 1934.

Distribution systems for alternating current have, to date, been ungrounded, 3-wire radial type with all feeders emanating from the main switchboard. The growth of the plants has resulted in increased complexity of the entire system, and in difficulty in properly co-ordinating the design.

Although predictions of future developments are often surprisingly wide of the mark, it appears inevitable that shipboard electric plants will continue to grow. This will probably result in attempts to increase voltages and frequencies

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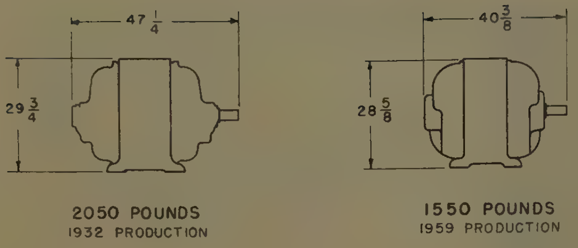

Fig. 1. Comparison of weights and major dimensions of a 150-horsepower 1,800-rpm motor showing trend toward smaller and lighter machinery

to effect weight savings in equipment and distribution cabling. Increased sophistication of equipment, particularly in electronics, will probably demand better voltage regulation at the equipment. Solid-state equipment is expected to fill this need in the relatively near future.

Present-Day Practice

Present-day marine electric plants as built in the United States are almost exclusively a-c systems, and have much in common with electric systems in industrial plants. All modern vessels and most industrial plants contain large numbers of low-voltage squirrel-cage induction motors. Power distribution in both cases is primarily by means of cables. Industrial plants may purchase their power from a utility while ships generate their own, but the stepdown transformers and substations of the industrial plant are generally equivalent to the ship's service generators and switchgear of the modern vessel. Shipboard a-c installations are fairly well standardized on the basis of employing 450-volt generating systems and 440-volt motors, whereas the land-based equivalent plant would undoubtedly employ a 480-volt system with 440-volt motors. The marine installation, being essentially very compact, can operate satisfactorily with the smaller voltage spread between generated voltage and utilization voltage. The industrial plant loads, on the other hand, would normally be more widely scattered and, consequently, would require a greater spread between transformer secondary voltage or generator voltage and the utilization voltage.

The similarities listed here are of a general nature, but a closer scrutiny of some finer details will show that there are areas in which the components of the two systems are practically identical.

The basic insulating materials employed in marine electric machinery are exactly the same as those used in industrial machines. Total operating temperatures, i.e., ambient temperature plus permissible temperature rise, are at least as high on marine machinery as they are on industrial machinery. Many marine auxiliaries operate on a continuous basis, 24 hours a day, until a voyage is completed, so it can be seen that the marine industry does not pamper the electrical insulation. On the contrary, in many instances it works the insulation to the rated limit.

Designers of rotating electric apparatus are continually striving to make more efficient use of the materials that are used in the machinery, particularly the copper and the iron. As an illustration of the reductions in size and weight that have been accomplished in the last 27 years, Fig. 1 shows the decrease in dimensions of a standard 150-horsepower 1,800-rpm motor.

Pyramiding of safety factors in the selection of motors for auxiliary drives used to be quite common but is done to only a limited extent, if at all, today since everyone is well aware of the fact that lightly loaded squirrel-cage induction motors cause low-power-factor operation.

In view of these three conditions (insulation worked up to its permissible limit, motors and alternators designed to make most efficient use of materials, and more rational application of safety factors in the selection of motors), it is appropriate that more consideration be given to the protection of modern shipboard electric equipment against abnormal voltages. One way in which increased protection can be afforded is by means of neutral grounding of the ship's service system.

A typical ship will have multiple generators, their number and rating depending on the expected electric load, available standard ratings, arrangements within the ship, and other design criteria. The generators will be 450-volt 60-cycle 3-phase machines, usually Y-connected, often with the Y point brought out to a terminal regardless of whether or not system grounding is intended. Until very recently it has been standard practice to design the system for ungrounded operation. Power from the generators will be led to one or more main switchboards, which will contain the necessary instrumentation and control facilities for paralleling of generators and monitoring the system, in addition to the necessary circuit breakers for primary distribution.

From the main switchboard, 3-phase feeders distribute power to the various distribution centers and to large single power-consuming devices throughout the ship. From the distribution centers, mains distribute power either directly to power-consuming devices, or to distribution boxes, from which submains and branches further distribute power to smaller loads. The distribution system is radial in nature, with only a single possible path at any one time from the main switchboard to the power-consuming device. Fig. 2 illustrates a radial distribution system.

Lighting systems are normally supplied 120 volts through a distribution system essentially separate from the power distribution system, but similar in arrangement. Lighting transformers rated at 450/120 volts are located at the main switchboard, or at load centers if the lighting load or ship configuration warrant it. On several recent ships these lighting transformers have been connected delta-Y, with the neutral of the secondary grounded. This has been necessitated by the need for certain rapid start fluorescent lamps to have a definite fixed voltage to ground for satisfactory operation. This is not to say that any appreciable ground current normally flows, but rather that the secondary neutral is prevented from "floating" as a result of phase unbalance.

With an ungrounded system, one of the instrumentation facilities required by the regulatory bodies is a set of "ground

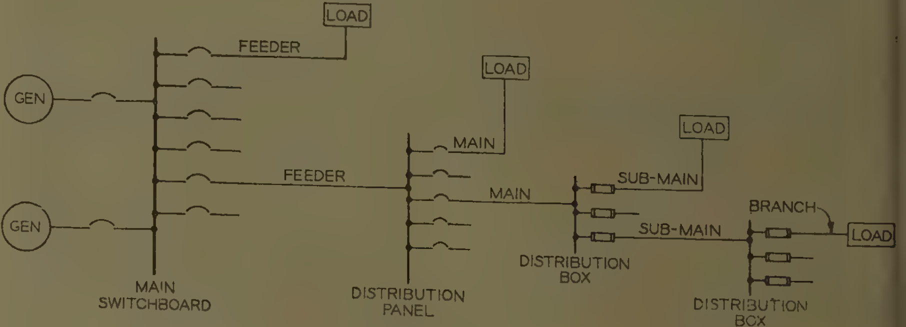

Fig. 2. Typical radial distribution system

mps," which are three small indicating ghts so connected that each lamp has cross it the potential difference between he phase and ground. If a ground occurs n one phase somewhere in the system, the oltage to ground decreases on that phase nd increases on the other two phases. The amount of change in voltage to round is dependent, of course, on the otal impedance of the ground path. This change in voltage results in a change n relative brilliance of the ground lamps, ndicating the existence of a ground, dentifying the grounded phase, and giving ome information about the impedance f the ground. Theoretically, these round lamps are regularly monitored by he crew, and grounds, when detected, quickly tracked down and eliminated. Tracing a ground, however, may often require the opening of vital circuit breakers and if the ship is at sea this may be deemed more dangerous than risking the chance of the occurrence of a second ground. In addition, human frailties being what they are, it is suspected with reasonably good evidence, that even if there is no danger involved the task will be postponed because it is irksome and time-consuming and because the existence of one ground does not noticeably affect operation. As a result, many a nominally ungrounded ship on the seas today operates a good deal of the time grounded, albeit improperly so.

To illustrate this point, the following will describe the procedure required to locate a ground fault on an ungrounded system.

First, the ground-detection lamps on the main switchboard indicate that a ground exists somewhere in the system, and also indicate which phase is grounded. The operating engineer must then:

1. Open the circuit breakers serving the various feeders one at a time, each time observing the ground lights to see when ground indication disappears. When it does, the feeder that is open at the time is the one containing the ground. It is important to realize that the man must be aware of exactly what loads are eventually served via each feeder, and must then be extremely careful of the time at which he opens each feeder, so as not to interrupt power to any service at a time when it might be vital. Even if the loads supplied are not strictly vital in the sense of being essential to the safety of the ship, there is certain to be a great deal of inconvenience involved in any power interruption. An even more arduous task confronts the operator when two or more grounds exist simultaneously on the same phase but not on the same feeder.

2. After reclosing the feeder breakers, proceed to the panel or load center supplied by the feeder identified as the one containing the ground and open each panel circuit breaker one at a time, observing precautions regarding power interruption as in item 1. Measure the resistance to ground of each phase of each main with a megohmeter until the grounded main is located.

3. If this main feeds another panel, distribution box, etc., proceed to that panel and repeat the procedure as above.

4. Eventually the location of the ground will have been narrowed down to a single power-consuming device. This equipment is then checked out in detail to pinpoint and correct the ground. It may be in a terminal box, in a controller circuit somewhere, or in the equipment itself. It is far more likely that a fault of this type will occur within the utilization equipment or its immediately associated circuitry than in the main feeder cables.

No matter how conscientious the engineer may be, he is justifiably reluctant to cause all these interruptions of power while at sea and so he will postpone this task until a more favorable opportunity exists, such as time in port.

Reasons for Grounding

Continuity

It is true that the ungrounded system has a theoretical advantage as regards continuity of service, since a single ground causes no interruption of service, and grounds are supposedly located and corrected as rapidly as they occur. In practice, however, as noted previously, there is a strong temptation to delay clearing the first ground because it is a distasteful task, and because it is causing no obvious harm to the system. Of course, the longer the single ground is permitted to exist, the greater the probability of a second ground developing. While the first ground fault remains on the system, the other two phase conductors throughout the entire system have their operating voltage stress increased 73% which makes the likelihood of another ground fault very much greater as will be explained later. When the second ground occurs, a line-to-line fault path exists through the hull, and an interruption occurs on at least one feeder, and more probably two, depending on the location of the second ground. With a grounded system, the first ground results in an immediate interruption, locating the ground and facilitating correction by the operators. In spite of the theoretical advantage of the ungrounded system, operating experience in industrial systems indicates[1] that greater continuity of service is experienced on grounded systems, since the ungrounded system's theoretical advantage is effectively negated by the common practice of ignoring the first ground. Since designers are not likely to succeed any better in the future than in the past in any attempt to alter human nature, it would appear that intentionally grounding the system is advantageous from the point of view of continuity of service.

Personnel Safety

A large number of the personnel casualties resulting from electric shock are traceable to the existence of a lethal potential on equipment enclosures or other places normally thought of as "safe." Unintentional grounding of live equipment to its enclosure, as a motor winding to its frame, will inevitably occur sooner or later. If the enclosure is effectively insulated from the ship's structure, as would be the case if it were supported on vibration mounts or on non-conducting materials, the enclosure would present a definite shock hazard to personnel contacting it. A ground system would lessen, to some degree, the hazard to personnel for two major reasons:

1. The voltage to ground with a grounded neutral system is fixed; in the case of the usual 450-volt system, it cannot exceed 260 volts. In the case of an ungrounded system the voltage may be the full line voltage of 450 volts if a ground exists on another phase elsewhere in the system, or it may be somewhere in between because of floating of the system neutral. True, the 260 volts is quite capable of killing, but under a given set of circumstances, with a certain skin, clothing, etc., resistance in the man, the lower voltage is always less likely to drive lethal currents through the man's body. Additionally the ungrounded system is subject to the possibility of very high overvoltages created by resonance, as discussed hereafter, which greatly increases the possible personnel hazard.

2. Probably more important, we are again faced with the nature of the human being. When a man thinks he is dealing with an ungrounded system, he is apt to consider himself safe so long as he contacts only one phase of a live circuit, and theoretically he is correct. However, as has been pointed out before, this is seldom the case. There may be a ground on another phase elsewhere, or even if not, it is possible that the unavoidable capacitive ground of the system may permit sufficient current to flow to harm the person seriously. This state of mind is not possible with a grounded system, as every man is aware that he can be killed by contact with *any* live conductor, and so is much more careful.

Overvoltages

Three types of overvoltages which can occur on ungrounded systems are eliminated by intentional neutral grounding.

The first of these is the type occurring when a single ground exists somewhere in the system. The normal voltage to ground of each phase is $450/\sqrt{3}$, or about 260 volts. If one phase develops a ground, the voltage to ground of the other two

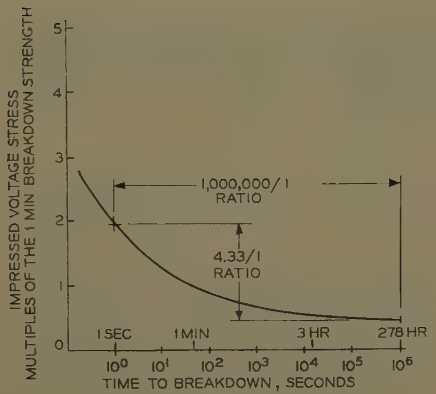

Fig. 3. Typical class A insulation breakdown strength

phases becomes full line-to-line voltage, or 450 volts. This is an overvoltage of 73%, which is not dangerously high, but which will definitely increase the probability of a second ground fault if maintained for any appreciable time. The influence of the time element is illustrated in Fig. 3, which is a plot of the impressed voltage stress against breakdown time for class *A* insulation. Note that an insulation which can withstand an impressed voltage of two scale multiples for 1 second can withstand an impressed voltage of one scale multiple for 1 minute. Similarly, the same insulation can withstand one-half multiple for a little more than 3 hours. It should be remembered that the tests illustrated in Fig. 3 were conducted under laboratory conditions and do not take into consideration such factors as the presence of moisture, dirt, or high ambient temperatures. Because of the habit of not clearing the first ground, the insulation may be subjected to this overvoltage for a large percentage of the time. This type of overvoltage is impossible on grounded systems since the voltage to ground throughout the system is firmly fixed by the intentional grounding of the neutral.

The second type of overvoltage possible on ungrounded systems is somewhat less probable, but also far more dangerous. As has been mentioned before, every "ungrounded" system is inevitably grounded through the distributed capacitance of the system. Additionally, every shipboard power system necessarily includes many inductive reactances; for example, controller coils. It is possible for a ground to occur at such a point that a series resonant circuit through ground is created, which results in extreme overvoltages to ground. These overvoltages could possibly reach 10 or 20 times normal, and some 4 and 5 times normal have, in fact, been observed. This type of overvoltage is usually characterized by numbers of simultaneous insulation failures throughout the system, with many grounds and burned-out coils, motors, etc.

Third, there is the intermittent or "sputtering" ground fault. Faults of this nature may be created by vibration causing an electric conductor intermittently to make contact with ground. Intermittent faults on low-voltage systems have been observed to create overvoltages of five or six times normal in several instances. A complete explanation of the mechanics of this type of fault is given in reference 1 and need not be repeated here.

All of these types of overvoltage can be effectively eliminated by grounding the system neutral through an appropriately selected impedance. Selection of an appropriate impedance is discussed in the following.

Methods of Grounding

Examination of the literature defining and describing grounding[2,3] practices in power systems shows that several different methods are recognized by the industry. Each one of the methods described possesses some advantages over the others when it is scrutinized for use in a particular system. The various methods are as follows:

1. Solidly grounded (directly grounded).
2. Effectively grounded.
3. Resistance grounded.
4. Reactance grounded.
5. Resonant grounded (tuned grounded).

Solidly Grounded

AIEE Standards define "solidly grounded" as follows:[3]

"Solidly grounded means grounded through an adequate ground connection in which no impedance has been inserted intentionally.

"*Note.* 'Adequate' as used herein means suitable for the purpose intended."

In industrial practice solid grounding refers to the connection of the neutral of a generator, power transformer, or a grounding transformer directly to the station ground. It cannot be considered as providing a zero-impedance neutral circuit because of the reactance of the windings of the grounded apparatus. Solid grounding of the neutral of a generator, for example, may cause the line-to-ground fault current in the faulted winding to exceed the 3-phase short-circuit current capability of the generator. Since, in normal practice, the windings of the machine are braced to withstand the mechanical forces created by the 3-phase short-circuit current, solid grounding of such a machine could result in damage in the event of a line-to-ground fault. On the other hand, solid grounding of the neutral of a transformer bank which is the only source of power for a particular system is usually correct since the reactances of a transformer will normally be such that no more current will flow during a line-to-ground fault than will flow during a 3-phase short circuit.

Effectively Grounded

AIEE Standards offer this definition of effective[3] grounding:

"Effectively Grounded. A system or portion of a system may be said to be effectively grounded when for all points on the system or specified portion thereof the ratio of zero sequence reactance to positive sequence reactance is not greater than three and the ratio of zero sequence resistance to positive sequence reactance is not greater than one for any condition of operation and for any amount of connected generator capacity.

"*Note.* When the entire system is not effectively grounded, a specified portion of the system may be said to be effectively grounded when for all points on the specified portion of the system the requirements are satisfied."

Grounding of low-voltage systems, 600 volts and below, is practically always accomplished by effective grounding. In the case of Y-connected generators, it is accomplished by the insertion of a low-ohmic-value reactor in the generator neutral circuit. The ohmic value of the reactor is determined primarily by the generator reactances and it is of such value that it limits the line-to-ground fault current to a value not in excess of the 3-phase short-circuit current. Thus, in an effectively grounded low-voltage circuit, the ground fault currents will be high. This is done deliberately for the following reasons:

1. Low-voltage switchgear invariably employs circuit breakers embodying direct-acting series-trip devices which require high currents for successful operation.

2. Correctly selected low-voltage circuit breakers are extremely fast in operation and clear the circuit before any significant damage can result from fault currents within their ratings.

Resistance Grounded

"Resistance grounded" is defined as follows:[3]

"Resistance grounded means grounded through impedance, the principal element of which is resistance.

"*Note.* The resistance may be inserted either directly in the connection to ground, o

dividually, as for example, in the secondary a transformer, the primary of which is nnected between neutral and ground, or in ries with the delta-connected secondary of Y-delta grounding transformer."

Resistance grounding of generators will, course, be accomplished by the insertion one or more resistors in the path between the generator neutral and ground. is chiefly employed in the medium-voltge (2.4-kv to 15-kv) range. In this nge the circuit breakers are not equipped ith direct-acting series-trip units and, onsequently, they do not require highagnitude currents. The circuit breakers re slower in operation than the low-voltge type. Therefore, it is considered esirable to restrict the amount of ground ult current in order to limit the amount f damage done prior to the opening of the ircuit breaker. Grounding resistors as ormally used restrict the ground fault urrent to a value somewhere between 5 nd 20% of the magnitude of the 3-phase hort-circuit current of the machine or ystem. The actual current value is etermined by the requirement of the ystem relays. The resistors dissipate nergy at a high rate during a ground fault. hey do cause the elevation of the system eutral above ground potential during a round fault, and because of this the lineo-ground voltage of the unfaulted phases s practically the line-to-line voltage of the ystem. Resistance grounding, therefore, ecause it is primarily designed to fill a articular need in medium-voltage intallations and because it does not keep he system neutral particularly close to round potential during fault conditions, s not attractive for shipboard use.

Reactance Grounded

"Reactance grounded" is defined in the ollowing way:[3]

'Reactance grounded means grounded hrough impedance, the principal element of vhich is reactance.

'*Note.* The reactance may be inserted ither directly, in the connection to ground, r indirectly by increasing the reactance of round return circuit. The latter may be one by intentionally increasing the zeroequence reactance of apparatus connected o ground, or by omitting some of the possile connections from apparatus neutral to round."

This definition is, of course, a perfectly eneral one similar to the definition given or resistance grounding and it technically mbraces the effective grounding of lowoltage generators as previously described s well as the resonant grounding prinriple sometimes employed in high-voltage ystems and which is described later. Effectively then, low-reactance grounding has been covered under the heading "Effectively Grounded." High-reactance grounding in a special form will be covered under the heading "Resonant Grounded." Except for these two cases, reactance grounding is not employed in industry for the simple reason that if high values of reactance are employed, high overvoltages will be developed in the system in the event of a ground fault. Since one of the primary reasons for grounding, is to prevent, as far as practicable, the creation of the high overvoltages on the system, the use of high-reactance grounding defeats its own basic purpose.

Resonant Grounded

Again, by AIEE Standard this is defined[3] as follows:

"Resonant grounded means reactance grounded through such values of reactance that, during a fault between one of the conductors and earth, the rated frequency current flowing in the grounding reactances and the rated-frequency capacitance current flowing between the unfaulted conductors and earth shall be substantially equal. In the fault these two components of the fault current will be substantially 180 degrees out of phase.

"*Note.* When a system is resonant grounded, it is expected that the quadrature component of the rated-frequency single-phase-to-ground fault current will be so small that an arc fault in air will be self-extinguishing."

Resonant grounding, or ground fault neutralizer grounding as it is often called, is a comparatively rare and relatively special type of grounding employed primarily in systems rated at 15 kv or higher and where faults through air are the predominant type of fault encountered. Briefly, it consists of a specially selected high value of reactance connected between the system neutral and ground. In operation, a line-to-ground fault impresses line-to-neutral voltage on the reactor and causes an inductive current to flow. When the reactor or neutralizer is properly tuned to the system the current that flows through the reactor will be approximately equal to the result of the charging currents of the two unfaulted phases. The two reactive components of the current will be approximately 180 degrees out of phase and hence will neutralize each other. The remaining fault current is due primarily to circuit resistance and hence is in phase with the driving voltage. When the voltage goes through zero the current will also go through zero and any arcs on the system will extinguish themselves.

The ground fault neutralizer is not suitable for shipboard electric systems since it is basically intended for use in systems where the insulating medium is self-healing and ready for service as soon as the arc has been extinguished. Furthermore, the neutralizer has to be quite accurately "tuned" to the system. This, in effect, prohibits the switching of any sizable sections of the plant or else it requires that the reactor be retuned each time any significant portion of the circuitry is connected or disconnected.

The characteristics that result from each of the various grounding methods described above are summarized in brief and contrasted with the system characteristics of an ungrounded system in Table I.

From this review of grounding practice in industry, the particular grounding method best suited to marine application, can be selected with, of course, the salient objectives of grounding in mind. These are

1. The automatic segregation of the faulted zone to eliminate the painful, time-consuming work that is now necessary with the

Table I. System Characteristics with Various Grounding Methods[3]

	Ungrounded	Essentially Solid Grounding: Solid	Essentially Solid Grounding: Low-Value Reactor	Reactance Grounding, High-Value Reactor	Ground Fault Neutralizer	Resistance Grounding, Low Resistance
Current for phase-to-ground fault in per cent of 3-phase fault current	Less than 1%	Varies, may be 100% or greater	Usually designed to produce 25% to 100%	5% to 25%	Nearly zero fault current	5% to 20%
Transient overvoltages	Very high	Not excessive	Not excessive	Very high	Not excessive	Not excessive
Automatic segregation of faulty zone	No	Yes	Yes	Yes	No	Yes
Remarks	Not recommended due to overvoltages and nonsegregation of fault	Generally used on systems (1) 600 volts and below and (2) over 15 kv		Not used due to excessive overvoltages	Best suited for high-voltage overhead lines where faults may be self-healing	Generally used on industrial systems of 2.4 kv to 15 kv

present ungrounded systems in the event that a ground fault has developed.

2. The elimination or reduction of the overvoltages which exist on ungrounded systems when a ground fault is present.

Recommended System

Reference to Table I indicates that the only two methods of grounding which will give the desired results and which are applicable to low-voltage systems are the solidly grounded and the low-reactance grounded methods, both of which come under the more modern category of effectively grounded. Because the power supply of a ship is derived from generators and not from large transformers, it is possible to ignore the solidly grounded method and concentrate on the one remaining choice, an effectively grounded system in which the generator neutrals are grounded through a low value of reactance.

Table II contains representative values of the significant reactances of each of five different ratings of typical marine generators. All values in the table are in per-cent ohms per phase related to machine kva. It should be noted that the widest variation of a particular constant occurs in the zero-phase-sequence reactance X_0. The zero-phase-sequence reactance of a synchronous alternator is primarily a function of the winding pitch of the machine and it becomes a very low value with a two-thirds pitch. Deviation from this particular value of pitch increases the value of X_0.

The value of X_0 becomes important when grounding the neutral of the generator since it has a great deal of influence on the magnitude of the current that can flow in the event of a ground fault on the system. It has no influence on the magnitude of a 3-phase short-circuit current.

The equation for calculating the 3-phase short-circuit current of a generator is

$$I\text{ (3-phase)}=\frac{E}{X_1} \tag{1}$$

where

E = line-to-neutral voltage
X_1 = positive-sequence reactance of generator in ohms per phase

(*Note:* A synchronous machine has several different values of positive-sequence reactance. Since we are interested in the maximum value of short-circuit current, we employ the lowest value of reactance in this equation. This is the generator subtransient reactance.)

The expression for line-to-ground fault current of a grounded neutral generator with no external reactance in the circuit is as follows:

$$I_G=\frac{3E}{X_1+X_2+X_0} \tag{2}$$

where

X_2 = negative-sequence reactance of generator in ohms per phase
X_0 = zero-sequence reactance of generator in ohms per phase

By using the constants of any one of the machines listed in Table II, it is found that the line-to-ground fault current will considerably exceed the 3-phase short-circuit current. The 600-kw machine will be considered as an example. In order to use the reactance values from the table and substitute them in equations 1 and 2, it is first necessary to convert them from percentage values to actual ohms per phase. This is done by use of the equation:

$$X\text{ (ohms)}=\frac{X(\%)\times \text{kv}^2\times 10}{\text{Base kva}}\quad\text{(in ohms per phase)} \tag{3}$$

thus

$$X_1=\frac{19.9\times 0.45^2\times 10}{750}=0.0538\text{ ohm}$$

$$X_2=\frac{23.3\times 0.45^2\times 10}{750}=0.0628\text{ ohm}$$

$$X_0=\frac{2.93\times 0.45^2\times 10}{750}=0.0079\text{ ohm}$$

Substituting in equation 1,

$$I\text{ (3-phase)}=\frac{260}{0.0538}=4{,}830\text{ amperes}$$

Similarly

$$I_G=\frac{3\times 260}{0.0538+0.0628+0.0079}=\frac{780}{0.1045}=7{,}460\text{ amperes}$$

Since the windings of a 3-phase alternator are braced to withstand the stresses occasioned by the 3-phase short-circuit current, it is easy to visualize the damage that might occur if the machine were subjected to the line-to-ground fault current.

Reactance can be easily inserted in the neutral path to limit the line-to-ground fault current and under these circumstances equation 2 becomes:

$$I_G=\frac{3E}{X_1+X_2+X_0+3X_n} \tag{4}$$

where

X_n = the reactance of the neutral reactor in ohms

Equating equations 1 and 4 and solving for X_n gives:

$$X_n=\frac{2X_1-X_2-X_0}{3} \tag{5}$$

Substituting the constants of the 600-kw generator in equation 5 gives:

$$X_n=\frac{2(0.0538)-0.0628-0.0079}{3}=0.0123\text{ ohm}$$

Calculation of the current rating of the neutral reactor involves the use of system reactances as well as generator reactances and is adequately covered in chapter 6 of reference 1. The current rating obtained in this fashion is a thermal current rating and it defines an rms neutral current in amperes which the reactor will carry under certain standard conditions for a specified length of time without exceeding the defined temperature limitations. It is assumed to be of constant value during the rated time. Neutral grounding reactors are available with 10-second ratings, 1-minute ratings, and 10-minute ratings. Reactors with a 10-second rating are normally used in industrial distribution systems and will be more than adequate for marine systems.

Since the typical marine vessel is equipped with at least two main generators, one of which is normally a stand-by unit, and any combination of these generators may be in use at some particular time, it will be best to equip each generator with its own neutral grounding reactor. By so doing the value of reactance in the neutral path will always be correct regardless of the number of generators in service.

The only significant difference, then, between an ungrounded system and a neutral grounded system is that the grounded system requires that a neutral point be made available on each genera-

Table II. Typical Values of Reactance for Marine Generators

Reactance, %	Generator Ratings (All 450 Volts, 3-Phase, 60 Cycles, 0.8 Power Factor), Kw 400	500	600	750	1,250
Positive sequence:					
Subtransient X_d''	14	11.3	19.9	16.5	20.0
Transient X_d'	24.5	19.0	33.2	21.8	24.0
Negative sequence X_2	16.1	12.9	23.3	21.1	26.0
Zero sequence X_0	0.686	0.72	2.93	4.57	2.5

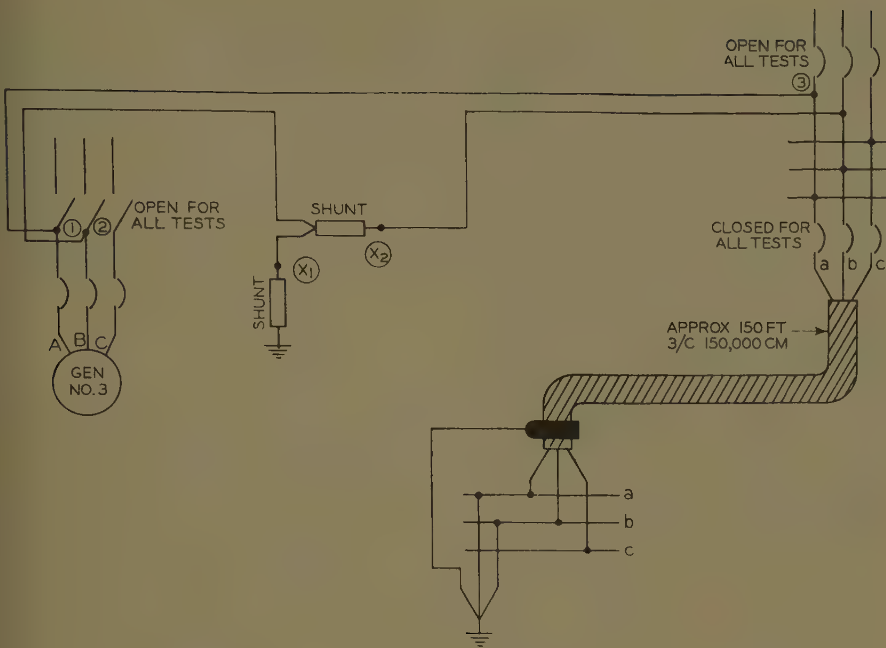

Fig. 4. Schematic arrangement of test setup on Submarine Tender "HW Gilmore"

tor and that a low-ohmic-value reactor be connected between this neutral point and ship's hull.

For purposes of complete isolation and, incidentally, to facilitate insulation resistance measurements on the generator itself, the installation of a disconnect link in the connection between the generator neutral and the reactor is recommended in order that the machine may be completely isolated when necessary.

The connection to the structure of the ship, of necessity, should be well made and in a location that will facilitate ready inspection. The connection stud, or studs, must be well bonded to a main member of the ship's structure. In the event that the ship's generators and their associated reactors are reasonably close together, then, for simplicity's sake, it would be logical to use but a single connection to the hull as the ground point. On the other hand, if the generators are fairly widely separated as they would be in the case of a ship with two engine rooms, then the generator or generators in each engine room should be grounded to a stud in each room. Since the fault current originating in a particular generator has to return to that generator there is certainly no advantage in creating a single ground point for all generators regardless of their location. To do so will result in unnecessary lengths of ground cable.

It should be appreciated at this point that, when a ground fault occurs, the return current will flow in a path which parallels as closely as possible the outgoing power conductor. This is due to inductive reactance. Theoretically, then, the metallic armor of the usual marine cable would provide an ideal return path. The armor, however, is made up of a large number of strands of fine wire having relative high resistance and, therefore, does not constitute an adequate return path. This apparent shortcoming does not create any problem in a shipboard installation because the cables are everywhere in close proximity to the metallic structure of the hull which provides an excellent return path, as might be expected.

To sum up the recommendations contained in the preceding paragraphs, they are briefly as follows:

1. Ground the neutral of each generator through a low-ohmic-value reactor.

2. Bond the grounding connection or connections solidly to a main structural member of the ship.

3. Provide a means for disconnecting the generator neutral from the associated reactor when it is desired to isolate a machine for any reason.

Summary of Tests

Various tests have been conducted by several interested parties in attempts to obtain reasonable values of hull impedance. This has been either to support or allay fears that hull impedance may be sufficiently high to limit line-to-ground fault currents in a grounded system to values too low to actuate protective devices, thus resulting in a dangerous situation. The test results, although in some instances slightly inconclusive and debatable, clearly indicate that hull impedance to a fault current is sufficiently low as to cause no concern. The tests are summarized in the following.

1. Tests on board the Submarine Tender *HW Gilmore*, June 16 and 17, 1959:

One of the ship's generators rated at 500 kw, 450 volts, 3 phase, 60 cycles, was employed for this series of tests. One of the three generator terminals was disconnected, one was permanently connected to the end of one conductor of the 3-conductor steering-gear feeder cable, and the remaining generator terminal was arranged so that it could be connected either to ground or to a second conductor of the feeder cable, or to both. In the steering-gear compartment, the ends of the feeder cable were all tied together and connected to ground. With this scheme of connections it was possible to send current out over the one conductor of the feeder and have it returned on another conductor, *or* the current could be returned via the steel structure of the ship, *or* the return path could be made to consist of the steel structure of the ship in parallel with one of the feeder cable conductors. These connections are illustrated in Fig. 4.

Values of current ranging from 423 to 3,475 amperes were obtained in a series of six different tests. The results of all of the tests indicated that the return path to the structure of the ship was of low impedance. From this series of tests an average impedance of $0.010+j\,0.035$ was obtained. This can be expressed as $Z=0.035$ at an angle of 72 degrees. Essentially, this means that the hull return-path impedance will have very little effect on the magnitude of a fault current and, conversely, indicates that there will be ample current to cause the tripping of the protective device in the faulted area

2. Tests on board a tanker at Quincy, Mass. (Bethlehem Steel Corporation) May 19, 1959:

A series of tests were made on board a partially completed tanker. Various sources of power were used such as welding transformers and soldering transformers. Current was sent over several configurations of cable and returned through the ship's hull structure. The results obtained from these tests indicate a hull impedance ranging from 0.020 ohm for low-current values to 0.027 ohm for higher currents on tests made with 190 feet of cable and 170 feet of hull return-path circuit. Similar tests were also conducted using 470 feet of cable and a hull return path approximately 470 feet long.

As should be expected, the indicated hull impedance increased. Hull impedance ranged from 0.0272 to 0.0318 ohm.

These tests, conducted at low to moderate values of current, essentially confirmed the tests made on the *Gilmore*.

Conclusions

1. Neutral grounding can easily be accomplished and, if done in accordance with recommended method, will result in the effective elimination of the sustained overvoltages that result from continued operation of "ungrounded" systems containing grounds.

2. There is no more probability of accidental ground on a grounded system than on an ungrounded system. During the life of a ship there should be fewer grounds on the grounded system than on the ungrounded system because of the reduction of strain on the insulation due to elimination of protracted overvoltage.

3. Based on tests that have been made, the hull return-path impedance of a ship will be low and will cause no significant reduction in fault current. In this connection it should be noted that, regardless of whether the system is grounded or ungrounded, there will be a reduction or attenuation of fault currents at the end of feeders due to the impedance of the feeders themselves. Also, the use of the hull as a path for fault current is not essentially different from the case on an ungrounded system wherein a ground occurs at each of two widely separated points in the system.

4. Faulted circuits will be automatically isolated with a minimum of delay, usually of the order of 2 or 3 cycles. The irksome, time-consuming, and often postponed task of hunting grounds will be greatly reduced.

5. There will be an effective reduction in shock hazard to personnel as a result of neutral grounding.

6. Continuity of power supply will be improved by neutral grounding.

References

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2. Grounding of Industrial Power Systems. *AIEE Publication No. 953*, Oct. 1956.

3. Neutral Grounding Devices. *AIEE Standard no. 32*, May 1947.

(See p. 352 for Discussion)

Advantages of Ungrounded Marine Electric Systems

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FOR MANY YEARS there has been a great deal of discussion and interest expressed concerning the possibility of grounding the neutral of marine electric systems. It is the purpose of this paper to consider the numerous theoretical and practical considerations accumulated over the years with regard to industrial and electrical utility plants[1–15] from the viewpoint that the ungrounded system is superior. In addition to a review of the literature, some field test work has been performed.

Almost universally, American ships presently use a 450-volt 3-phase 60-cps (cycle-per-second) 3-wire system with a secondary lighting system(s). The chief requirements of these systems are balanced characteristics of:

1. A low initial cost of electric plant while supplying the required loads.

2. A low secondary cost as reflected by elimination of unscheduled shutdowns leading to demurrage.

3. Low indirect cost arising from the use of a large volume of the ship which might instead be used for carrying cargo or meeting some other primary purpose of the ship.

The required loads, in order of priority, are communication and lighting, propulsion, auxiliary fans and pumps, and other auxiliaries such ascargo winches, capstans, ventilating blowers, and hotel equipment. While it is desirable to have an electric plant with a relatively low level of electrical maintenance, it should be remembered that the unique degree of autonomy of a ship with its complete electric system requires the presence of electrical maintenance personnel onboard ship. Therefore, system maintenance which does not require replacement of parts and does not necessitate demurrage is not as expensive as an industrial plant which may have to call in outside maintenance help for any but the simplest electrical work.

In considering the advantages of an ungrounded system, it is necessary to assume a particular grounded plant design, since none of any appreciable rating have been built in the United States to date, for purposes of comparison. Two possibilities could be considered for the 450-volt circuit: 3-phase 4-wire or 3-phase 3-wire systems. It is assumed that the 4-wire system can be eliminated from consideration as a main power system since the increased connection and equipment costs would far outweigh any advantages gained from the ungrounded system.

The lighting, communication, and other low-voltage equipment would use systems supplied through transformers using either hull or copper return.

3-Phase 3-Wire System Considerations

The following factors have been mentioned from time to time in discussing the advantages and disadvantages of the ungrounded 3-phase 3-wire system.

1. Overvoltages.
2. Fault location.
3. Safety.
4. Continuity of service.
5. Number of trip elements.
6. Difficulty of properly grounding.
7. Expense.

Overvoltage

The steady-state overvoltage condition which can occur on an ungrounded system arises from the parameters of the circuit illustrated in Fig. 1 (grounded system obtained by connecting Z_G to ground). Typical values of the leakage resistance and capacitance per phase are shown in Table I.

As an extreme case, a fault at x will cause the voltage to ground to be

$$E_{L1G} = \text{zero}$$

$$E_{L2G} = \sqrt{3}\, E_{L2N} = E_{LL}$$

$$E_{L3G} = \sqrt{3}\, E_{L3N} = E_{LL}$$

In a less extreme case the leakage resistance and line capacitive reactances R_N and X_N may be unbalanced because

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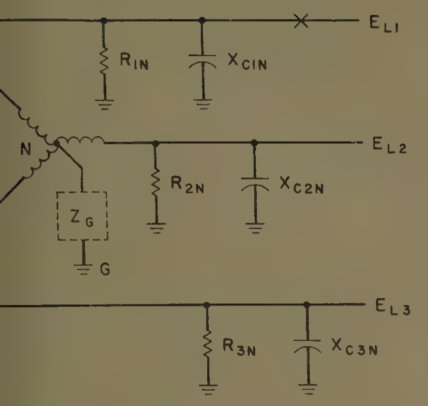

g. 1. Ungrounded 3-phase system with neutral grounding, Z_G, shown dotted

differences in length of cable runs, some symmetrically connected loads, or mply variation in insulation leakage.

In a typical case the latter effect is of practical importance unless the leakge becomes a fault. For instance, a pical case is

$$_c=\frac{1}{j2\pi\times60\times10^{-6}}=-j2{,}500 \text{ ohms}$$

hould the leakage resistance at some one oint in the insulation system fall as low s 2,500 ohms, which would be necessary r appreciable unbalancing, there would esult 27 watts or more being developed t the leakage point. This will lead to a emperature rise of the insulation in either ne grounded or ungrounded insulation ystems which will cause further deterioraon of the insulation (in 10 seconds one ould expect the insulation to rise approximately 2,000 degrees Fahrenheit bove ambient leading to burning of the ault clear or grounding).

As a generalization, then, one can state hat steady-state voltage stress on groundvall insulation may be increased from 260 olts up to 450 volts maximum.

Several causes of overvoltage under ransient conditions are possible. One of hese is a resonance (which under some onditions might approach steady state) f an inductance such as a contactor coil, enerator winding, or transformer coil in eries or parallel with the distributed apacitance of the line. (The possibility of traveling-wave resonance is not even a theoretical possibility because the frequency for a traveling-wave transient would have to be higher than 800 kc. At this frequency the damping of the circuit is such that this type of transient will be suppressed.) This inductive-capacitive reasonance may be induced as a simple linear or ferroresonance phenomenon. The various types of forcing functions which may cause this transient response are: (1) an arcing ground, (2) step change in line voltage arising from sudden grounding of a floating line, and (3) basic 60-cps line frequency of generator.

able I. Typical Marine Plant Capacitance and Leakage Resistance to Ground

Generating Capacity, Kw	Line-to-Ground Capacity per Phase, Microfarads	Line-to-Ground* Leakage Resistance, Ohms
400	1	18,000
1,250	8	1,700

Lowest measured leakage resistance per phase n any system is 6,000 ohm-microfarads.

Assuming 1 microfarad of capacitance (400-kw plant) and 100 henrys of inductance, this would give an undamped natural linear resonant frequency of 17 cps. The response of the circuit would depend, of course, on the amount of damping in the system. A commonly heard statement is that for such a system "theory" indicates a possibility of as much as six times normal voltage on the system. The damping used as a basis for this statement is not known. It is believed that realistic prediction of the maximum voltage under these conditions is virtually impossible because of the large variations in resistance in the infinity of possible circuit combinations. However, the results for an unloaded system with the transformer grounded are shown in Table II.

Step changes in supply voltage and restriking grounds can theoretically lead to results similar to those discussed for ferroresonance. It is believed that these effects are academic.

Another possible source of transient overvoltage is the "switching transient." Air (and other gases) exhibit a characteristic of constant voltage drop independent of current over a wide range of currents. However, for an enclosed or semi-enclosed volume of gas the pressure will increase. The voltage drop under these conditions increases approximately as the square root of the pressure. One of the techniques used in designing switches, circuit breakers, and fuses consists of generating a large arc drop by a pressure rise. It is possible that such a device misapplied or partially damaged by prior use, such as a fuse in which some of the parallel elements have been partially melted by a previous overload, may generate an excessive back voltage which might cause insulation breakdown. It should be emphasized that this phenomenon can be avoided by the use of properly applied circuit-switching devices and avoidance of fuses.

A similar pressure-rise phenomenon can also occur in a fault itself which may develop in an enclosed volume. The inductance of the circuit will prevent an instantaneous change in current while the fault arc drop may build up to a large value because of pressure build-up. Typical values are 1,000 volts per inch of arc length at 1,000 psi (pounds per square inch).

Because of the large number of causes of transient voltage which may theoretically exist and the large variation in, and number of, relevant circuit parameters, it is desirable to examine experience closely and make field tests. Table III gives the results of a series of field measurements made by the Bureau of Ships. Similar tests have been made on a large commercial ship. These latter tests revealed no cases of overvoltage.

In addition to the foregoing data, it

Table II. Ferroresonance Overvoltage Due to Grounding of Unloaded Transformer on Unloaded Electric System

Maximum Phase Volts (Per Unit of Normal 260 Volts)	Transformer Rating, Kilovolt-Amperes	System Capacitance, Microfarads
1.8	37 1/2	1
3.8	37 1/2	8
2.0	7 1/2	1
3.8	7 1/2	8
3.8	0.1	1
1	0.1	8

Assumptions:

1. 130% saturation level of transformer.
2. Circuit leakage resistances as in Table I.
3. Average transformer losses are assumed.

Table III. Voltage Surges to Ground Measured on Aircraft Carrier "Hornet"*

Date, 1954	Indicated Transient Peak Volts†	Equivalent Rms Voltage
4/15	Recording equipment put in operation	
4/15	1,500	1,060
4/16	1,000	707
4/22	1,000	707
6/17	1,000 (six separate occasions)	707
6/18	1,500 (two separate occasions)	1,060
6/19	1,000	707
6/28	1,000	707
7/16	1,500	1 060
7/27‡	2,500	1,770
7/30	1,000	707
8/2	2,500	1,770
9/1	1,000	707
9/2	1,000	707
10/15	2,500 (two separate occasions)	1,770

* Ship's generating plant consists of four 1,250-kw turbine-driven generators. There are four main distribution switchboards.

† Electrical plant operating records indicate transients occurred about the same time as plant changes were being made.

‡ Occurred at the same time as a single-pole generator disconnect switch was inadvertently opened under load.

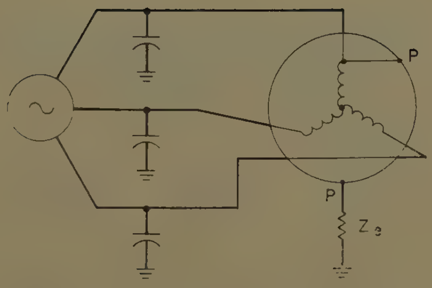

Fig. 2. Circuit with ineffective equipment ground

should be noted that the field trouble records of the Westinghouse Electric Corporation or the Navy do not record any instance of trouble with motors attributable to overvoltages. (In the case of Navy trouble reports, only 1/4% of installed and operating motor failures for 1955 through 1958 have been unexplained.)

An important practical consideration in discussion of overvoltage is the fact that 450-volt equipment is tested for dielectric strength at 2,200 volts rms. And with practical dielectric strengths of 2,000 volts per milli-inch of insulation, mechanical rather than electrical considerations determine insulation thickness. Therefore, in the case of 450-volt equipment, the insulation to ground is many times any recorded transient overvoltage.

It also seems pointless to discuss the adverse effect of aging of insulation because of increase in voltage since the life of the equipment is so grossly overdesigned from the electrical viewpoint. Thus, a common ground wall may be 0.010 inch which would support 20,000 volts for approximately 20 years.

Fault Location

In an ungrounded plant the first ground does not cause a fault current other than line-capacity charging current. (For 8 microfarads on a 1,250-kw plant there would be approximately 3 amperes charging current.) Normal procedure is to locate and remove this ground after annunciation by ground indicator light. The process of locating this ground can be simplified by using a sufficiently low impedance grounded system. In this case, in a properly coordinated system, the large fault current would cause tripping of the faulted section of the line.

Safety

With respect to safety of equipment, it appears that the ungrounded system has several advantages. First, there will be fewer faults with large fault currents on an ungrounded system. Fault current damage, even in properly protected circuits, may be appreciable. Thus, a fairly moderate 24,000-ampere fault current may melt 2 pounds of metal in a piece of electric apparatus. This same fault current may lead to pressures on the order of 1,000 psi generated in the equipment. These are destructive levels which it is best to avoid, if possible.

In the case of personnel, both grounded and ungrounded systems have sufficient voltage and low enough impedance to deliver well over the lethal level of current to a person touching one conductor while in contact with ground. It is not true, as sometimes argued, that the ungrounded system will not shock a person. The distributed line capacity is more than sufficient to give a lethaly low impedance.

It is also sometimes argued that the ungrounded system is unsafe since an inadvertent ground may cause a dangerously high voltage to exist on the equipment case if the equipment ground is in poor condition. Figs. 2 and 3 illustrate these conditions. Unfortunately, if Z_e is large enough (on the order of 100 ohms on a 1,250-kw plant) to cause E_{PG} to be dangerous, then there would be insufficient fault current to cause tripping of a grounded system. Thus, this danger is equally likely on either system.

There is one other respect in which the floating system is safer. If, in the grounded system an accidental short circuit to ground is caused by an electrician's screw driver, for example, he may be badly burned by the resulting arc. In the ungrounded system no more than 3 amperes (1,250-kw plant) line-charging current would result. This would not be dangerous.

Continuity of Service

In the floating system the first ground does not cause tripping of that section of the line unless the capacitive current leads to a 2- or 3-phase fault, in which case the damage will be no worse than for the ground system, or in case this inadvertent grounding should cause overvoltage failure of other equipment as discussed above.

As a worst extreme, then, if the electrician never removes the first fault, there will only be the minor improvement in operating time gained between the first and the second fault. On the other hand, in the case of conscientious maintenance, the floating system will reduce the inoperative time as the product of the probabilities. Thus, a grounded system inoperative 0.1% of the time would be inoperative only 0.00006% of the time if it were an ungrounded system.

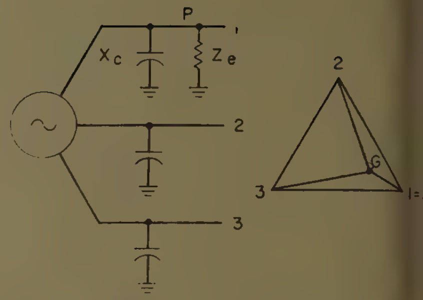

Fig. 3. Equivalent with ineffective equipme[nt] ground Z_e

Number of Trip Elements

Although some manufacturers do [no]t build some types of 450-volt breakers an[d] motor controllers with trip elements [o]n all three poles, there are enough speci[al] industrial cases which require grounded 450-volt systems that most manufacture[rs] are developing 3-element units.

Difficulty of Properly Grounding

In considering the advantages of th[e] ungrounded system, it is difficult to antici[-] pate the positive disadvantages of th[e] grounded system since, to the author[s'] knowledge, no grounded marine 60-cyc[le] 3-phase systems of appreciable size f[or] normal service have ever been built in th[e] United States.

Therefore, it is necessary to try [to] extrapolate from industrial and foreig[n] maritime experience with grounded sy[s-] tems in addition to attempting to antici[-] pate on a theoretical basis. In doing thi[s] it is well to keep in mind several signi[fi-] cant differences.

European ships use smaller, simple[r] plants, fuses instead of breakers, an[d] unarmored cable.

In the case of U. S. industrial plants, th[e] plants are connected to large power sys[-] tems through relatively low impedanc[e] transformers without "in-house" genera[-] tors. These plants are frequently larg[e] in area for a given rating. Lighting th[e] means of line-to-neutral connection is im[-] portant from an economy viewpoint. Since the average industrial plant does no[t] generate its own power and is always in [a] position to obtain outside help, it has n[o] need of electrical maintenance with th[e] capability required by an operating ship. With these factors in mind, the following problems are anticipated for a grounded marine system:

1. Marine generators are not now presently provided with differential protection to pro[-] tect generators from internal short circuit[s]. This gamble is made since it is believed and experience has proved, that internal faults to ground on different phases a[re] rare indeed (the authors have not hear[d] of a single case). However, with a grounde[d] system, a single internal generator fau[lt]

ithout differential protection of the generator would lead to destruction of the generator. Therefore, it is probable that differential protection would have to be added. Suitable marine equipment does not now exist.

To build a grounded system it is necessary to use a Y-connected generator with neutral brought out. This decreases the number of slot and winding combinations available to the generator designer. Therefore, the customer may be forced to buy a machine which has appreciably poorer waveform and is considerably larger (an extreme case of 15% increase in weight for some larger machines may be expected) than if the designer had the option of building a delta-connected machine.

. On the grounded system zero-sequence current may be induced in delta-wound transformers, leading to overheating.

. Because of the increased number of line-to-ground faults, the increased number of arcs will increase the probability of fire. Industrial experience in which the line-to-ground fault on lighting circuits is most common, indicates that this is the most commonly named source of electrically caused fires.

5. To obtain reliable tripping integrity of equipment, grounding becomes especially critical. Normal rusting conditions onboard ship would lead one to believe that Z_e of Fig. 2 may be appreciable in many cases.

6. It is well to remember a difficulty encountered in grounding of some industrial systems. Here, indiscriminate recommendations for grounding were made without considering the impedances of the circuit. This led to problems with arcing at conduit joints, and necessity of revision in the method of construction of bus duct used in industrial plants.

With regard to item 6, there is one new factor which needs to be considered in the design of a grounded system which is not a problem in the floating system. This is the problem of co-ordinating for zero-sequence fault currents. It has been recommended that $X_0/X_1 \leq 3$ and $R_0/R_1 \leq 1$ be used as a basis for selecting the values of a neutral reactor or resistor. These values are based upon a number of fault calculations for typical configurations of 13-kv and above transmission lines for which the circuit parameters and permissible overvoltage bear little relation to marine electric plants. Thus, as a first difficulty, reasonable criteria for selecting R_0 and X_0 are not known.

In addition, magnitudes of R_0 and X_0 are not known, although generator X_0 is known and may be as small as $1/3\ X_1$.

In order to determine Z_0, a series of field tests were made starting in November 1958. To date six field tests have been made. The results have given values of Z_0 which range from essentially zero up to 0.4 ohm. The fundamental reasons for the variation in these test results, i.e., dependence upon circuit configuration and distance, are not yet understood. However, in most cases it is believed that line-to-ground currents in a solidly grounded neutral system will be large enough to operate protective devices. For the grounded system, greater emphasis must be placed on providing fully selective protective devices because of the higher probability of line-to-ground faults.

Expense

Any change required by going to a grounded system would, at the worst require the addition of the following equipment or effort:

1. Differential protection.
2. Greater design time on sequential tripping.
3. Fault-current-limiting impedance.
4. Occasionally a larger generator.
5. Occasionally larger transformers.

120-Volt Circuits

The 3-phase portions of the 120-volt distribution systems supplied through transformer banks could either be 4-wire with grounded neutral, 3-wire with grounded neutral and hull return, or 3-wire ungrounded. Single-phase portions would be either 1-wire hull return, or 2-wire.

Some engineers are of the opinion that there is more justification for grounding the neutral of the low-voltage lighting system than there is in the case of the main power system. This reasoning is based on the fact that grounds occur more frequently in this portion of the system and maintenance time in locating grounds might be reduced. This opinion is also probably based on the assumption that continuity of supply for lighting is less important than the main power system since emergency power for lighting is available. The continuity of supply to lighting is important and disturbances should be minimized, especially under emergency conditions when such grounds are likely to occur. Also, in some cases electronics equipment which requires a high degree of continuity is also supplied from the lighting system.

The considerations relevant to the 450-volt system also pertain here. However. there are variations in degree as follows:

1. Overvoltage. Because of the much smaller extent of any given 120-volt circuit, the capacitance will be much smaller. Also, the number of connected inductance devices will be negligible. Therefore, there is almost no likelihood of overvoltage other than $\sqrt{3}\times 120 = 208$ volts which would have no effect on 120-volt devices.

2. Fault location. This is frequently more difficult with 120-volt circuits than 450-volt systems because of the lower quality of hardware used and the difficulty in visually inspecting small devices.

3. Safety. No difference between ungrounded and grounded systems.

4. Continuity of service. The same facts apply here as in the 450-volt case. Since lighting and communications are the most critical uses of electric power, the increase in reliability of a properly maintained floating system is even more important than in the 450-volt case.

5. Number of trip elements. No appreciable importance.

6. Difficulty in properly grounding. In the case of 120-volt circuits, the Z_0 of the transformer is the fault-current-determining parameter. Any external circuit effect, other than fault impedance, will be negligible.

7. Expense. If hull return is used, the 120-volt grounded system would be less expensive. However, difficulty has been experienced with starting instant-start fluorescent lights because of circuit impedance levels in the shipboard systems. The impedance for this type of starting would have to be investigated before hull return could be used.

It should also be emphasized that some brands of instant-start fluorescent fixtures require a grounded 120-volt system.

Summary and Conclusions

Summary

1. Overvoltage. There seems to be little evidence that this is a problem on marine shipboard systems. If, in fact someone believes this is a problem, a more desirable solution than grounding to prevent overvoltages would be the use of a damping resistor. It woud also be possible to design a surge suppressor for this application.

2. Fault location. Finding the location of a line-to-ground fault on a floating system is occasionally a tedious process. It appears that grounding the system may cure this problem.

3. Safety. The floating system is slightly safer but each should be treated with the same care and respect by personnel.

4. Continuity of service. The floating system has greater integrity.

5. Number of trip elements. No appreciable difference.

6. Difficulty of grounding. In other applications electric plants have had serious difficulties with grounded systems where there was a lack both of fundamental understanding of phenomenon and data, as well as experience. There are not sufficient data properly to design a co-ordinated, sequential tripping grounded

marine 3-phase 3-wire 60-cycle electric system.

7. Expense. The floating system will be slightly less expensive, although the difference will probably be minor.

8. Lighting. For isolated portions of the distribution systems, such as the lighting system, the ungrounded system is still preferred although the reasons are not as strong as for the main distribution system.

Conclusions

1. There are only two factors which can be considered to favor the use of the grounded systems. For the solution of one of these problems, the overvoltage problem, which apparently exists only in theory for most ships, techniques other than grounding are well known. Satisfactory hardware for elimination of this problem is also available.

2. The problem of locating the relatively few hard-to-find line-to-ground faults on a floating system can be solved by grounding. The saving of work seems relatively minor compared with the other problems generated. For this reason it would be desirable to develop some practical means of locating a ground fault other than relying on the drastic solution of using a large fault current generated by grounding the system.

3. All other factors seem to favor a floating system; i.e., safety continuity, simplicity of design, and economy.

4. Should someone decide to use a grounded system on a marine plant, it is strongly desirable that this action be postponed until better theoretical insight into, or empirical data for, Z_0 is developed. Premature use of a grounded system, especially in large systems for which there is not even a small amount of European experience, would be foolish.

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Discussion

Paper 60-145

R. H. Kaufmann (General Electric Company, Schenectady, N. Y.): The authors have presented a very convincing story about the operating benefits to be derived from appropriate system grounding practices in 450-volt 3-phase 60-cycle marine power systems. A tremendous amount of actual operating experience with both grounded- and ungrounded-neutral low-voltage a-c systems has been accumulated in the industrial and commercial building fields. It makes good sense to dip into this vast fund of experience and pick out those practices which will contribute to superior marine system performance.

The principal operational advantages sought in system grounded include the following.

Suppression of Overvoltages

Ungrounded systems are, without question, more susceptible to line-to-ground overvoltages. For the most part the development of excess voltage is unique to a-c systems. There is no point in looking to early marine experience with d-c systems for guidance. Fig. 3 of the paper shows how severely the insulation life ("time to breakdown") is shortened by moderate increases in voltage stress. The AIEE proof test demands that new insulation be capable of withstanding "twice rated plus 1,000 volts" (7.3 times normal on a 450-volt rating) for a time interval of *1 minute;* or alternately, a voltage 20% higher (8.77 times normal on a 450-volt rating) for a time interval of *1 second*. Operating a system for 1 day with a ground on one line (73% overvoltage on the other two lines) extracts insulation life equivalent to 3 months' operation at normal voltage.

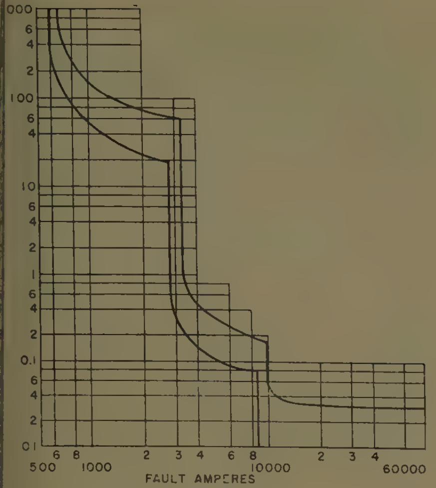
The overvoltage mechanisms described in reference 1 were created in the search for explanations of premature insulation failures observed in service, and not as a result of academic exercises in circuit analysis. There is nothing academic about the problem posed to a 480-volt ungrounded 60-cycle system operator who witnessed stator breakdown in more than 40 motors in a 2-hour interval due to a sputtering ground fault in a starting autotransformer. The overvoltage mechanism described on page 286 of reference 1 made this experience understandable. There is nothing academic about the problem which faced the operator of a 220-volt ungrounded 60-cycle system when a ground fault in a luminous-tube lighting unit resulted in insulation failure in several motor stators. The extended winding autotransformer overvoltage mechanism described on page 295 provides the explanation. To the plant maintenance man who saw two cable breakdowns occur on his 2,400-volt ungrounded 60-cycle system while making line-to-ground voltage checks with a transformer-coupled voltmeter, the problem was anything but academic. The *LC* voltage amplification mechanism described on page 281 took the mystery out of this experience.

The marine a-c electric system with its basic metallic structure, grounded machine frames, and grounded conductor enclosures, should closely parallel the behavior of typical industrial systems. The several overvoltage mechanisms described should find equal occasion for occurrence in marine systems. For military ships, the greatly increased circuit fault hazard during combat service should greatly exaggerate the overvoltage-producing tendencies.

Should future studies lead to the adoption of a primary distribution system of 2,400 or 4,160 volts, the 450-volt systems, if left ungrounded or merely high-resistance grounded, become subject to another severe source of overvoltage, namely unintentional physical contact between primary and secondary conductors (perhaps within the stepdown transformer for example); see reference 1, page 279.

Reduced Damage at the Fault and Lessened Bump on the Supply System

Electrical breakdowns are inevitable. Good system design and high-quality components can minimize but not eliminate insulation failures. If, when they do occur, their duration is needlessly prolonged, they result in severe burning damage at the fault location and impose a severe electric demand on the supply system, which may cause unnecessary tripout of healthy load equipments or even complete collapse of the power system. Some of the most devastating burndowns in electric systems result from the continued flow of fault current, even of quite moderate magnitude. An arcing fault condition tends to establish an arc voltage, which is nearly the same value whether the current magnitude be 2,000 amperes or 25,000 amperes. The energy released at the fault thus becomes the product of arc voltage (E_{arc}), current (I), and time (t).



ig. 5. Representative time–overcurrent tripping characteristics of the phase overcurrent ip units on a 600-ampere marine-system feeder air circuit breaker

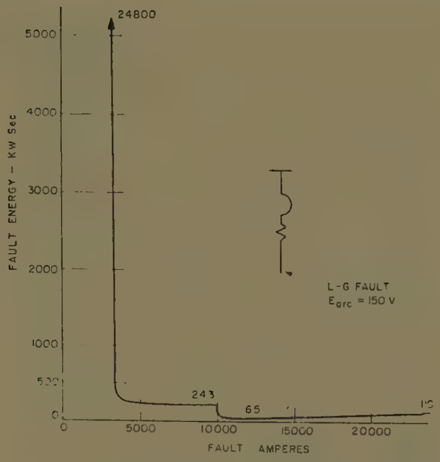


Fig. 6. Total arc energy released at the fault point as a function of fault arc current with 150-volt-fault arc voltage, a single line-to-ground fault, and duration controlled by the tripping characteristics of Fig. 5

To illustrate this point, let it be considered hat a feeder circuit is being protected by a ircuit breaker equipped with phase overurrent trip units with characteristics as hown in Fig. 5. Let it be assumed that the ault arc voltage (E_{arc}) is 150 volts. The nergy released at the fault point under the ontrol of this feeder breaker as a function f fault current magnitude is shown in ig. 6. Note that increased magnitude of ault current diminishes the energy released t the fault when controlled by phase overurrent trips. It is this fact which leads to olid grounding of the usual low-voltage ystems.

At once it is apparent that tremendous eductions in released fault-point energy for noderate-current arcing faults can be acomplished by fault detectors of increased peed and sensitivity. A neutral-grounded ystem paves the way for this added refinenent. The grounding circuit normally arries no current. The presence of current low in this circuit denotes the presence of a ault. Practically all faults will immediately r promptly involve ground. A fault-deecting relay responsive to ground-current low can thus be made very sensitive and xtremely fast. Thus, high-speed interrupion of moderate-current faults is entirely ossible which will tremendously lessen the ow-current burning damage indicated by ig. 6.

Additional relevant information may be ound in references 2–10.

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C. Raczkowski (Westinghouse Electric Corporation, East Pittsburgh, Pa.): "Low-voltage switchgear invariably employs circuit breakers embodying direct-acting series-trip devices which require high currents for successful operation," stated the authors in the paper. Further, in paragraph 4 of their conclusions referring to 450-volt effectively grounded systems, they say: "Faulted circuits will be automatically isolated with a minimum of delay, usually of the order of 2 or 3 cycles . . . ." The first statement is basically correct as far as today's practice is concerned. There are indeed very few low-voltage systems in the United States where overcurrent relays are used instead of series-trip devices.

The second statement, however, should be qualified. It is true as far as line-to-line or 3-phase faults are concerned. But in low-voltage effectively grounded systems, line-to-ground faults will not always be cleared with the minimum delay when conventional series-trip devices are used.

The experience in recent years with 480-volt solidly grounded systems brought up a very serious problem. It was found that arcing line-to-ground fault currents in solidly grounded systems can be anything but "high currents required for successful operation of direct-acting series-trip devices." In their excellent recent paper, Kaufmann and Page,[1] both from the same company as one of the authors of the paper, presented analytical proof that arcing line-to-ground faults in solidly grounded 480-volt systems might be as low as 19% of the 3-phase bolted fault. They give a tabulation of how fast different faults can be cleared by different protective devices, such as series trips and overcurrent relays. In the example they worked out, a 3,700-ampere line-to-ground fault would require 88 seconds to be cleared by the breaker equipped with series-trip devices even if the short-delay trips were set as low as 300% of the breaker rating. Obviously no industrial equipment, and more so, no equipment installed on shipboard, can survive such a "protection." They also quote an actual case where an electrician, "was severely burned and the equipment was virtually destroyed," because the instantaneous setting was higher than the value of the arcing fault current. To guard against this type of fault in solidly grounded systems, they recommend the use of ground relays on low-voltage breakers.

Our own experience with 480-volt solidly grounded industrial systems fully confirms what the Kaufmann–Page paper says about current-limiting action of an arc during line-to-ground faults.

There is no doubt that ground relays would provide the necessary sensitivity and speed, but they represent additional cost, require additional panel space, a source of energy for breaker tripping, and additional current transformers. It seems, therefore, imperative to take into consideration these facts before recommending 450-volt solidly grounded systems for marine use.

The authors of the paper, having their hearts set on effectively grounded systems, brushed aside, so to speak, resistance-grounded systems. Their only argument against resistance-grounded systems is that grounding resistors "cause the elevation of the system neutral above ground potential during a ground fault and because of this the line-to-ground voltage of the unfaulted phases is practically the line-to-line voltage of the system." With the insulation class provided for low-voltage cables and apparatus, 450 volts does not present any serious danger. The authors admit, however, that there are advantages of the resistance-grounded system: substantial reduction of transient overvoltages and a great reduction in equipment damage during the most common faults, that is, during line-to-ground faults

It seems that the main objection to ungrounded systems in industrial installations is a possibility of high transient overvoltages. Whether this is a problem or not on shipboard is beside the point. Let us assume that it is. However, having the system grounded through a high resistance will eliminate this danger and at the same time retain all the advantages of an ungrounded system. Furthermore, with the equipment available today, it is possible to selectively relay high-resistance-grounded systems. A Westinghouse *CWP-1* relay energized from a zero-sequence transformer is able to pick up, for example, approximately 2.5 amperes of primary ground current in a system grounded through a high resistance which limits the maximum line-to-ground fault to 7 amperes.

The relay can be wired to trip the affected feeder circuit breaker or to sound an alarm and drop the target to give the opera-

for an indication of which circuit is faulted, thus eliminating the "irksome and time-consuming" procedure of ground-fault location, so well known to the operators of ungrounded systems. Installation of *CWP-1* relays and zero-sequence current transformers represents an additional cost, of course, but it might be a very good investment. Therefore, before the final word is said in a present discussion about which system is the best for marine use, I am of the opinion that due consideration should be given to the high-resistance-grounded system.

REFERENCE

1. ARCING FAULT PROTECTION FOR LOW-VOLTAGE POWER DISTRIBUTION SYSTEMS—NATURE OF THE PROBLEM, R. H. Kaufmann, T. C. Page. *AIEE Transactions*, pt. III (*Power Apparatus and Systems*), vol. 79, June 1960, pp. 160–67.

W. J. O'Meara (The Atlantic Refining Company, Philadelphia, Pa.): The Atlantic Refining Company has been operating electrically propelled tugboats and tankers since the early 1920's, and all of our ships since those days have had electrically driven auxiliaries for the most part. In 1937 we built nine of our Van Dyke class turboelectric ships with 100% electrically driven ship's auxiliaries. Since that time we have built and/or operated *T2* tankers as well as conventional turbine gear tankers built both in America and Belgium. This has given us a long and continuous experience with electrically driven auxiliaries on cargo vessels of the bulk oil carrier type. In addition, we have two large refineries and numerous marine and land terminals with extensive electrical installations, all of which we feel entitles us to express an opinion on this subject.

We have consistently favored ungrounded systems on all of our marine and land installations. In our Atreco Texas refinery we found it necessary to install a reactance-resistance ground on our 2,300-volt system to eliminate simultaneous failures. This is the only exception to our policy of ungrounded systems. Our 440- and 110-volt systems at this and other plants are ungrounded except for ground-indicating lights.

PERSONNEL SAFETY

We do not agree with the authors' attempt to prove that grounded systems provide a greater degree of safety to personnel than do ungrounded systems. Very often we have to work our low-voltage systems "hot" and it has been our experience that any electrician who becomes even slightly contemptuous of 440 volts will regain his respect for it joltingly. We cannot support any argument advanced to prove one system to be less hazardous to personnel than the other system, and in so far as we are concerned, both are dangerous and must be so treated.

CONTINUITY

The principal reason we do not ground any of our 440-volt and most of our 2,300-volt systems on our land installations is because of the continuity of operation the ungrounded system assures us. We apply the same reasoning to our ships' distribution circuits. Grounding of one phase will not open the faulted unit breaker which is important on vital circuits such as steering gear, boiler fans, fuel oil service, etc., but the ground-indicating lights warn the watch engineers, that a circuit needs attention. Many of our vessels are in coastwise trade and spend a great deal of time in crowded East Coast and Gulf Coast harbors. In these waters we just cannot take chances with an unexpected shutdown of any of the vital auxiliaries.

I can speak only for the practices that maintain in my own company's plants and on our ships. When a ground is indicated on any circuit, it must be cleared at the earliest opportunity; that is, checks are made immediately to locate the fault and if possible to repair it. If, for some reason repairs cannot be made at that time, they are postponed until a more favorable time but they are not put off indefinitely. Reports of such casualties on board our ships are made to our Marine Headquarters, and if necessary, further investigation and repairs are made in port.

It appears to us that only the most careless type of housekeeping in any plant or on any ship would countenance the condition described by the authors.

At the present time we are operating 19 tankers with a total of over 800 motors rated 440 volts, 3 phase, 60 cycles, and ranging in integral horsepower size up to 400 horsepower. For the two years, 1958 and 1959, we had a total of 16 motor repairs for the following reasons: one ground in slot area, one short-circuited turns in end-turn area, four low megohmeter readings (which we needed only to dip and bake), ten roasted out due to single-phase operation or overloads.

When one considers that seven of these ships (Van Dyke class) have an average age of over 20 years, three of them are *T2*'s averaging over 15 years, and three others (Seamen class) are approaching an average of 10 years, he must agree that the record indicates these ungrounded systems are not and have not been subjected to dangerous, destructive, overvoltages.

The consensus of opinion among our electrical people, marine and land, is that we shall continue to operate our 440-volt systems ungrounded.

C. F. Maupas (Paris, France): In my experience with ship electrical installations equipped with an a-c system with an ungrounded neutral, so far I have not discovered any disadvantages due to the existence of overvoltages. On the other hand, neither has my experience with ships having a grounded-neutral system installed, such as the steamship *Kairouan*, indicated any particular disadvantages of this latter system. However, it is necessary to remember that with the present low voltages utilized on ships (maximum, 440 volts) ground faults are not frequent, and, also, the accidental existence of full line voltage between conductor and ground or even the appearance of greater overvoltages in case of inductive faults, which is very improbable, are not dangerous, particularly when the insulations are resistant to tracking (nontracking) and the creepage distances are sufficient.

My experience shows that a systematic supervision of the entire insulation of an a-c system (with equipment of adequate quality for humid environments) by means, for example, of a device which establishes a continuous low voltage between ground and the system, provides useful information concerning the system in operation.

It seems to me that the authors' considerations and conclusions concerning ungrounded-neutral systems are pessimistic. But, as it is generally advantageous to bring out the neutral of the alternator in the terminal box (I recommend even the installation in the terminal box at the neutral point of three current transformers for protection of faults between the generator and the switchboard), one can try to reserve the possibility of connecting the neutral to ground, in certain circumstances, by means of a low-impedance connection.

The difficult problem in regard to return currents by cable armor as indicated in the paper appears to me to be of less concern in an ungrounded-neutral system. Furthermore, my experience shows that a good ground to cable armor is difficult and costly. In a general way, I would today be tempted to advise the use on board of unarmored cables. Cables equipped with a very light protection (against rodents) by a thin-strip-wound metallic covering well insulated with a sheath of polyvinyl chloride or polychlorophene, are presently being tested aboard ship.

Paper 60-146

L. Spinelli (American Bureau of Shipping, Genoa, Italy): In Italy we have about 100 ships with a-c plants, 30% of which are Navy vessels, all built after World War II. All a-c plants are 440-volt 3-phase 3-wire systems with ungrounded neutral.

I communicated with the heads of the electrical departments of all the largest Italian shipyards asking information on the subject in question. All of them replied that, to their best knowledge, not one of the electrical troubles reported on board ships built by them could be ascribed to the fact that the electric system was designed for ungrounded operation.

A similar answer was received from the technical advisers to several important Italian ship owners, who confirmed, in addition, that their electrical engineers diligently enforce the instructions to eliminate any ground as soon as detected.

A. R. Cairone (Westinghouse Electric Corporation, New York, N. Y.): The authors are to be commended on their comprehensive comparison of an ungrounded system versus an effectively grounded system. Although the ungrounded system has certain undesirable features, it appears to be more attractive than the effectively grounded system. However, it is my opinion that the subject of grounding would not be complete without due consideration of a resistance-grounded system.

Inasmuch as there are no phase-to-neutral loads on the 450-volt system, the only apparent real reason for considering an effectively grounded system is the high ground-fault current required for the series trips on the available circuit breakers. Naturally this system suppresses transient

overvoltages; however, this can be accomplished by a much simpler method on a resistance-grounded system. All other advantages of an effectively grounded system have no apparent practical value on a 450-volt shipboard installation. The availability of a simple economical ground-relay system should eliminate the need for an effectively grounded system along with its complications.

The subject of transient overvoltages is controversial. Even though the authors present data which indicate the possibility of overvoltages to be very remote on an ungrounded system, it is my opinion that the minimum for any 450-volt system should be a high-resistance-grounded system, particularly since this can be accomplished simply and economically.

The use of a resistance-grounded system swamps the variations noted in zero-sequence measurements. Also, the question of good grounds appears to be a question of recognizing the problem. Copper pads welded to the ship steel, machine frames, and neutral-ground connection should provide a reliable path for ground-fault current as well as a safe ground for equipment frames. Resistance grounding is readily adaptable to Y or delta sources. The following possibilities could be considered:

1. High resistance, no selective tripping.
2. "Low" resistance with selective tripping.
3. Combination of 1 and 2.

1. High resistance: The basic function is to suppress transient overvoltages. To do this, the current through the resistance-ground source should be equal to or greater than the total 3-phase capacitive charging current to ground of the 450-volt system. As an example, for the case of 3 amperes mentioned in the paper, the current through the resistor ground source would be 3 amperes and the total ground current is $3 \times 1.41 = 4.23$ amperes. The loss in the resistor is $3 \times 260 = 780$ watts. Suppression of transient overvoltage is accomplished with a slight increase in total ground current. The probability of iron burning has never been clearly defined. The incremental possibility of iron burning would be hard to evaluate. Practically speaking, any precautions taken against iron burning should be the same for an ungrounded and a high-resistance-grounded system.

2. Low-resistance grounding: A low-ground-fault system has been developed for mining applications. Sensitivity for ground-fault current is accomplished by use of one "doughnut" current transformer surrounding the three phases. This gives a response to net 3-phase flux and hence is sensitive to ground currents. At point of installation (one per circuit breaker) the cable armor can be stripped back to insure that return ground-fault current does not go through the current transformer. Selectivity can be accomplished by time setting on the ground relays. The ground relay will make contact to energize a shunt trip on the circuit breaker. Inasmuch as the fault current can be limited to values in the order of 25 to 50 amperes (and possibly lower), there will be essentially no voltage dip, thereby insuring ample voltage for shunt trip operation. The shunt trip would be connected phase to phase. All phase faults would be cleared by the present series trips. Backup protection can be accomplished by a ground relay in the ground source.

3. Combination: This system lends itself to continuity of service. The grounding resistor can be made up of two sections consisting of a high-resistance low-ampere resistor in series with a low-resistance higher-ampere resistor. The two resistors in series limit the current to satisfy high-resistance grounding. A normally open breaker is connected across the high-resistance element. When the high-resistance element is short-circuited, the low-resistance element allows current to increase to satisfy selective tripping.

This system would operate as a high-resistance-grounded system. An alarm will alert the operator when a ground fault occurs. If the ground location is not readily apparent, the operator can make preparations to insure continuity of service on the essential feeders, or at least recognize that an essential feeder may trip when the normally open breaker is closed to accomplish selectivity. A further refinement could be installation of ground relays on nonessential feeders and ammeters only on the essential feeders. If there is no indication of ground current in the essential feeders, he can close the normally open breaker to produce fault current for selective tripping. If there is indication of ground current in an essential feeder, procedure can be carried out as dictated by the particular case.

I would appreciate the authors' comments on the above possibilities plus the following:

1. The authors give expected life on insulation. Does this reflect mechanical stresses due to motor inrush and short-circuit currents? These stresses are an important consideration as insulation ages.

2. The question of iron burning in machines is complicated. This seems to be a subject of opinion rather than facts. Many values of current versus time have been given at various times. Are there any practical test data available on this subject?

3. Why is it necessary to use armored cable? Present-day cable techniques should be able to offer equivalent serviceability without use of armor.

References

1. Electrical Transmission and Distribution Reference Book. Westinghouse Electric Corporation, East Pittsburgh, Pa., 1950.

2. Safety Features of AC Power in Underground Mining Operations, A. C. Lordi. Westinghouse Electric Corporation, 1958.

Robert A. Zimmerman (Westinghouse Electric Corporation, East Pittsburgh, Pa.): The authors have presented a thorough and comprehensive paper on the subject of ungrounded marine electric systems. I would like to commend them on their review and evaluation of the subject.

Industrial experience with ungrounded distribution systems indicates that insulation on these systems is subject to failure because of high transient overvoltages. Marine experience with ungrounded distribution systems indicates that these systems are not subject to insulation failures because of high transient overvoltages. The question then arises as to the differences in circuit parameters and conditions between the two ungrounded system applications that produce failures in one case and not in the other.

The theory of how these high transient overvoltages due to intermittent arcing ground faults may occur is well understood. If the dielectric strength of the arc path builds up subsequent to arc extinction as hypothesized, the voltage to ground on the unfaulted phases can build up to excessively high values with alternate restriking and extinction of the arc. What actually happens is that the zero-sequence capacitance of the system becomes charged to higher and higher values of voltage each time the arc is extinguished.

It is claimed in the literature that it is to be expected in general that these values will reach five to six times normal under practical system conditions. However, no indication is given as to how much damping is involved to limit the voltages to these values. A simplified study made to determine the effect of resistance damping on the transient overvoltage showed a maximum overvoltage on the unfaulted phases of 4.6 times normal. This study assumed 85% damping per half-cycle of the high-frequency oscillation. This is believed to be a practical value.

This damping is produced by the series resistance of the circuits, the leakage resistance of the system, and shunt resistance used in ground-fault indicators. The authors point out that they believe it virtually impossible to predict the maximum overvoltages because of the large variations in resistance and the infinite number of circuit combinations that are possible. Their reference is to all ungrounded systems. However, this statement may not hold true for ungrounded marine systems. Physically these systems are limited to relatively short runs because of the restrictions of length and beam of ships. The charging current of the system will be low, and the damping effect of system resistance, both series and shunt, may be high. These system parameters and conditions, which would be fairly common to all marine systems due to their similarity, may well be the reason why transient overvoltages are not a problem on ungrounded marine electric systems.

These ungrounded systems are not ungrounded but are, in effect, high-resistance-grounded systems. If transient overvoltages became a problem on ungrounded marine systems, it would appear to me that intentionally grounding these systems through a high resistance would solve the overvoltage problem without introducing any other problems.

J. R. Batcheller (Philip F. Spaulding and Associates, Seattle, Wash.): It would be interesting to have the authors of this paper answer a few questions.

First, they state that "it should be remembered that the unique degree of autonomy of a ship with its complete electric system requires the presence of electrical maintenance personnel on board ship." This may be true for Navy ships, but not all Merchant Marine ships carry an electrician on board. It is usually the job of the chief engineer and his assistants to make any electrical as well as mechanical repairs. Human nature being what it is, and the difficulty of isolating a ground on an ungrounded system

being what it is, the clearing of a ground is delayed often for many months even when the service permits the vessel to tie up for several hours every night.

Under "Difficulty of Proper Grounding," they indicate that for a grounded system the provision of generator differential protection is necessary, and conclude with the statement that "suitable marine equipment does not now exist." Ordinary land-side reverse power relays are used on marine switchboards, and since Westinghouse has manufactured *CA* and *HA* relays for generator differential protection for many years, it would appear that suitable equipment is available.

Also, under this same heading they appear to be disturbed over the fire hazard of arcs from line-to-ground faults on a grounded system. Modern breakers are sufficiently fast to clear these faults without serious damage unless they have been incorrectly applied. The modern practice is to isolate faults rapidly with breakers and not attempt to set them so high that the fault "will burn itself free." That philosophy has disappeared from good land-side operations and should not be used in setting breakers on a ship either.

Under "120-Volt Circuits" they state: "5. Number of trip elements. No appreciable importance." It is a well-known fact that with a solid neutral system only single-pole breakers are required for single-phase circuits, while ungrounded 120-volt delta circuits require double-pole breakers. Therefore, with a 208Y/120-volt grounded system the number of trip elements will be just one half that of an ungrounded 120-volt delta system.

At this time the grounding of the 120-volt lighting system is more important than the 450-volt system. When the U.S. Coast Guard will accept for Merchant Marine Service 265-volt fluorescent lighting (which is now standard in industrial buildings), then the full advantages of the grounded Y system can be realized at that voltage also.

Joint Discussion

O. T. Estes (Commander, U. S. Coast Guard, Washington, D. C.): In Paper no. 60-145, Mr. Hall and Mr. McSweeney have proposed a scheme of grounding the generator neutral through a low-impedance reactor and ground-disconnect switch. The rest of the system apparently is left as it was when ungrounded. Then they infer that ground detection is not needed now since ground faults will be self-locating. This is all very good if we still have a satisfactory system, but several questions arise: Since in the ungrounded system ground faults would eventually be self-locating, why have ground detection on ungrounded systems? I have asked this question of many in the marine field. They all agree that ground detection should be provided for ungrounded systems because it continuously gives a measure of the insulation condition in the electric system and operating personnel have the opportunity to take corrective action before complete insulation failure occurs. This leads to my second question: Why should ground detection be provided for an ungrounded system and not for a grounded-neutral system?

As the authors have stated, marine electric plants continue to increase in size. In today's marine electric plant the possible short-circuit current at a point in the system may exceed the interrupting rating of the usual circuit breaker or fuse. In this case a backup breaker (or current-limiting fuse) is furnished with adequate interrupting rating and with an instantaneous trip setting at not more than 90% of the interrupting rating of the device protected. Often one breaker will back up several lesser rated breakers. This arrangement may not be unsafe in an ungrounded system, where ground detection affords the opportunity to correct impending faults and lessen backup breaker operation, but where full fault currents are more probable, as in the proposed system, operation of backup breakers is encouraged. The opening of a backup breaker will disrupt power to several services at one time, may result in damage to the backed-up breakers, and, in fact, could cause the loss of the entire plant. Here a third question arises: Shouldn't the use of backup breakers be restricted in a grounded-neutral system?

The answers to these questions would be of great assistance.

In Paper no. 60-146, Mr. Bruning and Mr. Lusby made the statement: "No grounded marine 60-cycle 3-phase systems have ever been built in the United States." Whenever someone makes a definite statement like that, someone will always come up with an exception. Several vessels in the offshore oil industry employ a 3-phase 4-wire a-c distribution system. The neutral is solidly grounded at the switchboard. These installations are small, in the order of 80 kw, and are reported to operate satisfactorily.

E. Gott and **H. Arnemo** (Allmanna Svenska Elektriska, Vasteras, Sweden): In the paper by McSweeney and Hall no mention is made of possible circulating currents of overtone frequency which may occur between the neutral connections of generators operating in parallel. Admittedly, the disconnecting link between the generator neutral and the reactor referred to can be used to disconnect any generator neutral to give a suitable combination of generator neutral connections for minimum current but, should the grounded generators be isolated from the system by their generator circuit breaker, action must be taken to connect another generator reactor in order to maintain a grounded system. This may be forgotten, or take some time, thus leaving the system ungrounded in the meantime. In all circumstances it is essential that a check, under load conditions, be made of the magnitude of the circulating currents in the generator neutral connections, especially in the case of generators which are not identical, to determine if any disconnecting action is necessary.

With regard to the Bruning-Lusby paper, we wish to point out that at the introduction of alternating current on board ships in Swedish shipyards the ungrounded 3-phase 3-wire power system was used. The first ships were for 50 cycles but quite soon 60 cycles was chosen and, consequently, our practice corresponds to that employed in the United States. In our experience the ungrounded system has proved to be very good and effective, causing no trouble. We have no intention of abandoning this system.

With respect to the section in the paper dealing with overvoltages, we can report that figures of leakage capacitance, measured by us on actual installations, have given values of about 1–2 microfarads per phase on the power system as well as on the 110-volt lighting system, the generating capacity being maximum 1,200 kw. The values include estimated capacitances of motors in service simultaneously and it is interesting to note that practically half the capacitance relates to the rotating machines. We have thus not found that there are such great variations in capacitance due to the size of plant, as the paper indicates. For the lighting system higher values can, no doubt, be reached if wireless sets with radio-interference suppressors are connected, as such suppressors may commonly be of the order of 2–4 microfarads. As to the values of the leakage resistance, given in Table I, there appears to be some error (we regard values of the order of megohms as normal). As pointed out in Paper no. 60-146, concerning European practice, we strictly adhere to the practice of using fuses in outgoing groups under, say, 300 amperes, including motor contactor protection. This system is employed almost exclusively in our land installations as well. We have not found that this has been detrimental in any way and have experienced no trouble due to overvoltages in the apparatus or machines belonging to the system.

Regarding the standard test for dielectric strength applied to 450-volt equipment, we may say that we have found from practical experience that, for a medium-voltage transformer, a 1-minute 50-cycle voltage test corresponds to an impulse test value approximately 2.3 times higher. Admittedly, the two tests are not equivalent as the 50-cycle test concerns the dielectric strength relative to earth but as an indication of the relationship the value may be used. With switching surges, however, it was found that this value should be reduced to approximately 1.9. Assuming this to apply roughly to other equipment, the figures quoted would give an impulse test value of about 4,000 volts which covers the reported values of overvoltages with considerable margin.

Indicator lights may be used for ground-fault annunciation but we have used a method of superimposing a d-c measuring current on the different systems and are thus able to obtain a continuous check on the insulation level of the systems by means of an appropriate ammeter with a megohm scale. We consider this possibility to be a great advantage of the ungrounded system as information of variations in insulation level can, with time, be of value in preventive servicing.

We have always maintained that internal protection of generators is vital despite the fact that also in our experience internal faults to ground on different phases have not occurred; furthermore, we consider that demagnetization must take place to lessen any damage as, with alternating currents, the prime mover does not become overloaded in the case of a short circuit on the generator side as it does with direct current. We have, therefore, always brought out the generator neutral, but with the winding methods used by us have found no difficulty

selecting windings for Y-connected nerators to give satisfactory waveform d minimum size of generator. The point erred to in the paper as a disadvantage the grounded system has, therefore, no l significance in our opinion.

In our case the 120-volt distribution sys-n is also ungrounded which enables us to aintain a continuous check on the insula-n level of this system as indicated.

J. Tuke (Shell Tankers, Limited, Lon-n, England): I wish to comment first on e McSweeney–Hall paper, no. 60-145. view of my later statements on Paper no. -146, it is rather difficult to avoid wide-read conflict with the views of these thors, although again the variation arises the relative emphasis placed on the guments rather than with the arguments emselves.

The general statement in the first-men-oned paper that "in many instances the sulation works to the rated limit" indeed rikes a strange note. I do not believe is is generally the case and would cer-inly not accept equipment for our own ips if it assumed temperatures on test hich indicated that when the ship would in zones of high ambient temperature the sulation would reach its maximum safe mperature, this being due partly to the treme difficulty in estimating closely the cal air temperature surrounding any par-cular item of equipment in the ship. In neral, it is more worthwhile for the ship wner to pay for a reasonable margin of fety in the machine ratings (particularly ith regard to insulation temperatures) than "clutter up" the installation with quanti-es of purely industrial-type relays, some which do not stand up to marine condi-ons and which themselves create interrup-ons in the supply rather than afford pro-ction against legitimate faults.

With regard to ground lamps, while shar-g the authors' view that there is the anger that ground-lamp indication may ot receive the degree of attention which it hould, we have taken a number of steps with a measure of success) to overcome this ifficulty. In the first place, all ships' per-onnel are instructed that ground lamps are arning devices rather than indication de-ices and, in the second place, ground lamps ust be energized at all times as we have ever understood the wisdom of using push-uttons or switches spring-loaded to the off" position for a device which is intended r continuously monitoring a safe condi-ion.

Another feature which counters this diffi-ulty is the arrangement of the ground-etecting circuits so that the monitoring oint can be changed by a single switch from ne part of the ship to another to facilitate ocation of faults at the 440-volt load-dis-ribution centers. In practice this means hat the ground lamps on the main switch-oard are arranged so that the ground con-ection is led to the midships subswitch-oard and connected to ground via a nor-ally closed contact of a switch on the mid-hips subswitchboard. This same switch, hen closed, disconnects the ground lamps n the main switchboard and completes the ircuits to a second set of ground lamps on he midships subswitchboard. Therefore, a ground should occur on the midships section of the installation, location is very much facilitated.

Grounds on the 440-volt system are, happily, infrequent and the possibility of a second ground occurring before the first is clear is sufficiently remote in practice to be almost disregarded. These facts strongly support the adoption of the ungrounded system for important loads.

Summarizing then, having expressed support for the views expressed by the Bruning-Lushy paper, it is difficult to avoid conflicting with the views of Mr. McSweeney and Mr. Hall, particularly in respect to their conclusion no. 6.

It is appreciated that the subject is still in its infancy and although there has been a sizable installation completed in Great Britain for a passenger ship (*M.V. Bergensfjoord*, 1956) with a grounded-neutral system, and various publications issued, these have not influenced our conviction that essential loads should be operated on an ungrounded system.

With reference to the Bruning-Lusby paper, I find that while I agree with many of their arguments of a technical character, the evaluation of these arguments and hence the final conclusions which must depend upon the amount of stress attributed to them are not completely in line with our present tendencies, particularly in the case of the McSweeney-Hall paper discussed previously. As my company is basically concerned with operating a tanker fleet, the following remarks are necessarily made with a bias towards tanker installations, extrapolated here and there to cover larger installations. The remarks are, however, confined to practical operating features rather than to theory.

All vessels in our fleet are equipped with installations of which the neutrals are not grounded, since this is a special requirement of Lloyd's Register for oil tankers. Even if this were not the case, however, I would quite definitely oppose any change in this policy.

At the beginning of the paper, Mr. Bruning and Mr. Lusby give an order of priority for required loads with which I am unable to agree. The emphasis underlying the electrical installations on our ships is that those auxiliaries directly associated with steam raising, i.e., forced draught fans, fuel-oil pumps, etc., and those associated with the auxiliary turboalternator for the maintenance of auxiliary power supply such as turboalternator condenser cooling-water circulating pumps and condensate extraction pumps, are of first priority. Communication and selected lighting loads, while obviously important, are, in our estimation, of secondary priority compared with those just enumerated.

This may also be associated with a rather fundamental difference in concept for distribution systems which exists between American practice and our own. In the former case, for a ship with two turboalternators and one diesel alternator as in a tanker, regulations require that the auxiliary diesel alternator shall start automatically and operate in conjunction with a self-disconnecting emergency bus system throughout the ship, ensuring that only selected loads are supplied by the auxiliary diesel alternator; these selected loads cover in general the lightning and communication circuits. Our approach to this problem is based on the concept that the auxiliary diesel alternator can supply the main bus system and the distribution of its power as determined by the engineroom staff which decides which loads should assume highest priority for the limited power available if the main power supply is lost under any given set of circumstances. The decision depends, for instance, on whether the loss of power occurs in open waters or in confined waters. In conjunction with this system, there is a battery-operated emergency lighting system in the machinery space which functions automatically on the loss of normal lighting supply.

In summary, then, in the American system, the auxiliary alternator is regarded as an emergency alternator mounted outside the engineroom on boat deck level. This system tends to reflect slightly more faith and depends to a lesser degree on the ability of the operating personnel than does our system in which the auxiliary alternator is deemed to be more in the category of an auxiliary to the main plant, and is mounted in the engineroom.

Perhaps I should add that we have a number of American-built ships in our fleet and both systems give satisfactory results, but the preference is for the elimination of automatic features as far as possible.

Under the heading "3-Phase 3-Wire System Considerations" we would classify "continuity of service" well above all other factors listed and would not accept any system which would be capable of suddenly depriving one of the services of some vital auxiliary machine without warning in the case of a ground developing. Of the considerations listed, it appears to us as though overvoltages, safety, number of trip elements, and expense can produce arguments both ways, but in our case continuity of service alone would decide the issue even if the classification societies permitted any choice in the matter.

It is noted also that "switching transients" are more likely to occur, in the authors' view, with fuses than with "properly applied circuit switching devices." Hereby lies another difference in outlook since we tend to use the HRC fuse more widely than the circuit breaker for distribution circuits and have had only one suspected case of switching transients in approximately 250 ship-years of operation. Again, based on experience with the American-built ships in our fleet, we must acknowledge that circuit breakers provide similarly reliable service.

In the paper, Mr. Bruning and Mr. Lusby refer to the possibility of adopting a grounded system for lighting where faults are more likely to occur. Our indications are that we would expect to be faced with approximately 50 faults on the lighting system (115 volts) for every ground on the main system (440 volts) and we do support the view that nonessential circuits of lighting systems only may benefit by operating with grounded neutral. It is felt, however, that for essential low-voltage circuits such as communications, navigation lights, etc., the need for maintaining continuity of supply supports the adoption of ungrounded systems. In short, it is suggested that the low-voltage distribution network should be divided into essential and nonessential categories with the ungrounded system being used for the former and the grounded system for the latter.

I find myself also in almost complete agreement with the four conclusions of this paper.

D. Gray (Lloyd's Register of Shipping, London, England): We have had very little experience with the grounded system in Europe. Almost all a-c systems are fully insulated. I have read Paper no. 60-145, as well as Paper no. 60-146, and while we would agree with the writers in their comments on fault currents, overvoltages, shock risk, and difficulty of fault location in the insulated system, the prime reason why the insulated system is favored in England is "continuity of service," i.e., two faults are required before a service is lost. This, without doubt, is considered to be an outstanding advantage.

Regarding the difficulty of fault location, this is not as difficult as Mr. McSweeney and Mr. Hall imply. It is possible to have the ground lamps switched so that they can detect ground faults on any section of the system, all indication being on the main switchboard.

I strongly disagree with Mr. McSweeney and Mr. Hall on their conclusion no. 6.

In general, I think that current thinking here is as follows: Where continuity of service is essential the insulated system should be used. Where continuity of service is not essential, the grounded system could be used. As most ships have both essential and nonessential equipment, a combination of both could be used if desired, e.g., insulated for power and grounded for lighting services.

I must add that this view does not agree with views held in Germany, where single-wire hull return is held to be safer and more reliable, i.e., the reverse of views held in the United Kingdom.

G. O. Watson (Chartered Electrical Engineer, London, England): Having followed the discussions on this problem for several years, I can say without hesitation that I consider Papers no. 60-145 and 60-146 as comprising the most complete and exhaustive appraisal of the subject yet presented.

I regret that the space at my disposal for discussing these papers does not permit a detailed reply and analysis of the arguments which would approach the high level which they merit and I must be content with a few random comments.

1. It must be assumed that from the point of view of human safety neither system is safe: 260 volts can be fatal, especially under shipboard conditions, and a 450-volt "insulated" system will always have sufficient leakage current plus the risk of a ground fault to make it lethal.

2. Mr. McSweeney and Mr. Hall imply, unless I have misinterpreted them, that in modern machine design insulation is "worked up to its permissible limit" and that an increase of operating voltage stress of 73% is likely to result in failure if allowed to continue.

If this is true then I would say categorically that it is cutting things too fine for the degree of safety we expect in ships. British Standards for machines engaged on priority services is that the windings must be insulated from ground with mica or the equivalent and this insures a high standard of insulation from ground.

3. On a grounded system a ground fault may be such that the fault current plus the load current is less than the setting of the protective device whether it is fuse or circuit breaker, and the fault current may be considerable and such as to constitute a serious fire risk and yet the protective device will not operate.

To cope with such risks it is usual in land installations to provide ground-leakage protection which will disconnect the supply when the leakage exceeds a relatively small value. To fit each circuit in a ship with such protection would considerably complicate the installation and increase the liability of frequent interruption of supplies.

It may be said that similar risks exist with insulated systems when there is an uncleared ground fault. However, then the risk is not continuously present and persists only until the personnel locate and clear it. The difficulties entailed in doing so are very well described by McSweeney and Hall but neglect to act should not be tolerated and the importance of taking remedial action should be strongly impressed.

Incidentally, this factor stresses the need for insistance on first-class apparatus and materials and workmanship during installation to reduce the hazard of ground fault.

It is a mistake to assume that all grounds will be of low resistance. They are just as likely to be arcing faults or faults in windings such as contactor coils, transformers, etc. The fault current must find its way back to the generator via the hull and if there should be a weak spot in this return path local heating will occur. This is liable to occur with apparatus in the superstructure and particularly in aluminum superstructures which are insulated from the main structure in order to inhibit corrosion.

I have always understood that American marine engineers were strongly opposed to d-c systems based on hull return. It would seem to me that a-c systems with grounded neutral, depending on the hull for the return in the event of faults, are very closely akin to the unwanted d-c system.

4. If fuses are used on a grounded 3-phase system, a fault on one phase will result in single-phasing. Fuses are used almost universally for circuit protection in European ships.

5. I would appreciate an explanation of the statement under the heading "Solidly Grounded" in the McSweeney-Hall paper in which the bracing of windings is referred to. Should it read "are *not* braced withstand" etc.? The statement has the effect that *because* they are braced they will suffer damage in a line-to-ground fault.

6. In a grounded system leakages to ground can occur throughout the system and remain undetected until they either become of sufficient magnitude to trip a protective device or start a fire.

It was the difficulty of monitoring or meggering d-c systems with hull return which made them unpopular with classification societies.

7. As there is one large passenger ship in commission with a grounded neutral, *M. V. Bergensfiord*, it would be interesting: (a) to have a report by the owners of their operational experiences; and (b) to examine the circuitry and protection system and compare it with a normal ungrounded installation.

The main agrument against the ungrounded system is the possibility of overvoltages. Mr. Bruning and Mr. Lusby discount these risks in their paper and say that evidence that they are the causes of breakdown is lacking, and they give some reasons. This seems to me the crux of the whole matter. Are overvoltages a valid risk in marine equipments? And is not the risk of an outage due to a single ground a greater hazard to vital services with a grounded neutral?

Where a grounded connection is necessary for rapid-start fluorescent lamps there is no objection to grounding the neutral on the secondary side of the lighting transformer. This is a relatively small local system and the main arguments against grounding the main distribution system do not apply. No vital services are involved and the current ratings of the circuits are comparatively small. However some further factors are dealt with by Bruning and Lusby.

J. M. Apple (Bureau of Ships, U. S. Navy Department, Washington, D. C.): From Paper no. 60-145 by McSweeney and Hall it appears that the following advantages for a grounded-neutral a-c system are claimed over an ungrounded neutral system:

1. Elimination of sustained overvoltages.
2. Fewer number of grounds during life of ship.
3. Reduction of maintenance task in locating grounds.
4. An effective reduction in shock hazard to personnel.
5. Improved continuity of power supply.

Since these claimed advantages are not supported by known experience or data, it appears doubtful that many ship designers or operators, who have had numerous years of successful experience with an ungrounded system, will agree with the authors on the desirability of a grounded system. It is recognized that any data or experience on actual grounded-neutral a-c marine electrical installations is difficult to obtain, because of the apparent preponderance of use throughout the world of ungrounded systems. There is one notable exception in that it is understood the Germans have successfully used 380/220-volt 3-phase a-c neutral grounded hull return systems. However their claimed advantages for such a system appear to place the major emphasis on reduction in fire hazard. Mr. McSweeney and Mr. Hall's comments on this claimed advantage would be of interest, since they do not list this as a reason for grounding.

Although this question of relative safety from fire hazard, and, in the case of tankers, from the explosion hazard, is too broad to cover fully in this short discussion, a few pertinent comments are submitted for consideration. (The Germans apparently consider the fire hazard to exist in moisture faults in wooden panels of accommodation spaces and in the case of explosion hazard on tankers, an arc in either type of system can cause an explosion.)

The fire hazard can be considered in two parts: first, those fires caused by a continuous high-impedance fault generating heat, and second, those caused by an arcing

ult. In the first case there are many ariables contributing to the fire hazard and nder the right conditions these can result a fire in any type of electric system. hese variables include the capacity of e energy source, the temperature rise in e fault, and the presence of combustible aterials. Briefly, the argument for an ngrounded system to reduce this hazard is redicated on the basis that features are rovided which permit the operators to aintain the leakage to ground on the unulted leg to a low value that reduces the robability of reaching the necessary comination of variables on the faulted leg to ause a fire. In effect, this precept provides for a limitation of the capacity of the nergy source.

As to arcing grounds, reference 1 is a ery good presentation of the problems and he potential damage that can be caused y an arcing fault on a grounded-neutral system with direct-acting overcurrent protective levices. Here again, the maintenance of a ow leakage on the unfaulted leg of an ngrounded system will provide, in effect, high-impedance energy source thereby esulting in self-clearing of the arc in 1/2 cycle.

This same reduction in arc energy by the use of an ungrounded system is also considered a factor in reducing the explosive hazard on tankers.

The second paper, by Bruning and Lusby, covers the items of safety and continuity of service which are considered to be of the utmost importance in designing an electric system for shipboard application. However, under the safety aspects only the electric shock hazard and the personnel danger from electric arcs are discussed. In ship designs great stress has always been placed on making ships as safe as possible for personnel and reducing probabilities of property damage. The reduction of potential fire and explosion hazards of electrical origin have been very important in achieving these safety goals. In most cases increased equipment cost and more maintenance are fully justified if a reduction of these fire and explosion hazards can be achieved. In item 4 under the heading "Difficulty of Properly Grounding," the authors indicate an increase in probability of fires due to the increase in the number of line-to-ground faults on a grounded system. Assuming that the number of faults will be the same on the two types of systems, does the authors' lack of detailed discussion on these aspects indicate that they consider there is no choice between a grounded or ungrounded electric system in regard to relative safety from fires and explosions?

Conclusion 2 of the Bruning-Lusby paper states that the problem of locating the relatively few hard-to-find line-to-ground faults on a floating system can be solved by grounding. This will only be true where the impedance of the fault is low enough to provide for sufficient current to trip the protective device.

With direct-acting overcurrent protective devices the resulting current will probably result in damage to the equipment in which the fault is located. It might be inferred from the authors' conclusion that it would be desirable to operate a system ungrounded until the ground detector indicates a ground on one line and then a switch could be closed to connect the neutral to ground, thereby causing the current to trip a protective device isolating the faulted circuit. Although such a scheme is possible, it would probably result in more damage to equipment, necessitating replacement, and increase the probability of fire than would be involved in the conventional method of ground location in an ungrounded system. Experience has shown that the majority of such single line-to-ground faults on an ungrounded system can be located in the conventional manner and cleared without the necessity for replacement of damaged equipment. This reasoning leads to the conclusion that even where you have systems where continuity of supply may not be important, such as low-voltage secondary systems, the use of a grounded system to locate line-to-ground faults is very undesirable.

REFERENCE

1. ARCING FAULT PROTECTION FOR LOW-VOLTAGE POWER DISTRIBUTION SYSTEMS—NATURE OF THE PROBLEM, R. H. Kaufmann, J. C. Page. *AIEE Transactions*, pt. III (*Power Apparatus and Systems*), vol. 79, June 1960, pp. 160–67.

H. P. Walker (Bureau of Ships, U. S. Navy, Washington, D. C.): In conclusions 1 and 2 of Paper no. 60-145, Mr. McSweeney and Mr. Hall indicate that neutral grounding will eliminate sustained overvoltages and that fewer grounds will occur because of the reduction of strain on the insulation due to elimination of protracted overvoltages.

Referring to the second paragraph on overvoltages and to Fig. 3, the authors infer that the difference between 260 and 450 volts will reduce the strain on the insulation. This comparison is like stating that 3 volts will kill you quicker than 1 volt. Actually, Fig. 3 is a relationship of time and voltage in the breakdown area and is not believed to be significant in the range of 260–450 volts. If actual values are considered for a typical insulation having a dielectric strength of 740 volts per mil using the one-minute step-by-step test, then a 0.010 insulation would have a dielectric strength of 7,400 volts. Following the extrapolation of Fig. 3, to say, 10 years would yield a 40% value of breakdown strength of 2,960 volts. Other data show a decline to 25% for the same period of time which would yield a dielectric strength of 1,850 volts. In either case it would appear that an adequate level of dielectric strength is maintained.

The third paragraph under overvoltages discusses overvoltages due to resonance. It would be interesting to know in what shipboard installations the 4 to 5 times overvoltages have been observed and what type of inductive devices caused the resonance condition, and also the specific type of insulation application and location of the equipment that failed throughout the system under these conditions.

The fourth paragraph under overvoltages discusses arcing faults. It would be interesting to know of specific cases in shipboard systems where this has occurred and what happened to the insulation used.

In paper no. 60.146, under the heading "Undervoltages," Mr Bruning and Mr. Lusby discuss insulation dielectric strength for 450-volt equipment and state that a common ground wall may be 0.010 inch, which would support 20,000 volts for approximately 20 years. No explanation is given as to the materials and construction of the ground wall or the characteristics of the voltage as to time of application. Although the statement by the authors that "for 450-volt equipment, the insulation-to-ground permissible gradient is many times any recorded transient overvoltage" is considered to be correct, the following data are submitted to substantiate that the type of insulation used for Navy shipboard electric equipment is satisfactory for use on ungrounded systems.

Moses, Clokey, and Wolford, in an unpublished conference paper, state that a satisfactory insulation system should have, when new, not less than three times the breakdown of the air spacing to qualify it as a satisfactory dielectric barrier and at the end point of useful life it should still be able to maintain at least $1^1/_2$ times the dielectric breakdown of the equivalent air spacing even under conditions of severe humidification.

If the breakdown of air is established as 1,500 volts, for a 0.010-inch spacing using a-c rms voltage,[1] then a satisfactory 0.010-inch insulation should have a breakdown value of 4,500 volts and an end-of-life value of 1,650 volts. If this criterion was adhered to, then the equivalent rms voltage surges reported in Table III by the authors of 1,770 volts maximum would indicate no practical margin of safety.

Actual Government laboratory tests on many slot-cell flexible materials indicate dielectric strength values as follows after conditioning, heat aging, and humidification:

1. Class *A* composite materials average about 1,800 volts for a 0.007-inch thickness or 260 volts per minute.
2. Class *A* composite materials with plastic film cover sheath average about 7,500 volts for a 0.007-inch thickness or 1,070 volts per minute.
3. Class *B* materials using mica average about 7,100 volts for a 0.008-inch thickness or 890 volts per minute.

The thickness of ground insulation is usually decided upon by the equipment designer and is selected on the basis of the voltage requirements and mechanical handling ability. In general, most designs utilize thicknesses in excess of 0.010 inch. It is evident then that most electric equipment is designed with an adequate margin of safety and the lack of shipboard overvoltage failures indicates that the designs are satisfactory in service. It is believed that the authors' statement would be more correct if the ground wall of 0.010 inch were said to support 8,000 volts for a normal life expectancy.

REFERENCE

1. ELECTRICAL INSULATION, ITS APPLICATION TO SHIPBOARD ELECTRICAL EQUIPMENT (book), G. L. Moses. McGraw-Hill Book Company, Inc., New York, N. Y., 1951.

V. W. Mayer (U. S. Department of Commerce, Maritime Administration, Washington, D. C.): Mr. McSweeney and Mr. Hall, the authors of Paper no. 60-145, deserve commendation for the excellent manner in which all of the advantages of the grounding systems have been developed.

They have pointed out in the section on

personnel safety that a large number of casualties have been caused by the existence of a lethal potential on equipment enclosures. In our opinion, on a properly constructed merchant ship all equipment enclosures should be positively grounded, both through the cable armor and through the equipment mounting. Equipment installed on vibration mounts or otherwise insulated from the hull of the ship are required to be equipped with ground connections.

We cannot agree that a qualified man dealing with an ungrounded system will consider himself safe to the extent of carelessly contacting one phase of a live circuit. It is believed he will have equal respect for the voltages under discussion regardless of whether or not the system is grounded.

We agree with the theory expressed in the description appearing in the second paragraph under heading "Overvoltage." The authors are requested to advise what damaging effect could be expected with this overvoltage on a 450-volt distribution system.

Reference is made by the authors to numerous simultaneous installation failures throughout the system with many grounds and burned-out coils, motors, etc., caused by creation of a series-resonant circuit through ground. Since the Maritime Administration has been unable to obtain evidence which would support even moderate failures of equipment as a result of this type of overvoltage on merchant ships, it would be helpful if Mr. McSweeney and Mr. Hall would provide more specific detailed information as to the merchant ships on which such failures have occurred and the extent of replacement or repairs which were necessary.

The value of an ungrounded system on shipboard should not be overlooked since the operation of the system would not be affected by the first ground. Thus at least two ground faults would be required to cause any interruption of service.

Paper no. 60-146 by Bruning and Lusby covers an excellent presentation of a very interesting and timely subject. It is considered to be quite informative and in the course of our review we have made the following observations.

The damaging effect of a moderate 24,000-ampere fault current which may lead to pressures of 1,000 pounds per square inch generated in the equipment has been pointed out in the section on safety. While we agree that such levels should be avoided whenever possible, we believe that 24,000-ampere fault currents should not cause appreciable damage on a properly protected circuit.

In general we do not disagree with the discussion under this section and we are not convinced that the grounded system for shipboard installations is superior to the ungrounded system. However, since the National Electric Code recommends that systems be grounded and because safety is the primary reason for the existence of the Code which is nationally accepted for shore installations, the possible advantages of grounded systems cannot be overlooked. It would be helpful if the authors of this paper could clarify why the same safety reasoning for selection of the grounded system for shore installation by the National Electric Code should not apply for shipboard installation.

It is not believed the application of differential protection for marine generators would present any problems because of suitable equipment not being available as indicated in the section which describes the "Difficulty of Properly Grounding." We have no record of multiple grounds causing the destruction of a marine generator on shipboard, and because of the large number of machines which have been in service over the years it is our opinion that differential protection for generators on ungrounded systems cannot be economically justified and that the present practice should, therefore, be continued.

In connection with the size of Y-connected generators, investigation reveals that practically all marine generators installed under the cognizance of the Maritime Administration have been Y-connected and recently some of the machines have had all windings brought out to terminals. It would, therefore, appear that grounded systems should not greatly affect the machine design.

P. Smith (The Admiralty Engineering Laboratory, West Drayton, England): I and some of my colleagues have studied Papers no. 60-145 and 60-146 with considerable interest. We are not able to offer any serious comment on this controversial subject because our experience in the British Navy with a-c ships is limited to about 10 years, and although a-c systems are now being installed in liners and tankers, the tendency is to follow naval practice which is to use an ungrounded system.

We adopted the ungrounded system for reasons which are dealt with in the Bruning-Lusby paper and in order to standardize with the United States, and to date have found no tangible reason to doubt the wisdom of our decision. It is interesting to note that a number of 10,000-ton maintenance ships were built in Canada for the British Navy during the war and they had a modest a-c installation consisting of one 300-kw steam generator and four 150-kw diesel generators with the neutral connections grounded at the generators. The connections were made individually to the stator frame of the steam generator and the terminal box of the diesel generators. Unfortunately these ships were little used as the war had ended when they were completed, and while a number have been modernized and converted for other purposes, this has generally resulted in a complete remodeling of the electric system and the neutral grounds have been disconnected to conform with our current practice. We have no information regarding the performance of these systems with grounded neutral, but there were apparently no troubles which could be traced to this.

There is, I think, quite a body of opinion in the United Kingdom which supports an ungrounded system for generation and 440-volt power distribution, but with grounded secondary systems for lighting, small power, etc. This, I think, is a good compromise arrangement, but I have not had any first-hand experience with such a scheme.

J. Garagnon (Service Techniqu Des Constructions et Armes Navales, Paris, France): I understand the interest that would be aroused in the discussion of Papers no. 60-145 and 60-146 by the presentation of the results of experiments on the exploitation of ships equipped with a system grounded to the hull of the ship. A few liners of French construction fall within this category, but I would like to point out that on French Navy ships, the electric installations of which fall within my jurisdiction, the neutral of the electric system is always insulated. I cannot, therefore, furnish any details of personal experimentation on the behavior in actual service of a system with a neutral grounded to the hull of the ship. As far as I am concerned, I remain partial to the principle of insulating the neutral of a-c distribution systems on board ships, whether it relates to the power system at 440 volts or to a lighting system at 115 volts.

With regard to the problems raised in the paper by A. M. Bruning and E. W. Lusby, I would like to make the following comments.

Overvoltage: To date it has not been established that damage has been caused by transitory excess voltage on our ships.

Location of faults: Our ships are provided at each "power" switchboard and at each "lighting" switchboard with an instrument for checking insulation resistance, relative to the ground, of the insulated system. (A direct voltage is applied between an artificial neutral point and the hull of the ship, through a high level of impedance, and the continuous or direct current is measured by means of milliammeter graduated in ohms.) I believe that the possibility of following the development of insulation failure is of great importance in connection with the operation of a system.

As a matter of information, general insulation to ground, as read from the various switchboards on our ships, is on the order of 100,000 ohms. Any insulation below 50,000 ohms is considered insufficient, without necessitating an immediate search for the weak point in the system. Continuity of service remains the primary consideration. In general, the search for a fault is greatly facilitated by maintenance of logs of periodical insulation measurements made by the electrical personnel on board ship for each part of the system.

Safety: It is evident that, considering the usual extent of the electric system of a ship with an insulated neutral, the capacitive currents between phase and ground are dangerous as far as personnel safety is concerned.

Difficulty of properly grounding: The statement that "European ships ..." is inaccurate in this general form. On the other hand, certain systems exist for the differential protection of generators on board ship against internal faults.

Harry G. Parke (Marine Electric Corporation, Brooklyn, N. Y.): These two papers (and the discussions) present an admirably thorough and vigorous discussion of most aspects of the grounded versus ungrounded auxiliary-power-system controversy. It seems to me, however, that the question of personnel safety has been far too quickly dismissed with the undeniably true statement that both systems are dangerous. Granted, but the question is: "How dangerous?" It is this that seems to be one of the strongest arguments for the use of the grounded-neutral system.

From the papers under discussion the llowing conclusions may be drawn:

In a fairly large system the capacitance ground is sufficient to permit a current of nperes to flow through a man connected om a line to ground. Therefore, any ipposed safety deriving from the insulated eutral is imaginary.

In the grounded-neutral 450/260-volt ystem the maximum phase-to-ground voltge is 260 volts.

In the ungrounded-neutral system the hase-to-ground voltage may be 450 volts if iere is a phase-to-ground fault on another hase. Even without a fault, high phase-o-ground voltages may exist in the case of nequal capacitance to ground among hases.

Therefore, the question of the relative afety of the two systems reduces to the uestion of the relative deadliness of 260 nd 450 volts.

As is well known,[1-3] the physiological ffects of electric shock are determined by ne current flowing through the body and ne organs through which it flows. Deaths rom voltage below 1,000 volts are almost ıvariably the result of ventricular fibrilla-ion, a destruction of the heart's rhythm. 'he basic data on ventricular fibrillation vere obtained by Dalziel[1] by experiments on everal species of animals. He found that atal currents were proportional to body veight. Applying his data to man yields he following conclusions: For a 3-second hock, a current of 95–107 ma (milliamperes) vill be lethal to 1/2%, of 258–290 ma to 0%, and of 420–470 ma to 99$^1/_2$% of 70 ilogram men.

The current that will flow through the ody from contact with a given voltage de-ends on the voltage, individual skin char-cteristics, and area of contact. The basic ata were taken by Gilbert[2] and have een further analyzed by Schecter.[3] Body esistances show a wide range of variation ut are generally in the range of one to everal thousand ohms for small area con-acts and further decrease considerably at igher voltages.

By comparing data from the references sted it may be seen that a 2-square-centi-neter contact with a sweaty hand (such as night be produced by a finger accidentally rushing a live wire) would leave the ictim with 200-to-1 odds in favor of survival the voltage were 260, and a 50% chance the voltage were 450. A somewhat better ontact would result in a 50% chance of urvival from a 260-volt contact, and almost ertain death from a 450-volt contact. ince most contacts would be expected to be lancing and in a small area, a 450-volt line-o-ground voltage must be regarded as far nore deadly than a 260-volt line-to-ground oltage.

The situation shown in Fig. 2 of the runing-Lusby paper is especially dangerous ince it presents the likelihood of large area ontact to a supposedly safe surface. As Mr. Bruning and Mr. Lusby rightly point ut, a sufficiently poor equipment ground to ermit a dangerous use of voltage on the quipment frame will draw insufficient cur-ent to trip the circuit breakers. It is true hat danger is equally likely in both systems, ut the danger is greater with the un-rounded system since the voltage is likely be higher.

The grounded-neutral system is not dependent on the overcurrent tripping of circuit breakers for detection of light grounds. Sensitive ground-current ammeters are now available which are capable of detecting ground currents of under an ampere and would certainly appear to belong in any such system. In the case of ungrounded systems the frequent European practice (mentioned by several previous discussers) of having a resistance-measuring device constantly operating between neutral and ground would similarly protect against faults in equipment with high-resistance local grounds and seems to me to be far superior to ground lamps.

REFERENCES

1. DANGEROUS ELECTRIC CURRENTS, Charles F. Dalziel. *AIEE Transactions (Electrical Engineering)*, vol. 65, Aug.-Sept. 1946, pp. 579–85.

2. BODY RESISTANCE, T. C. Gilbert. *The Electrical Review*, London, England, vol. 119, Oct. 2, 1936, pp. 435–46.

3. PREVENTION OF ELECTRIC SHOCK HAZARD, E. Schechter. *Electrical Manufacturing*, New York, N. Y., Jan. 1960, pp. 120–24.

Closing Remarks

R. J. McSweeney and **W. A. Hall:** We would like to express our appreciation and thanks to all discussers for their interesting and discerning comments, and are gratified at the amount of interest shown in this topic. So many of the points debated are apparently irresoluble without operating experience that it is felt the final verdict regarding grounded or ungrounded marine systems must await the construction and operation of some ships with grounded electric systems.

With respect to certain specific discussions, we would like to comment as follows:

We should like to express our thanks to Mr. Kaufmann, who has presented an excellent discussion which supports and effectively supplements our paper.

Commander Estes, U. S. Coast Guard, has posed questions in two areas: ground detection and backup circuit breakers. It is true that we inferred that ground-detection means are not necessary on a neutral-grounded system. We did not, however, intend to preclude the use of ground-detecting schemes. Consideration of this matter shows that some advantage may accrue from the use of a ground-detection instrument such as an ammeter in the neutral path. It is possible that such an instrument could show that ground current was higher than normal and could therefore, at least in theory, provide a warning of the existence of a condition which, unless corrected, would lead to a fault and a shutdown. In our opinion this is a relatively minor detail and should not influence a decision on the subject of operating "grounded" versus "ungrounded." Regarding the question on backup circuit breakers, we can see no reason why a neutral-grounded system should cause frequent opening of backup breakers. Almost invariably, the fault currents at the ends of feeders will be so attenuated that only the circuit breaker of the faulted circuit will open. Only in the event that the branch circuit breaker is unable to perform its normal function will the backup breaker be called upon to prevent further damage. It should be remembered that a "backed-up" breaker on any system may be damaged to the point of uselessness if it attempts to open a fault current beyond its capabilities. If the opening of a "backup" breaker does result in loss of an entire plant, as mentioned by Commander Estes, then the plant layout was not correct in the first place in that either (1) backup breakers were not supplied in sufficient number, or (2) the division of vital loads under each backup breaker was incorrect.

Mr. Raczkowski has correctly quoted a portion of the paper by Kaufmann and Page when he states that in order to protect against arcing faults it would be necessary to add extra equipment such as ground overcurrent relay or bus-ground differential protection, but Mr. Raczkowski failed to point out the fact that Kaufmann and Page further concluded as follows: "Regarding ungrounded systems, about the only practical supplemental relay protection available would be full 3-phase differential protection."

In effect then, both systems are subject to the same hazards in this regard, but it is more expensive to cure on an ungrounded system. Faults of this general type are not likely to happen on shipboard because generally speaking they are caused by working "hot" equipment and this is seldom done aboard ship. In favor of the ship's installation is the fact that it is thoroughly checked out before the vessel leaves the building yard and is not disturbed thereafter by the ship's personnel. Marine gear is very well protected mechanically which eliminates some of the other hazards.

We did not "brush aside" the high-resistance-grounded system because of any great affection for the effective grounding, but we dismissed it because it does not give automatic segregation of the faulted area which we consider to be an advantage of an effectively grounded system.

Mr. Arnemo and Mr. Gott raise the question of high-frequency circulating currents which can obtain when neutral-grounded alternators are operated in parallel. Before discussing this particular matter, it seems necessary to correct a conclusion erroneously drawn by the discussers from our paper. It was never our intention that the disconnecting link between the generator neutral and the associated grounding reactor be used to adjust the system for minimum circulating current. The link was only intended as a means of complete isolation of a given machine to facilitate insulation-resistance readings. Since insulation-resistance readings are taken infrequently, the disconnect link would normally remain closed so that each active alternator would be grounded through its own reactor. It is true that the neutral grounding reactors and their associated circuitry do provide a path for the circulation of third-harmonic currents when the alternators are operating in parallel. If a vessel were to be equipped with alternators of widely differing design, then the problem of circulating current might be a real one and certainly should be given consideration. In American practice, to the best of our knowledge, such an installation has never been considered. If then, we consider only vessels containing practically identical machines with low third-harmonic voltage content and realize, first, that the circulating current is only created by the difference

in the third-harmonic voltages, and second, that the neutral grounding reactors by virtue of their high reactance at triple frequency offer considerable impedance to the flow of such current, we can see that the problem is more of an academic nature than of a real nature. We have calculated this current for two paralleled generators assuming that they both had high third-harmonic content voltages and that by a stretch of the imagination the voltages added up to 20% of system voltage, and then checked the heating that this current would cause. The result indicated that, even under the highly unfavorable assumptions we made, the generator heating would be increased by approximately 2%.

Mr. Tuke questions the point that insulation in marine equipment is not pampered. We did not intend to imply that there is no safety factor in insulation design, but rather that, because of the press of competition, more precise design engineering, and greater confidence in the quality of modern insulating materials, the safety factors are no longer excessive, and therefore more care must be taken to avoid subjecting the equipment to conditions for which it was not designed. We cannot agree with Mr. Tuke's claim that grounds on the 440-volt system are so infrequent that the possibility of a second ground occurring before the first has been cleared can be disregarded. This point is discussed briefly under "Reasons for Grounding."

Mr. Gray mentions a method of simplifying the location of faults on an ungrounded system by switching the ground lamps "so that they can detect ground faults on any section of the system, all indication being on the main switchboard." It is not understood what Mr. Gray has in mind, since a ground on one phase anywhere in the system will provide the same ground lamp indication regardless of where in the system the ground lamps are connected. It is not understood what further information is gained by switching the ground lamps from one location in the system to another.

Mr. Watson brings up a good point regarding neutral grounding when circuit protection is provided by fuses. A single phase-to-ground fault will blow only one fuse, resulting in single-phasing of any loads on that particular circuit. The authors had in mind common American practice, where 3-phase circuit protection is almost exclusively by means of circuit breakers. This point must be carefully watched during design of a grounded system to ensure that any loads which would be harmed by single-phasing are protected by circuit breakers rather than fuses.

Mr. Maupas is apparently one of the few people who has had operating experience with a grounded-neutral ship. It would indeed be interesting if Mr. Maupas could elaborate on his experiences with this ship.

Mr. Maupas feels that there may be some advantages to the use of unarmored cable, eliminating the return path of fault current through the armor. We concur that the cable armor is a poor, in fact unnecessary, return path, but believe the principal reason that armor is used at all is for mechanical protection during the process of installation.

Mr. Apple feels it is highly unlikely that ship designers or operators, who have had years of successful experience with ungrounded systems, will agree on the desirability of grounded systems until such desirability is proved by known experience. While this may be true, we feel it is unfortunate, since there is such a great deal of favorable operating experience available on grounded systems in small industrial plants, which are, as we have pointed out, very similar in all respects to shipboard electric plants.

Mr. Apple also brings up the point of fire hazard: grounded versus ungrounded. While it is true that relatively high impedance faults may remain uncleared and create a fire hazard by local heating, and arcing faults are of course obvious fire hazards, these situations can occur on either grounded or ungrounded systems, so we feel there is no significant difference in fire hazard between the two systems. Additionally, as noted in our reply to Mr. Raczkowski, the cure for arcing fault hazards is less expensive on a grounded system than it is on ungrounded systems.

Mr. O'Meara's comments concerning the practice of the Atlantic Refining Company were most helpful to our case for neutral grounding. We note that in one of the two refineries it was found "necessary to install a reactance-resistance ground on our 2,300-volt system to eliminate simultaneous failures." We contend that shipboard systems are plagued with the same basic troubles as beset land installations and we are consequently grateful for this support. We agree with Mr. O'Meara that both grounded and ungrounded systems leave a lot to be desired in the way of personnel safety. Mr. O'Meara's description of the maintenance practices that obtain on board the ships of the Atlantic fleet confirms our findings in this matter, namely, that if a repair cannot be made immediately it is put off until a more favorable opportunity, such as time in port. Concerning continuity, we note that all of the drives mentioned (steering-gear, boiler fans, fuel oil service, etc.) are installed in duplicate on most ships or with steam-driven standby equipment to protect against mechanical or electrical failure, so we do not quite see the validity of this argument against the use of a grounded system.

Mr. Walker, by the nature of his comments, first called to our attention the fact that our explanation of the meaning of Fig. 3 was inadequate and we are grateful to him for so doing. In one sense, at least, he supports a part of our agrument because he has extrapolated our curve to about the 10 years' point and arrived at a breakdown strength, and then he offered another reference to show that even our curve is practically optimistic.

This reduction in breakdown strength is caused by aging of the insulation and not by the presence of overvoltages. The point that seems to be commonly missed, and we must take the blame for not sufficiently emphasizing this feature, is that overvoltages impressed on the system hasten this aging effect.

Mr. Kaufmann in his discussion of our paper pointed out that "operating a system for 1 day with a ground on one line (73% overvoltage on the other two lines) extracts insulation life equivalent to 3 months' operation at normal voltage." In other words, repeated application of overvoltages accelerates the breakdown of the insulation.

To answer Mr. Walker's other question, we have never personally observed the four to five times overvoltage on any shipboard installation; all the examples in our paper were drawn from industrial plant experiences. The same is true of the intermittent or "sputtering" fault mentioned under the heading of "Overvoltages." This, again, was drawn from industrial plant experience. In the event that Mr. Walker is still interested in further detail of these examples, we will be happy to supply additional data.

We certainly agree with Mr. Mayer's contention that all equipment enclosures should be firmly bonded to the ship's hull by means of ground connections when mounted on insulators for vibration isolation or other reasons. Unfortunately, however, we feel sure Mr. Mayer has seen, as we have on many occasions, bonding straps, either completely broken free or so badly corroded as to provide a high-impedance path to ground. This is a situation which will inevitably occur after the ship has been in service for a time. The grounded system will then lessen the shock hazard as explained in the paper.

With regard to the question regarding the effects of 73% overvoltage, Mr. Mayer is referred to the discussion of Mr. Kaufmann where this question is specifically answered.

In reply to Mr. Mayer's question regarding simultaneous failures caused by overvoltages, as previously noted, all the examples in our paper were drawn from industrial plant experience, not from shipboard installation.

We would like to thank Mr. Parke for his excellent and informative discussion of the relative personnel hazards afforded by the grounded versus the ungrounded system. We also agree that it might be very desirable to provide a ground-current ammeter in grounded systems to provide general state-of-the-insulation indication, similar to the function now performed by ground lamps on ungrounded systems.

A. M. Bruning and **E. W. Lusby:** We want to thank those who have taken the time to comment. The following are answers to the questions raised.

1. Possibility of "high-resistance" grounding: This is certainly a possibility. It does add necessity for further equipment and, more important, requires that more engineering time be spent on design of plant to provide proper relaying without any strong advantage.

2. Practical effects on insulation life: Mr. Walker's data take into account practical effects of mechanical deformation in construction and use rather than idealized laboratory measurements. These results also reflect fabrication procedures.

3. Iron burning due to fault current: We know of no quantitative experimental data.

4. Need for armored cable: New types of insulation are considerably more abrasion resistant. For this reason it is probable that armor braid will gradually be used less and less.

5. Stray capacity and leakage resistance: The data presented are for two specific cases, not an average of many. However, both ships had complex electric systems with a large number of connected feeders. In addition, the large rating plant was on

which had longer and a larger number eeders, rather than simply larger ratings, hat it is reasonable to expected the ca- itance to increase with rating.

Ve agree that a single device or feeder uld have a leakage resistance on the er of megohms.

The possibility of using continuous leak- resistance monitoring instruments is iguing. A periodic quantitative read- routine would be nice. However, we nder if this might not be ignored, in ich case the simple annunciator system v used would be superior.

. Surge voltage of fuses versus breakers: tainly, the basic principle of the two ices (current-limiting action of the fuse forcing current to zero with a relatively ge $L\ di/dt$ as compared with circuit inter- ption at current zero of the properly de- ned circuit breaker with $L\ di/dt=0$) gests the possibility of higher surge tage with a fuse. (Note that all surges Table III were switching surges.) However, in the 450-volt application it is our feeling that this effect is not important in view of the dielectric strength of the 450-volt insulation as discussed previously.

7. "European ships use smaller, simpler plants . . . .": This is based on the fact that European practice does not incorporate nearly as many automatic bus transfers and frequently, and perhaps more realistically, uses lower ratings for drive motors.

8. "Safety of ungrounded versus grounded systems": Certainly, Mr. Parke has presented an interesting syllogism to prove that the grounded system is safer. We believe that before the conclusion is valid, data on how much of the time the average ungrounded system has a voltage above 260 volts are required. The resulting advantage in favor of the grounded system would have to be compared with the lack of safety arising from the possibility of losing the running lights, illumination, capability of steering because of an unexpected first fault on the steering gear pump system, or even the greater possibility of fire as suggested by Mr. Apple. It should be noted that the last possibility is very real; in fact, unilateral proponents of the grounded low-voltage system are beginning to recognize this possibility as emphasized by such recent papers as that by Kaufmann and Page.[1]

It is for safety reasons that the National Electrical Code recommends grounding. However, for the systems to which the National Electrical Code applies, there is the possibility of transformer breakdown permitting the kilovolt primary to raise an ungrounded low-voltage secondary to an extremely dangerous level. And even in this case, there are reasons, in some cases, for preferring the floating system.

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Air Cleaning Features for Traction Equipment

P. G. LESSMANN
MEMBER AIEE

THE EFFECT of dirt on the life and general performance of electric uipment has been the subject of pre- ous papers and discussions. Dirt is ot a unique phenomenon on traction uipment alone. Industrial installa- ons have the same or similar problems. ention need be made only of electrical stallations in such places as cement fac- ries, chemical plants, mines, etc., here the dirt probelm is also serious. owever, what makes the traction field fferent is that the electric equipment part of a moving vehicle, that it is for e most part perhaps the least accessible all electrical installations, and that ilures of electric equipment will enerally stop the traction vehicle some- here remote from help and replacement arts, and thereby tie up a transportation stem. Hence, there arises the interest and need for features to keep trac- on equipment clean and prevent break- owns because of dirt contamination.

This paper deals with the means of emoving dirt from the ventilating air needed to keep the temperature of traction equipments under control. The means discussed here are all based on the principle of using centrifugal action to eliminate dirt from ventilating air. The principle underlying this cleaning action has been used in so-called cyclone-type air cleaners, which are well known. Fig. 1 shows schematically how these dirt eliminators work. Air is blown into the top of this device and is given a rotating motion by the shape of the eliminator housing; the centrifugal force of this rotating motion moves dirt particles toward the periphery of the casing. Gravity and a small air discharge separate the dirt down toward the discharge opening at the bottom. The main air volume is discharged through the top opening. These dirt eliminators have been very successful.

The motion of dirt particles through the air stream caused by centrifugal force and gravity can be analyzed and thereby the performance of the device can be predicted if the sizes of the dirt particles are known. According to theory and tests, dirt particles of a given size attain a terminal velocity when falling free in air i.e., when subjected to the acceleration of gravity. They are affected in similar fashion by the acceleration of centrifugal force. A plot of accelerations, velocities, and distances travelled can be made for typical particle sizes, or simple experiments can be set up to establish the mechanism of the travel of particles. Terminal velocities in free-fall and a general discussion of these phenomena can be found in reference 1 and other works. A method of particle separation by an air stream and gravity is mentioned in reference 2. This same article also discusses motion induced by rotation. It sets up differential equations which are useful in studying these phenomena and evaluating designs.

A similar approach was used to design new means of eliminating dirt from the ventilating air of traction equipment. In all cases, the dirt conditions were studied and dirt samples were analyzed as to particle size, distribution, and specific weight. Samples of these analyses are shown in Fig. 2.

On the basis of this sample, an analysis and design was made to eliminate dirt at the air inlet of a traction motor.

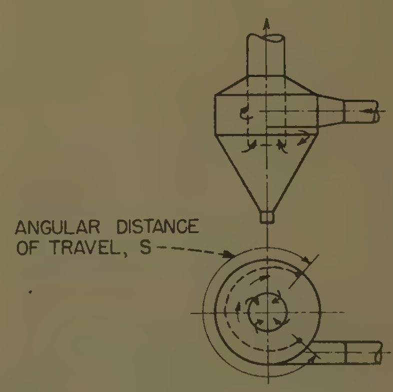

Fig. 1. Cyclone-type air cleaner

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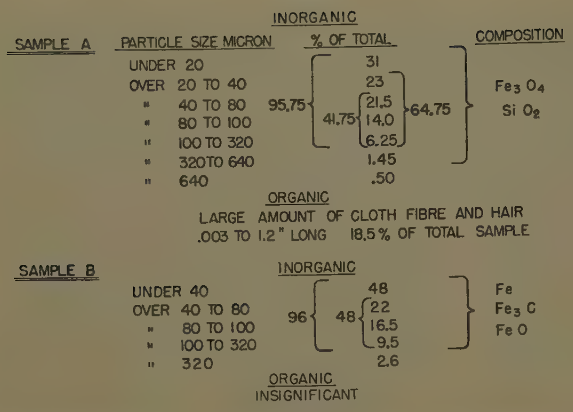

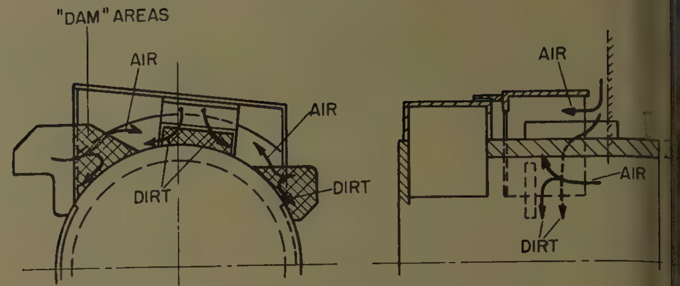

Fig. 2 (left). Analysis of dirt samples

Fig. 3 (above). Dirt elimination in air inlet by centrifugal force around 90-degree bend

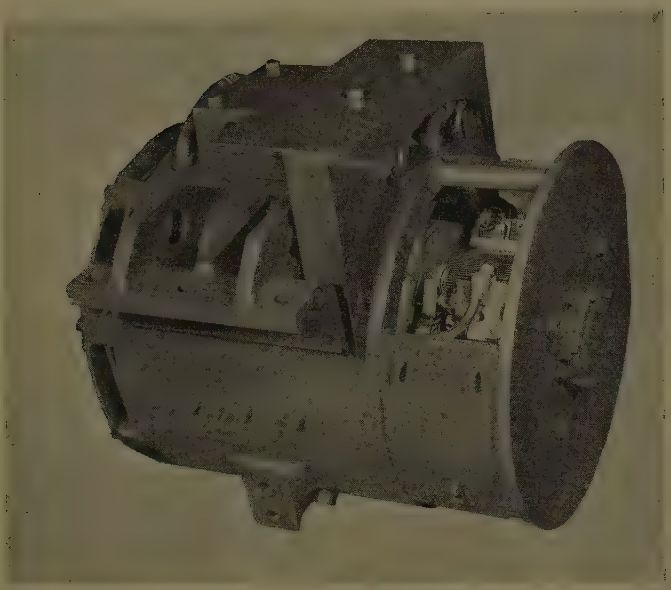

Fig. 4. Dirt elimination on air inlet (view with actual air inlet cover omitted, see Fig. 3)

Fig. 5. Dirt elimination on air inlet (view of motor with air duct complete, see Fig. 3)

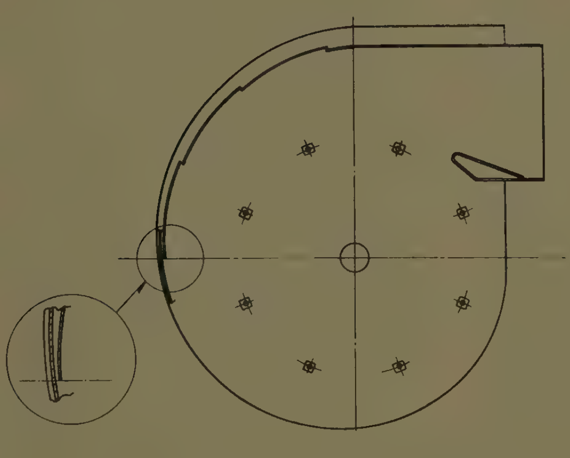

Fig. 6 (left). Centrifugal fan with dirt eliminating feature

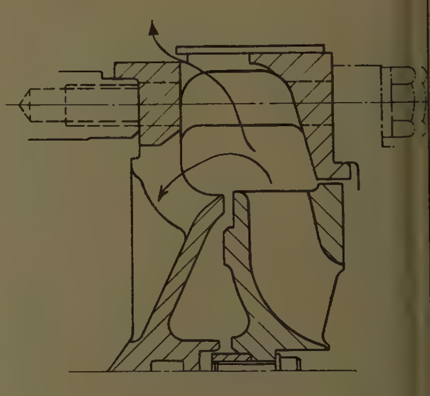

Fig. 7 (right). Dirt eliminating feature on shaft-mounted traction motor fan

Fig. 3 shows the principle which is used to move the air around a "dam" so as to produce at least a 90-degree bend and sufficient centrifugal action to separate most of the dirt out and deposit it outside the dam where gravity will assist in moving it down and to both sides of the motor inlet. It should be noted here that this eliminator works on the inlet side of the fan, whereas the cyclone-type dirt eliminator works on the discharge end of the fan which moves the air.

Dirt elimination on the inlet side is always more difficult than on the outlet side for obvious reasons, principally because the air-pressure gradients and flow directions of dirt in the case of the cyclone-type cleaner are in the same direction, but they are in opposite directions in the case where dirt is to be separated out on the inlet side of the fan or air circuit.

Figs. 4 and 5 show a motor with this type of dirt eliminator. The dams are not recognized readily since part of the structure surrounding the motor is utilized as "dams" or air guides and thereby form a part of the dirt eliminating feature.

A ventilating fan with built-in dirt elimination feature was brought out several years ago. Its principle is illustrated in Fig. 6. The principle is obvious from

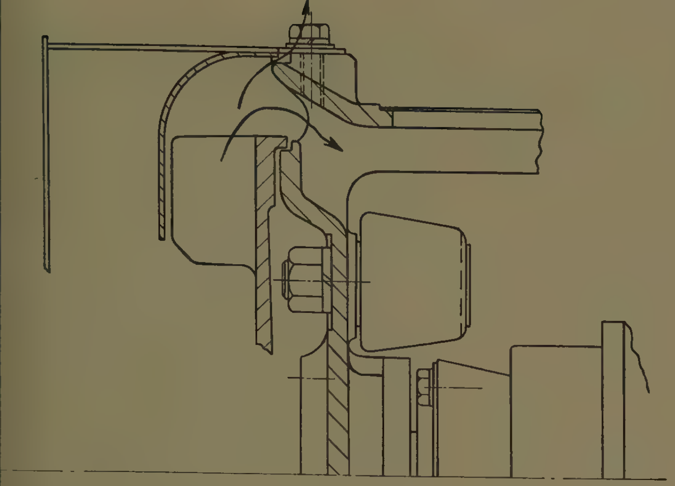

Fig. 8. Dirt eliminating feature on shaft-mounted motor-generator set fan

he figure. A certain amount of the otal air volume is not used for ventilaion and is used instead to carry the dirt particles out. About 15% extra volume hould be allowed for the dirt elimination eature.

The same principle was used on shaft-nounted fans of traction motors and generators. In the case of the traction notor, of course, the fan must operate equally well in both directions of rotation. Likewise, the dirt elimination feature must also work in both directions of rotation. The scheme uses a fan with radial blades, a discharge chamber around the fan, and a radial discharge slot all around the discharge chamber at the end where this chamber connects to the motor inlet. This slot and the configuration of the fan discharge chamber or volute housing effect the dirt elimination. (See Fig. 7.)

A dirt eliminating fan of similar type was also designed for a motor-generator set. Here, the fan operated only in one direction of rotation, hence inclined blades could be used. In addition, because of the nature of the dirt, all passages or their obstructions were made well rounded. The dirt samples in this case contained a large amount of fibers, as much as 1/2 inch long. These fibers rule out the use of filters or screens because, in a very short time, these would be clogged up by the matting of the fibers against the filters and around the screen wires. (See Fig. 8.)

The two types described here have been in operation for about 2 years. Recent inspection shows the respective machines to be appreciably cleaner than duplicate machines using conventional forms of ventilating fan arrangements.

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Economic Trends Make Railroad Electrification Inevitable

L. B. CURTIS
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ELECTRICAL operation of railroads is the most economical form of motive power known today. There are many signs that point to additional electrification in North America soon.

This paper is written to call the attention of the AIEE to at least 13 areas where electrification will effect economies over present diesel-electrical operation and where the first cost of electrification will be reduced. The fact that there has been no new electrification in this country since 1938, there has indeed been some minor shrinkage, should not cause any thinking engineer to come to the erroneous conclusion that electrification is an obsolete form for use in transportation. In this country, existing major electrifications are still maintaining their record of dependability, flexibility, and economy, while abroad electrification of railroads is growing by leaps and bounds.

The 26 engineers on Committee 13, Railway Electrification, of the Electrical Section of the Engineering and Mechanical Divisions of the Association of American Railroads (AAR), are virtually unanimous in their belief that many of the railroads will regard dieselization as a stepping stone to electrification of lines of heavy-traffic density. This committee consists of electrical engineers representing 13 railroads, consulting representatives from industry, including the Edison Electric Institute (EEI), manufacturers of electric apparatus and materials, the telephone industry, the aluminum and copper industries, and consulting engineers.

The committee has three major objectives:

1. To keep the railroads up-to-date on electrification practices and statistics from all over the world.

2. To recommend and develop for this country a standard system of electrification for through service on main-line tracks.

3. To encourage the faith of many engineers who believe that the future of railroad electrification in this country is bright.

In order to further these objectives, Committee 13 for several years has been co-operating with the International Union of Railways in Europe. Statistics and facts from around the world are also developed. The committee solicits the help of the AIEE and related electrical groups to aid it in its mission to demonstrate the economic advantages of electrical operation, where feasible, over other types of motive power. The fields in which the AIEE can help will become apparent in later parts of this paper. Incidentally, there are at present eight electrical engineers who are serving on both Committee 13 and the AIEE Land Transportation Committee.

The basic reasons for railroad manage-

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The author wishes to express his appreciation to L. W. Birch of the Ohio Brass Company; to H. F. Brown, R. Paul, and J. Stair, Jr., of Gibbs & Hill, Inc.; to C. J. Henry, J. W. Horine, and J. E. Hogan of The Pennsylvania Railroad Company; to A. G. Oehler of Railway Locomotives and Cars; and to T. F. Perkinson of the General Electric Company for their helpful suggestions in the preparation of this paper.

ment to give attention to railroad electrification are the measurable economic advantages. Management must be shown that electrification offers the most efficient and least expensive form of motive power for certain applications. It will then direct its attention to this field. Committee 13 needs the co-operation of the AIEE, the EEI, manufacturers, and other interested parties. Certainly the effort to obtain better and more efficient transportation at lower cost is a worthy objective.

Railroad electrification offers many advantages, which have a bearing on the over-all economics, over other modes of motive power. Among them are the following:

1. Power generation in large centrally located plants is more efficient than in many small, space-limited units such as locomotives.
2. More traffic can be economically handled over the same number of tracks.
3. Electric locomotives have the highest availability and the least stand-by losses.
4. Electrical operation is the cleanest and least noisy.
5. There is less terminal congestion.
6. Higher speeds are available for less cost.
7. The electric locomotive is the most compact in length, particularly at the higher horsepower (hp) ratings.

Thirteen Areas of Influence

Since the economics of electrification compared with diesel-electric operation (now over 95% of all operation in the United States) is the major theme of this paper, the thirteen areas where electrification can and will produce savings and an improved financial condition for the railroads are investigated in the following sections.

Conditions Which Would Result in Improved Load Factor

1. Largely as a result of competition from private automobiles, airplanes, trucks, and buses, there will be a trend toward more freight and passenger trains that are shorter, faster, and have more frequent runs. This is already the situation abroad. In North America, freight and passenger trains have become longer in order to reduce costs, mainly labor costs. This has resulted in fewer trains, more costly yards, terminals, and stations, delays in delivery, and a mounting chorus of complaints on the part of the public. The railroads' competitors operate more frequent, and usually faster, service in smaller units. Railroads must meet this competition to get their fair share of the business.

2. The increased use of CTC (centralized traffic control) makes possible the operation of increased number of trains on fewer tracks with the result that excess trackage is abandoned.

3. The trend toward railroad mergers is increasing, particularly with railroads whose tracks parallel each other. With each merger will come abandonment of duplicating tracks and the consequent increase in the number of trains operating on remaining tracks.

As a result of these three factors, there will be more trains operating on remaining tracks and their usage spread over 24 hours of the day, thus producing a higher load factor at public utility generating stations with electrical service. This type of load with its good diversity and high load factor is attractive as a base load to the power companies and would result in favorable rates for electricity.

Electric Locomotive Maintenance Cost

Maintenance costs for electric locomotives are less than for diesel-electric, or any other type, of motive power. As diesel-electric locomotives become older, maintenance costs increase more quickly than maintenance on electric locomotives of the same age. Until railroads have been 100% dieselized for several years, diesel-electric maintenance costs will be misleading. The diesel-electrics have been acquired so rapidly between 1945 and 1955 that the average age has been kept low. The cost of maintenance of the new locomotives is averaged in with the older ones. This averaging of costs has hidden how fast the repair costs of the older units have risen. Now that most railroads are entirely dieselized, the rapid rise in maintenance costs will become apparent. In general, diesel-electric locomotives receive major overhauling in 3-year cycles, with the cost curve rising until, after about 15 years (compared with 25–30 years for the electric locomotive), the old locomotives have been replaced or actually scrapped.

A recent paper[1] shows in graph form that "... the general trends indicate that diesel-electric maintenance, reduced to the same common denominator, will be for locomotives 10 years of age at least twice the maintenance costs of electric locomotives."

A paper, reference 2, in discussing maintenance costs over a period of 9 years of use of electric locomotives on the Virginian Railway states, "The adjusted or constant-dollar cumulative cost to the end of 1956—23.2 cents per mile—represents a figure of 3.41 cents per 1,000 rated rail-hp miles, a value approximately 1/4 to 1/3 of comparable diesel-electric locomotive maintenance costs for the same type of service performed."

The 25- to 30-year life of the electric locomotive is well substantiated. On the Pennsylvania Railroad the 25-year-old *GG-1* locomotives are still giving successful service. The New York Central and the Chicago, Milwaukee, St. Paul & Pacific Railroads have still older electric locomotives in satisfactory service. On the Swiss Federal Railways electric locomotives placed in service as early as 1920 are still in operation.

The lower cost of maintenance and the longer economic life of the electric locomotive, compared with the diesel-electric, spell substantial savings in favor of the former.

Cost of Oil Expected to Rise Faster Than Cost of Electricity

One study[3] on the future cost of oil stated: "By 1970 diesel-fuel costs are expected to increase by possibly 25 to 50 per cent." Certainly, as the consumption of oil in this country and abroad increases, it will be necessary to search harder, drill deeper, and go farther to find additional oil reserves. As the reserves decrease the need for synthetic oil increases. The net result is higher oil costs.

On the other hand, indications are that with increased efficiency the cost of electric power will rise at a slower rate. W. H. Sammis, former President of EEI said in 1954, and it is just as true today, "Our industry has established a unique record during this era of inflation when the purchasing power of the dollar has dropped 50%, when construction costs, federal taxes, labor rates, and the price of fuels have more than doubled. Through engineering achievements, improvements in operations, and well-directed sales efforts, prices for electricity are lower today than they were before World War II. Where can you find another such example of accomplishment?"

At present the cost of oil and electricity at the drawbar is, in general, approximately the same. As the differential between the cost of oil and electricity increases, the savings of electrification over diesel-electric operation increase.

Use of Commercial-Frequency Power

The use of commercial-frequency power will reduce the initial costs of electrification below those of the older established systems.

Recently, Committee 13 has recom-
ended that the United States adopt the -kv 60-cycle system as its standard for ain-line electrification. This system 0 cycles in most countries) is already in eration in the Republic of the Congo elgian Congo), England, France, Ger-any, Hungary, Japan, Luxembourg, rtugal, Russia, and Turkey, It is anned for Argentina and India. These reign installations will be of great aid to is country as the experiences abroad ill help solve many of the problems that ill confront United States railroads in e event of 60-cycle electrification.

Among the problems which will arise e the following:

Load balancing on commercial 3-phase stems. A fine study[4] was made on this bject in this country by the American as and Electric Service Corporation on e Norfolk & Western Railway. The IEE and the EEI can be of great service furthering these studies, particularly by couraging young engineers to interest emselves in this and other railroad prob-ms.

. Inductive interference on communica-on and signal systems. Committee 13 ith the help of the American Telephone nd Telegraph Company has made a omprehensive report on this subject. Iore work needs to be done.

. Semiconductor rectifiers for locomotives. n unpublished AIEE paper was recently resented on the subject of semiconductor ectifiers for electric traction substations. Iuch more must be accomplished.

. Improved pantographs.

. Interpenetrating (intersystem) loco-otives. In Europe locomotives[5] have een developed that operate on four different ystems: 3,000 volts d-c; 15 kv $16^2/_3$ cycles -c; 25 kv 50 cycles a-c; and 1,500 volts -c. The United States will need loco-otives that will operate on at least two ifferent systems.

The large increase in North America f high-voltage interconnecting lines which cross and recross the railroads akes the use of commercial-frequency lectrification all the more feasible. This liminates the necessity for railroad-wned transmission lines on railroad tructures, as the contact system can be irectly fed, with or without transformers, rom these power lines. Co-operative tudies would have to be made by the ailroads and the power companies to nsure a tolerably balanced load on the -phase lines with a minimum of disturb-nce to other loads fed from the same nes.

In addition to the elimination of the ailroad-owned transmission lines, the 5-kv 60-cycle system would effect the ollowing savings over existing types of lectrification in this country:

1. At higher voltages a lighter catenary system is possible.

2. With no transmission lines and lighter catenary the structures and foundations will be smaller and lighter.

3. At higher voltages substations can be installed farther apart.

4. Transformer stations can be wholly or partially eliminated, depending upon the commercial voltage available.

5. With higher voltage the current will be lower, thus reducing the ampere rating of substation apparatus.

6. Frequency changers at supply points will be eliminated.

7. By using rectifier or motor-generator locomotives, d-c substations will be eliminated.

These savings are offset in part by the following additional costs of this system:

1. Somewhat greater clearances are required.

2. Increased insulation is necessary over some existing forms of electrification on overhead, substation apparatus, and motive power.

3. There is the possible requirement of some load-balancing apparatus to maintain balance on commercial 3-phase systems where the latter are small and isolated.

4. Additional engineering will be required initially in developing this system.

Electrification Standards Will Reduce First Costs and Maintenance Costs

Standardization of electrification will reduce engineering, material, construction, and locomotive and multiple-unit car costs.

Engineering costs. With a standard pattern for extensions to existing electrifications and new ones, standardization will be possible for catenary systems, structures, substations, locomotives, etc. Basic engineering will have to be performed regarding catenary makeup, spans, pole and foundation sizes, and spacing and details of substations and sectionalizing points. For similar conditions of load track curvature, and number of tracks to be electrified on any railroad, the same design will apply. Therefore, except for special problems the engineering costs after the first study will be at a minimum. Much of this engineering would be applicable to other fields as well. Here is a good source for future AIEE papers.

Material costs. After the initial engineering has determined wire and pole sizes, hardware, apparatus sizes, and other standard parts, these same items can be used on any standard electrification interchangeably. With all railroads using the same type of material, quantity buying will reduce first costs, also the cost and amount of material to be stocked for maintenance. This should stimulate manufacturers to develop better and lower cost materials and apparatus.

Construction costs. With a standard specification, standard methods of construction will be developed, labor-saving devices employed, and packaged substations, such as the unit type, installed, all of which will reduce the cost of installation.

Since material and construction costs will require standard material and apparatus that can be used in other applications as well as electrification, the National Electrical Manufacturers Association (NEMA) in particular should look ahead and design the best and most economical products for this future market.

Locomotive and multiple-unit car costs. Nothing reduces first cost more than quantity production. In the past, electric locomotives have usually been custom-made. When standard electric locomotives are accepted and used by all railroads, quantity production will greatly reduce both the first cost of motive power and maintenance costs. The same reasoning applies to multiple-unit cars.

Now that the North American railroads are almost completely dieselized replacements will be purchased in reduced quantities with the accompanying loss in the economic advantages of high-volume mass production. Rebuilt diesels have not and undoubtedly will not show any advancement in technology. About all that can be expected are refinements in the present design which already has proved inadequate to provide an economic life comparable with the electric locomotive.

Conversion of Diesel-Electric to Electric Locomotives Will Save Investment Costs

The immense investment in diesel-electric locomotives can be conserved by electrification. Fortunately, those diesel-electrics that have not reached the end of their economic lives can be converted to electrics by replacing the diesel engines and the d-c generators with rectifiers, or the diesel engines alone with a-c motors plus necessary transformers and control. Thus, as has been predicted by others, the diesel-electric system would appear to be an intermediate step between steam operation and electrification.

Higher Speeds Favor Electrification

As mentioned in the first section under Conditions Which Would Result in Im-

proved Load Factor, higher speeds for passenger and freight trains are sure to come in order to compete with airplanes, trucks, and buses. The higher the average speed of trains the greater the financial advantage electrification has over any other type of motive power. In a recent study of a large section of an eastern railroad, it was demonstrated that if speeds were increased an average of approximately 15% over present schedules the annual savings of electrification versus diesel-electric operation would triple. The following illustration demonstrates the advantage electric locomotives have over diesel-electrics at higher speeds.

The maximum train a locomotive can start depends upon weight on drivers and per-cent adhesion. At running speeds there is the additional limitation of horsepower available. For both types of locomotives, reasonable figures are 25% adhesion in starting and about 16% at 60 mph (miles per hour). At 60 mph in the example that follows the limitation for each locomotive is horsepower, not the 16% adhesion.

A typical modern electric (rectifier) locomotive rated 4,000 hp (weight on drivers 174 tons) can start and accelerate on level tangent track a train of about 6,000 tons, and at 60 mph can haul a 2,800-ton train continuously; or, for a short time because of its overload rating, can haul a 4,400-ton train at 60 mph.

A typical modern diesel-electric-unit locomotive rated 2,500 hp (weight on drivers also 174 tons) has approximately 82% or 2,050 hp available at the rails, as the 2,500-hp rating refers to the diesel-engine input to the main generator. This locomotive can also start a train of about 6,000 tons, but it can accelerate a train of only 1,200 tons to 60 mph. The diesel-electric locomotive has no overload hp rating. As a matter of interest this locomotive can haul a 2,800-ton train at 38 mph and 4,400-ton train at 29 mph.

On this basis it would take three (2.33) 2,500-hp diesel-electric units to haul a 2,800-ton train at 60 mph and four (3.67) units to haul a 4,400-ton train at 60 mph. In terms of money the electric locomotive in the first case would cost approximately $440,000, based upon a relatively few custom-built units; and the diesel-electric locomotive approximately $810,000 (3×270,000), based upon mass production. In the second case the electric locomotive would still cost $440,000 compared with a diesel-electric locomotive cost of $1,080,000 (4×270,000).

Obviously, the more units of diesel-electrics compared with the electrics that are necessary, the higher in proportion will be the initial investment, the annual fixed charges, and maintenance and operating costs for motive power.

It should be stated that the cost of the electrification fixed property, that is, catenary, structures, and substations, would be substantially the same regardless of the speed.

CTC Will Reduce Number of Tracks to Be Electrified

Since the installation of centralized traffic control, which is on the increase, permits more trains to be operated on the same track thus reducing the number of tracks required by at least one, and in some multitrack areas by two or more, the resultant number of tracks to be electrified is minimized. In many cases 1-pole bracket structures can be used instead of 2-pole structures, the number of miles of catenary required is lessened, and the sectionalizing and substation requirements are reduced. This situation, brought about by the increasing use of CTC, will reduce the first cost and later the maintenance cost of electrification.

Joint Use of Right of Way Possible with Electrification

As areas are being built up and cities and suburbs are expanding, utilities, particularly the power companies, are finding it more and more difficult and expensive to find rights of way for their transmission lines. In many areas they are looking to the railroads for joint use of their rights of way. This is an ideal arrangement for the power company as the railroad right of way is already cleared and protected. Hence, a natural procedure would be for the utility to erect structures that are suitable for attaching electrification wires if and when the railroad is electrified. Provision for just such an arrangement has already been included in some existing agreements. Since the rentals to the utility take into account future electrification, such a joint use of the structure is of mutual benefit and would result in a considerable reduction in the cost of the fixed portion of the electrification to the railroad. As this arrangement is also of economic advantage to the power companies, this is another field where the AIEE and the EEI can assist Committee 13.

Electrification Permits Additional Income From Air Rights

In congested areas one of the valuable assets a railroad has is its air rights. Previously, with steam and now somewhat less with diesel-electric locomotives, these valuable air rights in heavily populated areas, could not be utilize[d] since ventilation is essential. With elec[trification], areas of air rights can be sol[d] or rented, bringing to the railroads addi[tional] needed revenues. The value o[f] this product of electrification should no[t] be overlooked.

Future Use of Atomic Power

Vice Admiral Rickover had this to sa[y] in an after-dinner speech before th[e] Scientific Assembly of the Minneso[ta] State Medical Association in 195[7]: "Another limit in the use of nuclear powe[r] is that we do not know how to employ i[t] otherwise than in large units to produc[e] electricity or to supply heating. Becaus[e] of its inherent characteristics, nuclear fu[el] cannot be used directly in small machine[s] such as cars, trucks, or tractors. Rathe[r] than nuclear locomotives, it might prov[e] advantageous to move trains by elec[tricity] produced in nuclear central sta[tions]."

During August of 1959 the author ha[d] the privilege of inspecting the new Dre[s]den Nuclear Power Station which wen[t] into service early in 1960. This plan[t] will add 180,000 kw to the Common[wealth] Edison Company of Illinois reser[voir] of electricity. It is estimated tha[t] by 1975 the United States will be usin[g] nearly four times as much electricty a[s] today—a tremendous drain on the re[serves] of coal, oil, and gas. Hence atomic power must be used to help mee[t] future electricity needs. It is expecte[d] that after the high initial developmen[t] costs are absorbed, the cost of generatin[g] electricity by atomic power will compete with that using conventional fuels. Wit[h] the experience gained from this large ex[perimental] plant, other more efficien[t] atomic power plants are already in th[e] building and planning stage.

Thus atomic power will do its part i[n] producing abundant amounts of electricit[y] at reasonable costs to take ample car[e] of the future needs of electrification.

Possible Shortage of Oil

In the event of war or some oth[er] catastrophe that would bring about [a] shortage of oil, rationing, and prioritie[s], the railroads, with all their motive pow[er] dependent upon oil, could very well be i[n] trouble. It would be a decided ad[vantage] for rail transportation, unde[r] such conditions, to have its major line[s] electrified, for the generation of electricit[y] is not dependent upon oil alone. Oth[er] fuels and sources of power such as wate[r], coal, poorer grades of oil, gas, and atom[ic] power would give a diversity that cou[ld] keep the railroad wheels turning.

Effect of Foreign Market on Electrification

Another item that will undoubtedly reduce the cost of electrification in this country is the competition of the foreign market. Unfortunately for our home suppliers of material, apparatus, and equipment, our high cost of labor is driving prices up to the point where foreign goods are being sold in this country in increasing amounts. Unless something is done this situation will become worse. In such an event it is possible that electrification in the United States will come almost entirely from foreign material and apparatus and with trains pulled by foreign-manufactured locomotives. At any rate, the foreign competition will tend to lessen the cost of future electrification. Of course, foreign competition will tend to lower the cost of diesel-electric locomotives too, but anything that will help reduce the first costs will make electrification more attractive.

Summary and Conclusions

To summarize, the following factors point to the ultimate reduction in cost of electrification facilities and reduction in the cost of operation of railroads with electrification to the point where electrification must be considered:

1. A high electric load factor.
2. Reduced locomotive maintenance with electric locomotives.
3. Favorable cost of electricity versus oil.
4. Use of commercial-frequency electricity for electrification.
5. Standardization of electrification.
6. Conversion of diesel-electrics to electric locomotives.
7. Higher speeds.
8. Number of tracks reduced by CTC.
9. Joint use of rights of way.
10. Increased sale of air rights.
11. Use of atomic power.
12. Shortage of oil in times of national emergencies.
13. Effect of foreign market.

It is the firm opinion of the author that these factors will not only produce a form of motive power that is more economical to operate and maintain than the present diesel-electric system, but will reduce first costs of electrification to a point where they are equal to or below the cost of the additional diesel-electric motive power required to give equivalent electrical service.

The foregoing evidence points to a more favorable economic position of electrification compared with other forms of motive power. This is of prime importance to the railroads themselves as they are forced to compete under the present unfair conditions of subsidy, taxation, and antiquated regulations. It is of national importance that the railroads be strong and efficient, for in times of national emergency no other mode of transportation in existence can replace them. Electrification is also of importance to the power companies, the electrical manufacturers, and other related industries. It will contribute immensely to the nation's prosperity. It is a project worthy of the best efforts of the AAR, AIEE, EEI, NEMA, etc. With sufficient united effort and unanimity of opinion on the part of the engineers, railroad management will be glad to "Stop–Look–Listen." Their investigations will convince them that electric motive power will give them the most efficient and economical operation obtainable, and then the resurgence of railroad electrification will be inevitable.

References

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2. Virginian Railway Motor-Generator Electric Locomotive Maintenance Costs, T. F. Perkinson. *Ibid.*, vol. 79, Mar. 1960, pp. 33–35.

3. A Preliminary Investigation of the Possibilities of Expansion of Railroad Electrification in the United States. *Report*, Battelle Memorial Institute, Columbus, Ohio, Apr. 23, 1952, p. 2.

4. Technical Aspects of Providing Service to Single-Phase 60-Cycle Railroad Loads, T. J. Nagel, A. F. Gabrielle. *AIEE Transactions*, pt. II (*Applications and Industry*), vol. 77, July 1958, pp. 172–76.

5. Four-System Electric T.E.E. Trains. *Railway Gazette*, London, England, Apr. 10, 1959, p. 419.

Discussion

R. M. Stacy (Fairbanks, Morse and Company, Beloit, Wis.): Mr. Curtis, together with Messrs. Birch, Brown, Stair, Horine, Oehler, Perkinson, and others, deserve a vote of thanks for continuing to furnish the "motive power" for this type of paper, even in the face of considerable odds.

Although I do not agree with all of the statements or conclusions, the very fact that the paper stimulates thinking and discussion makes it worth while.

Many call this paper purely academic in view of the fact that railroads can't even get money for new boxcars, let alone electrification. What is the answer? At the American Society of Mechanical Engineers–AIEE Railroad Conference in April 1959, V. E. McCoy, Chief Purchasing Officer of the Milwaukee Railroad, pointed out how capital was not available to railroads even for improvements calculated to yield as high as 40% return.

When comparing operating costs of electric versus diesel-electric locomotives, the cost of power supply and distribution must be included in the cost for the straight electric or the two figures are not comparable. However, the investment in about 1,500-central-generating-station hp will do the same work as 6,000 hp in self-propelled electric locomotives, because a certain percentage of locomotives always lay over or only work part-load. Reviewing this, John W. Barriger, President of the Pittsburgh and Lake Erie Railroad says that on dense traffic lines electrification can yield a sizeable return on the investment even including cost of repair and maintenance of overhead. It would be interesting to see an analysis of this.

It is gratifying to see this paper push the advantages of the locomotive with high horsepower per axle. Railroads have not been convinced that the high hp locomotive is to their economic advantage. In fact, the trend has been the opposite, with all builders constructing 1,600- and 2,400-hp locomotives with ballast and extra traction motors added to increase maximum tractive force for a given hp rating. A reversal seems to be at hand, however, with the expansion of operation of lighter, faster freight trains. If reduction of unneeded members of train crews is accomplished, this trend should accelerate. However, high locomotive horsepower alone, particularly on grades, is limited by weight on drivers.

A *GG-1* or new 4,400-hp electric locomotive if operating on the Pennsylvania Railroad's (PRR) Pittsburgh division for instance, would require a helper locomotive where present 3-unit 4,800-rail-hp diesel power does not; for example, the Broadway Limited when hauling over 15 cars. The same is true when dynamic braking capacity is considered. Also, high-speed wheel slip on flat country can be a problem. Demonstrating a 4,800-hp 2-unit diesel-electric on the Broadway Limited in 1950, and running 4,500-rail hp at the time, we lost running time because of high-speed wheel slip at 60 mph on wet rail west out of Fort Wayne, with the Limited pulling 16 cars with 506,000 pounds on drivers.

The operating flexibility of electric multiple-unit trains, with their fast acceleration and ability to operate without firemen even under present labor agreements, is another electrification potential.

Particular issue is taken with the statement, "Rebuilt diesels have not and undoubtedly will not show any advancement in technology." Actually the very opposite is true. In fact, the 66 new electrics on order for the PRR include many advances made on the diesel, such as the underframe, swivel-type trucks, d-c traction motors, control equipment, road-switcher-type carbody construction, and dynamic brake.

Also, diesel engine technology has made enormous advances since 1946.

The statement that diesel-electric locomotives receive major overhauling in 3-year cycles is considerably in error, the true figure being 6 to 12 years. Three years is generally true for the diesel engine, which, if desired, can be unit-exchanged without tying up the entire locomotive.

On a particular group of 8-year-old 1,600-hp 4-motor diesel locomotives recently overhauled for a railroad by one builder, using nearly all original components, it is interesting to note that the cost followed the familiar formula of 34% engine and auxiliaries, 33% carbody, and 33% electrical. Adhesion is the limiting factor in these locomotives' service. Thus if this group of locomotives were electric, the overhaul cost would still run 66% of the diesel-electric cost, plus any overhaul costs for pantographs, transformers, rectifiers, and associated auxiliary equipment.

E. B. Shew (Philadelphia Electric Company, Philadelphia, Pa.): The title of the paper is pleasing to the utility engineer. It is my hope railroad electrification is inevitable, not only for selfish reasons but also because the economic health and reliability of the railroad industry are important to our national economy and to the national defense. I believe these will be enhanced by a progressive program of railroad electrification.

The author has supplied one good reason for this belief by stating that the cost of diesel fuel will rise faster than the cost of electricity, gradually improving the competitive position of utility sources. Aside from the economics of the situation, it is not difficult to visualize unusual conditions associated with a world war or disruptive events in the Middle East which might affect the availability of oil at any price. Is it desirable for the railroads, with their important responsibilities, to rely on one source of power when in contrast the electrical utilities offer five primary sources of power: coal, oil, gas, water, and nuclear energy? Under normal conditions, these are used as determined by the economics of the prevailing situations but in case any one of the sources is not available, the others are used. All of this economy, flexibility, and dependability is available to electrified railroads supplied from commercial-power systems.

Considering the additional facts that the diesel locomotive must provide and transport its own power plant while the electric locomotive fundamentally is concerned only with utilization; that the diesel must provide its own reserve capacity while the electric locomotive shifts this responsibility to the electrical supply system; and the better efficiency, better reliability, and longer life of the electric locomotive, the electrified railroad has many advantages.

It was interesting to note that Mr. Curtis believes there are many signs that point to an early revival of railroad electrification in North America. We cannot quite agree, however, that dieselization is a hopeful sign. While, as stated in the paper, there is some possibility of converting diesels to electrics, the utilities have regarded dieselization with considerable apprehension and we would have selected a more direct approach to electrification. Since so many other favorable factors are mentioned by the author, we hope there will soon be developments which will lead to a revolution in the position of railway electrification in the United States.

One favorable factor may be the use of the commercial-frequency system of supply. The advantage here is to eliminate the costs associated with conversion equipment of various kinds affecting substantially the economics of electrification. While there are indications the attractive principle of commercial-frequency electrification can be worked out practically, it seems necessary to emphasize in this connection that there are substantial problems associated with the idea which will require a great deal of time and money for the necessary intensive studies by the utilities, the electrical manufacturers, and the railroads. It may be interesting at this point to quote from reference 1: "Substantial technical problems are involved in the proposal to supply single-phase railway electrifications at commercial frequency from three-phase power systems. There are indications these problems can be resolved but it would not be reasonable to proceed with the expensive and time-consuming investigations which would be necessary to confirm this unless the railways are interested seriously in such a project. The Task Group on Railway Electrification therefore will not pursue the matter further until there are developments in other areas of sufficient promise to justify the resumption of activity."

The electrical utilities stand ready to proceed aggressively if there should develop some indication of practical interest on the part of the railroad industry. At present there seems to be none and, in fact, there are strong indications based on recent events the tendency is more than even toward dieselization rather than electrification. This, of course, leaves the electrical utilities without practical motive to pursue the matter of commercial-frequency electrification. This is a regrettable situation indeed because progress under these circumstances will be limited.

To summarize, the utilities are much interested in any consideration which will lead to a revival in electrification but it will be necessary to proceed deliberately with some of the associated technical problems. The problems are not yet solved and their solution will not be easy or quick.

Reference

1. *Advance Reports*, 7th Annual Meeting, Electrical Section, Association of American Railroads, June 1959, p. 159.

N. E. Funk (Consulting Engineer, Philadelphia, Pa.): The author is to be congratulated on the presentation of a very timely paper since the fashion seems to be to add more diesel-motive power without any apparent consideration of other means of power supply. I wish to state that I am in favor of railroad electrification, where conditions make it economical for both the railroad and the electrical utility, should that be the source of the power supply, because the following comments may otherwise be misconstrued. Moreover, this is more a discussion of electrification in general than of Mr. Curtis' paper.

This problem must be approached in a very realistic manner with any wishful thinking completely eliminated. The serious consideration of 60 cycles on the trolley wire removes the expensive item of conversion to 25 cycles single phase from the polyphase 60-cycle system of the electrical utility system. However, there are other problems that are not too easily solved. I refer to the supply of single-phase service directly from a polyphase system. The power of a polyphase system is continuous while that of a single-phase system passes through zero twice every cycle.

I am familiar with only two methods of converting the supply of single-phase power into a continuous draught of power from a polyphase system: first, by storing mechanical energy in a rotating body; second, by storing energy in a magnetic and electric field, both during the time that the single-phase load passes through zero and then releasing this stored energy during the peak part of the single-phase load. The storing of mechanical energy for this purpose has been in operation for many years. In cases where the single-phase load was supplied at 25 cycles from 60-cycle systems, frequency changers have been used, a 60-cycle polyphase motor driving a 25-cycle single-phase generator. This is a simple automatic means of obtaining the desired result without the need of any complicated controlling devices; however, because the frequency relations limited the speed to 300 rpm, it resulted in very large and expensive machines Where the 25-cycle lead was supplied from a 25-cycle polyphase bus, phase balancers have been successfully used. These machines are not as complicated in their construction as in their theory and have the advantage of being limited in speed only by design characteristics and the frequency on which they operate. The control however is quite intricate and requires numerous auxiliary machines, which again adds to the cost of service.

Personally I do not know that the method suggested some years ago using a network of reactors and capacitors to balance a single-phase load on a polyphase system has ever been tried, but it is evident that it will be necessary to arrange some method of varying both with variations in the single-phase load, at least down to some low-load value, which, on the face of it, would seem to make the practical application much less simple than appears at first glance.

Because of these costs, suggestions have been made that single-phase railroad load be carried on a utility transmission line erected along the railroad right of way without any attempt to balance the railroad load in any one section of the railroad but to supply adjacent sections from different phases of the polyphase transmission line. This was tried on the original electrification of the Pennsylvania Railroad Company's suburban lines. The Chestnut Hill branch was supplied from one phase and the Paoli branch from the other phase of a 2-phase system with a result that was far from satisfactory since a complete balance never was obtained because of different road-bed characteristics, train lengths, schedules, etc. Another result was that the load went completely haywire if either branch got off schedule. This type of supply was abandoned when through main-line electrification was established.

The argument that the utility load is

made up in large part of single-phase loads does not hold for the railroad load as these other utility loads are very small units and are balanced locally and, more important, stay in the same location while the railroad load moves along the railroad right of way.

The ratio of the railroad load to the total load of the power company is not the problem, but rather the ratio of the railroad load to the polyphase load on the affected transmission line. Every polyphase synchronous or induction machine supplied from the unbalanced line will act as a phase balancer (not to the extent that a machine designed for the purpose would) in sufficient amount to cause overheating unless considerably overrated for the loads they are carrying. It is not right that these polyphase customers should supply this service free of charge to the railroad or that the utility company should be called upon to correct the situation at no cost to the railroad.

This discussion may seem to indicate that I feel that the problems involved are so great that a lower cost power service to the railroads is out of the question. This is far from true. All I wish to do is indicate that, because of the problems involved, there is no general solution that can be applied to all cases. Each railroad electrification is the occasion for a special study that takes all the variables of that particular situation into consideration and comes up with the right answer, not one that is arrived at based on assumptions that do not exist.

C. W. Maloney (Stone & Webster Engineering Corporation, Boston, Mass.): It was a pleasure to read the fine article written by Mr. Curtis who so ably presents the cause for expanding railroad electrification which could be an attractive source of revenue for electric power companies.

Standardization of power characteristics would go a long way toward attracting the electrical utilities. With systems at 25 kv and at commercial frequency the railroads could then be classed with all large industrial power users. In this case the service would be at the high-voltage level and all transmission and substation costs would be borne by the power company. High-voltage rates, generally lower than utilization voltage rates, would prevail.

In one important respect as far as power supply is concerned, railroads differ from manufacturing plants in that the location of the latter definitely determines the point of service. With railroads these connections could be selected at points most convenient to the power company. As Mr. Curtis points out, the existing frequent transmission-line crossings and recrossings might influence the choice of feed points which at 25 kv could be in the order of 30 miles or so apart. Thus, by giving the utility the option of service location, they may be induced to establish special railroad rates lower than those applying to industrial users.

Where present crossings are too far apart or are not in a convenient location, joint use of railroad right of way should be of mutual benefit. Joint use might be extended to permit the power company to serve other users; thus, the railroads might work hand in hand with the public utilities in developing new manufacturing sites, complete with rail and power facilities.

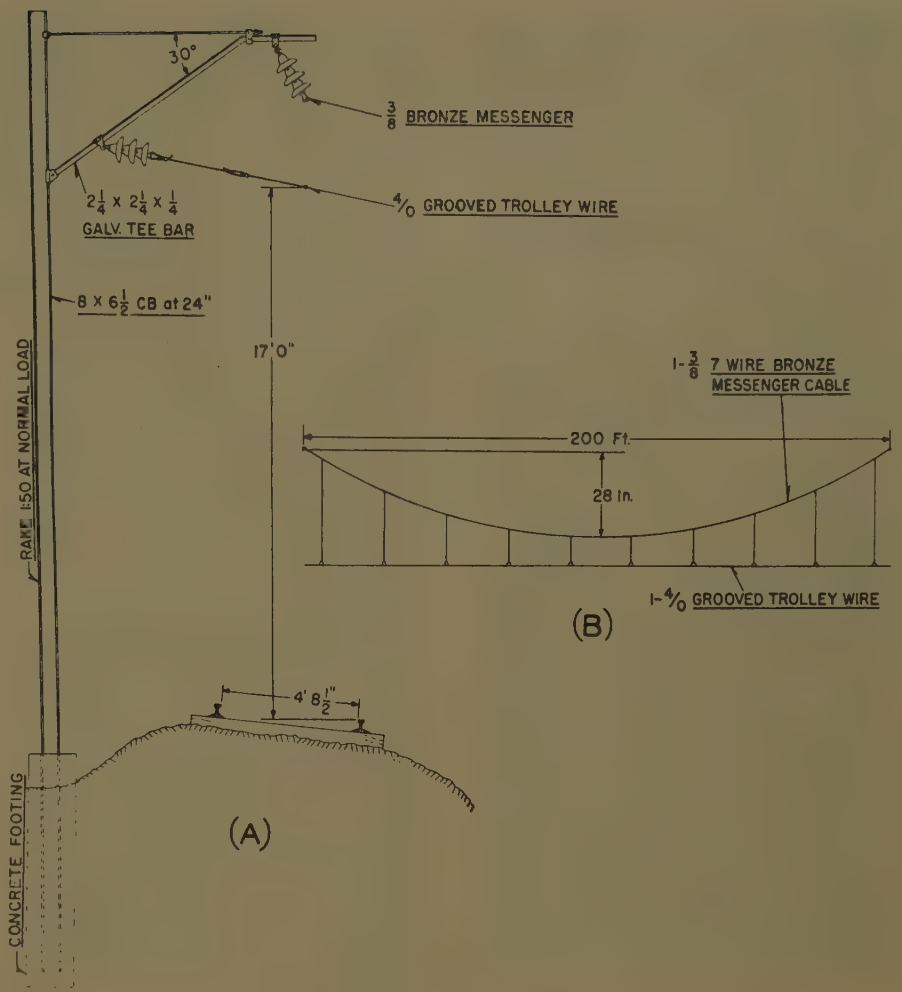

Fig. 1. Belgian Congo 25-kv catenary design

A—Single-track bracket pole, 25-kv 60-cycle construction
B—Catenary span; total conductivity, 300,000 cm; weight per foot, 1.1 pounds

With all these considerations, it appears that by the means of electrification and through the efforts of such forward thinking individuals as Mr. Curtis, railroads can regain the important part they once played in the development of this country's industrial progess.

L. W. Birch (Ohio Brass Company, Mansfield, Ohio): I would like to add the distribution design, Fig. 1, to the discussion of Mr. Curtis' paper. This single-track structure and simple catenary span, although designed for an operation in the Belgian Congo, seems to satisfy most of the requirements for a standard 25-kv commercial-frequency overhead system for standard-gage United States and Canadian railroads. This figure has already been included in the 1960 report of Committee 13, Electrical Section, AAR.

Economy of electrification starts before actual construction of an overhead system. There are certain existing railroad standards that reduce the cost of electrification but, on the other hand, the lack of certain standards can waste the railroad's money. For example, the single-track structure in Fig. 1(A), assembled with standard steel or aluminum shapes and clamps, may satisfy a number of railroad requirements but the cost of the structure can vary greatly if certain dimensions are not standardized. The height of the trolley wire affects the moments in both the columns and the trusses of multiple-track structures, as well as the column in single-track structure. The separation of tracks affects the length of trusses and, as a consequence, necessitates heavier structures.

Vertical dimensions of the structures bearing on trolley-wire height have been relieved due to the restrictions against men riding on tops of boxcars. Also, horizontal dimensions of structures have been greatly standardized due to specifications on separation between track centers. The Electrical Section of AAR is recommending a 19-foot trolley-wire height for main-line electrification.

Another means of reducing structural costs is by eliminating duplication of engineering drawings. In my AIEE paper, reference 1, an effort was made to emphasize the duplications that take place in the engineering of a railroad electrification, particularly the duplication of working drawings. First, the railroad produces an excellent set of loading drawings for each structure, then proceeds to design the steel structures necessary for handling these loads. Second, the railroad details the structures preparatory to sending

to the fabricator and the fabricator, in turn, duplicates these drawings. On many jobs, particularly foreign, the loading drawings go direct to the fabricator, no detailing by the railroad is attempted. These drawings cost money.

REFERENCE

1. ARE THE OVERHEAD DISTRIBUTION COSTS RETARDING RAILROAD ELECTRIFICATION? L. W. Birch. *AIEE Transactions*, vol. 67, pt. I, 1948, pp. 243–57.

F. O. Billings (Industrial Analyst, Newport, Wash.): The encouraging future for railroad electrification in the United States as envisioned by Mr. Curtis is bound to interest electrical utility men in the Pacific Northwest.

Electric power in the Pacific Northwest, both privately and publicly owned, is very low cost, and there would be plenty of it available for railroad use should the author's thesis prove correct. Of course, we don't have the traffic density that exists in the east, but the lower cost of electricity here would help make up for that to some extent. Should the railroad companies form an association or corporation that would do the power buying for all of them and would be responsible for all matters of distribution to trackside delivery points, further savings would result. Finally, it should be mentioned that Pacific Northwest power is water generated: it doesn't escalate in cost with time as does fuel-generated power.

Many utility executives in this region will want to keep in close touch with the work of Committee 13.

H. F. Brown (Gibbs & Hill, Inc., New York, N. Y.): This paper presents a comprehensive summary of the economic factors which are gradually turning attention toward further railway electrification in this country. Most of these factors have been given careful consideration and adoption in other countries where the railways are more closely identified with the national economy than they are here.

It is somewhat incongruous to observe the progress made in railway electrification for economy in war-devastated Europe since 1947, a substantial amount of which has been financed by the United States taxpayer, and then to note and compare the economic difficulties in which many of the American railways are involved today. In this country, insofar as transportation is concerned, the taxpayer seems to think only in terms of more improved highways and terminal parking areas for his automobile. Apparently very few taxpayers or their elected representatives consider the railways to be a necessity today.

The author of this paper is known for his energetic leadership as chairman of the Committee on Railway Electrification of the Electrical Section of the AAR. One function of the AAR committees is to study conditions and methods wherein and whereby railway operations may be improved. This paper is not only an excellent example of such AAR activity, but also of that of the Land Transportation Committee of the AIEE in encouraging the presentation of papers directed toward these same ends.

Naturally, a great deal of railway operating economy is involved with motive power. The diesel-electric locomotive, in the United States, was evolved from the electric locomotive. Electrical engineers have possibly had as much to do with the development and success of the diesel locomotive as the mechanical engineers who improved the diesel engine. This type of locomotive would not have been applied so extensively without the electric drive as a torque converter. There are now types of mechanical torque converters which some day may replace the electric drive.

Electrification has already been used on parts of the railways of the United States for more than 50 years. Where it has been applied for economy over steam operations, it has never been replaced. It has been replaced by the diesel in most of the short operations where the diesel, with additional ventilation, can also overcome the former limitations of steam operation in long tunnels.

The renewed interest in railway electrification is based on the fact, now becoming apparent to those who are studying motive-power economics, that electrification has the same economic potentials, applied to dense traffic now operated with diesel-motive power, that it formerly had with steam operation of such traffic.

This may be disputed, because there is a great deal of "mythology" which has been developed along with the application of diesel-electric motive power to American railways. Greatly exaggerated claims for its economies have been made, some of which are gradually being shown in their true value as the time progresses. Mythology always fades under the light of truth.

The railways of the United States cannot be called a growing industry at the present time. Any industrial enterprise not growing has difficulty in attracting new capital. Yet the railways are continually requiring and seeking new capital for improvements and replacements. With little total increase in traffic since the end of the war, the total investment of the class I railways has been increased by one third.

The railways would require less new capital if they could reduce their present large annual capital consumption. The class I railways of this country have spent an average of $200 million annually since 1945 for new diesel-motive power alone. Most of this, it is now becoming apparent, has an average economic life of but 15 years. Therefore this capital expenditure and renewal must continue.

The maintenance of this diesel-motive power in road service alone on these railways is now costing over $1 million per day; $365 million in 1956, and $377 million in 1957. Thus, the maintenance and replacement of diesel-motive power on our class I railways are costing, and will continue to cost, more than one-half billion dollars per year.

Reduction of these large maintenance expenses, and the conservation of the large consumption of capital for motive power, are the two major and immediate economies electrification can make in railway operations today.

Where locomotive operations per mile of track are sufficiently large, that is, with sufficient traffic density, on carefully selected routes, the total investment in electrification and electric-motive power can be not greater than for the total investment in diesel-motive power alone, over the life of the electrification. With such electrical operation, the capital consumption will be actually less over a 30-year period than with the continued investment in diesel power. The operating savings exclusive of fuel, now on a par, can be quite attractive compared with diesel operation.

It is not necessary to wait for some tomorrow for advances in fuel costs for their additional savings. These advances will surely arrive in due time and will then add to the continued high cost of diesel operation, or to the savings with electrical operation where this has been substituted.

It is extremely short-sighted for the railways to state their inability to plan for more than 10 to 15 years ahead, the expected life of their present motive power. Surely the tremendous expenditure of more than $40 billion planned for our national highways during the next 15 years visualizes more than an additional 15-year life and use. The railways must start soon to plan how this new competition will be met. It can be met by higher speeds, which are only possible with electrical operation.

The railways cannot look to the taxpayers for financial assistance unless they want government ownership or government operation. They are entitled to increased rates where necessary to meet rising costs, but they must also search for further economies in their own operations to enable the increased revenues to meet the total expenses.

The engineering professions: civil, mechanical, and electrical, are all vitally involved, and will continue to aid the railways in this search for more economical operations. This paper gives emphasis to one method now being used extensively all over the world, except in the United States.

T. J. Nagel and **B. A. Ross** (American Electric Power Service Corporation, New York, N. Y.): We would like to commend Mr. Curtis for his forward thinking with regard to railroad electrification. Electrified railways represent a substantial load to the power industry which, if realized, would be of benefit to the railroads, the coal industry, and ourselves.

New railroad electrification in this country has been confronted by several obstacles, the most important of which have probably been the relatively high capital cost of installing the overhead catenary system, the need for expensive conversion or balancing equipment, and, in many cases, the need for a separate railway transmission system.

The catenary problem needs new approaches in order to simplify and reduce the cost of the overhead system installations. The current use of relatively high voltages, e.g., 25 kv, is a step in that direction. Improvements in overhead insulation techniques, both with regard to motive power and adjacent structures, is another area in which reductions in this cost may be achieved.

The conversion equipment and transmission obstacles can be met by direct service from power-company transmission systems at commercial frequency, without phase balancing or conversion equipment. Recent studies by the American Electric

ower Company and the General Electric Company have dealt with the capabilities f central station 3-phase generators for upplying single-phase loads. The power ystem in many areas may already be able to upply the railway electrification load from xisting facilities. This is particularly true f an attempt is made to provide multipoint ervice from separate phases, thereby obaining a relatively good balance of load. Our own studies have indicated this to be easible.

Further proof of this power-system apability is offered by recent European xperience, particularly in France, where ittle difficulty has been experienced in suplying relatively large railway single-phase oads. Continued growth of power systems vill further enhance their ability to supply nbalanced loads of this type.

We urge the railroads to join the power ndustry in keeping alive an aggressive pproach to the railway electrification probem. It will require co-operative and incere effort on both our parts if we are to ealize the goal of extensive railway electriication in this country.

. B. Curtis: The author appreciates the nterest of those who submitted comments on this paper. The discussions are contructive and help focus attention on this mportant subject.

Mr. Stacy asks where the money will come rom to electrify even if the savings are subtantial. In 5 to 10 years American railoads will have to replace the bulk of their diesel-electric fleet of locomotives as they will have passed their economic life. It might pay some of these railroads to consider electrification. Also it is hoped that by that time the railroads will be able to compete on more equal terms with their competitors on land, water, and air, and thus be more economically sound. The author believes the electric locomotive will do as well or better than the diesel-electric on grades or in regard to high-speed wheel slip. Experience has shown that in electrified territory passenger trains hauled by diesel-electric locomotives cannot maintain the schedule assigned the electric locomotive trains. In my statement that rebuilt diesels have not and undoubtedly will not show any advancement in technology, the word "diesels" is loosely used to refer to the diesel-engine part of the locomotive.

The items Mr. Stacy lists as improvements are common to both types of locomotives. Again, in the matter of 3-year cycles of major overhauling, the word "locomotives" was used loosely for the diesel portion of the locomotive. Mr. Stacy has helped considerably to clarify statements in the paper.

Mr. Shew has misinterpreted the statement "...railroads will regard dieselization as a stepping stone to electrification of lines of heavy traffic density." It is not the intention of the author to recommend dieselization as an intermediate step; the step is already an accomplished fact and has made many railroad men electric conscious.

Both Mr. Shew and Mr. Funk emphasize the problems involved in tapping the utility's 3-phase system with single-phase railroad load. This is acknowledged in the paper. However, the problem has been solved in many foreign countries and can be solved here when the time comes.

Mr. Maloney's remarks are very pertinent and emphasize how teamwork, which could also include the manufacturers, could render a real service in the development of this country.

Mr. Birch, a great exponent of electification, emphasizes the advantages of standardization of both engineering and material. This point cannot be stressed too strongly in the problem of reduced initial costs of the fixed structures and catenary system.

Mr. Billings suggests the intriguing idea of several railroads considering electrification pooling their purchasing of power and taking advantage of cheap water-generated power. With ultrahigh-voltage transmission lines this might reach a very large area. AAR Committee 13 on Railway Electrification should follow up this thought.

Mr. Brown is undoubtably the number 1 man on electrification in this country and has studied most thoroughly the pro and con of electric versus diesel-electric motive power. All the author can do is say "amen" to Mr. Brown's comments and to point out that he answers in part Mr. Stacy's question as to where the money will come from for electrification.

I appreciate the comments of Mr. Nagel and Mr. Ross, particularly as AAR Committee 13 collaborated with their company in demonstrating that, at least in the area tested, the power system could take the single-phase commercial-frequency railroad load without expensive balancing apparatus.

Semiconductor-Magnetic Overvoltage and Underfrequency Protection Circuits

R. R. SECUNDE
ASSOCIATE MEMBER AIEE

THE ELECTRIC POWER requirements of advanced manned aircraft are such that 3-phase 400-cycle a-c electric power systems are used on many of the recently developed aircraft. Airborne equipment, such as radar and navigational aids, is of such a nature that the performance requirements for the a-c power systems are more stringent than those normally encountered for d-c power systems. If the limits for system voltage or frequency are exceeded as a result of a malfunction in one or more of the system components, some of the equipment supplied by the system, as well as the system components themselves, would be damaged. Such damage can be eliminated or minimized through proper isolation of the affected area of the system. This isolation, and thereby protection, is obtained through the use of supervisory panels which incorporate fault sensing, time delays, and appropriate relay circuitry. The stringent requirements plus the need for tolerance in normal operating conditions for a-c generators, regulators and constant-speed drives necessitates protection circuits which can sense an out-of-limits condition within ±1 or 2% and initiate proper isolation. This sensing accuracy must be maintained over temperature ranges of −55 to +120 C (degrees centigrade) up to 70,000 feet of altitude, and under other severe environmental conditions. To accomplish this end, circuits utilizing the latest circuit components and techniques must be employed.

In this paper, two protection circuits which operate under these conditions are described. The first is an overvoltage circuit which senses the presence of an overvoltage condition, and initiates proper isolating action after a time delay which is inversely proportional to the overvoltage value. The second is an underfrequency circuit which will initiate proper isolating action whenever the system frequency becomes less than a predetermined value. Both of these circuits are presented as unrelated protection schemes. In prac-

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The author wishes to thank Dr. C. H. Grace and Mr. A. W. Pratt formerly of Jack & Heintz, Incorporated, for their assistance in the development of the circuits described in this paper.

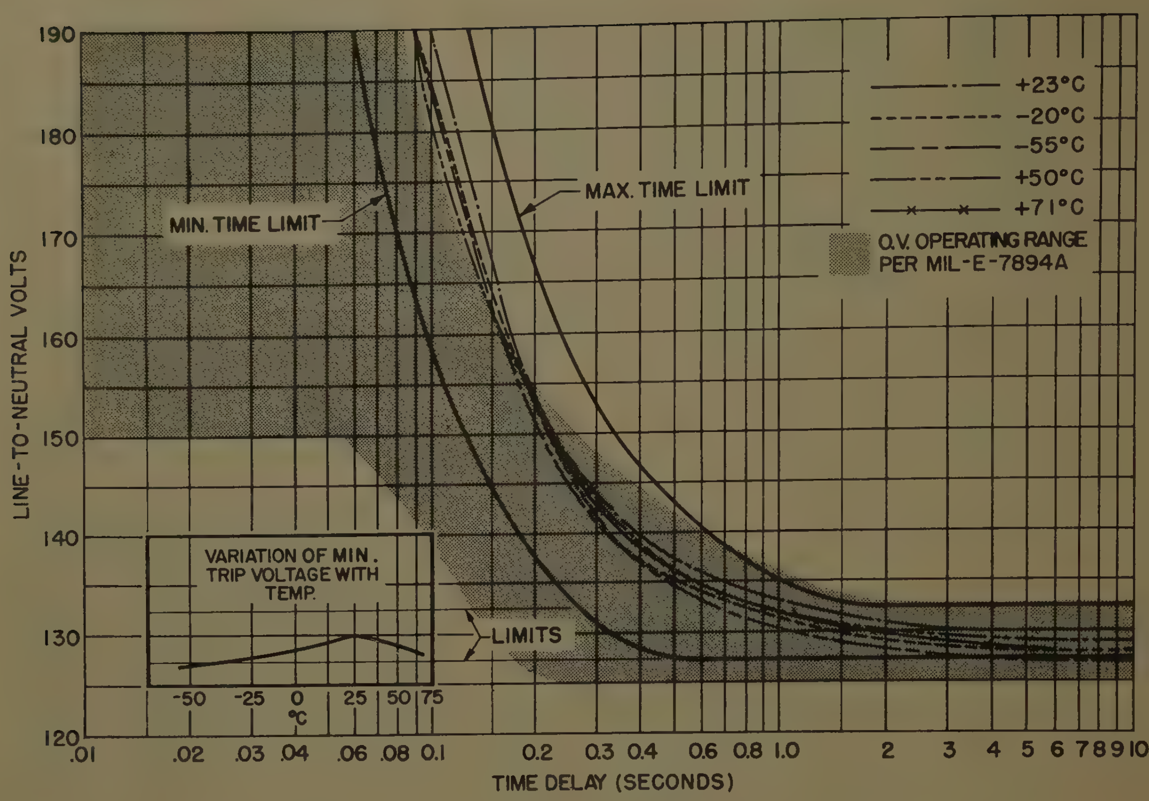

Fig. 1. Over- voltage: time delay characteristic limit and actual per- formance of over- voltage circuit

tice, they are combined in integrated protection systems.

Overvoltage Circuit

Malfunction of the generator or voltage regulator, and transient conditions which cause high voltage in an electric power system can cause damage to system components if allowed to exist beyond a time limit determined by the design of the affected components. An overvoltage circuit is used to sense the presence of a condition in which the system voltage exceeds a predetermined maximum safe level and then causes proper isolation or generator de-energization after a specified time delay. Hereafter, the action of initiating isolation or de-energization will be referred to as "tripping." The maximum limit line in Fig. 1 indicates the maximum length of time which an overvoltage condition can be allowed to exist in a particular system without possibility of damage. Normal voltage transients, however, must not cause the overvoltage circuit to trip. Therefore, a minimum time and trip-level limit is also shown in Fig. 1. To provide satisfactory operation the overvoltage circuit must not trip when the system voltage is below the minimum limit, or faster than the minimum time when it is above. It must trip within the proper time when the voltage is above the maximum limit. Hereafter, the value which the system voltage must exceed in order for an overvoltage condition to be present will be referred to as "the minimum trip level." As can be seen from Fig. 1 a typical tolerance on the minimum trip level is ±2.5 volts, or 1.9%.

Circuit Description

Fig. 2 is a single-phase overvoltage protection circuit which consists of a saturable core reactor, Zener diode, and other solid-state components which will sense the presence of an overvoltage condition and provide the required time delay before actuation of the relay. Three-phase protection, with sensitivity to the highest phase voltage, can be obtained by using three sensing and timing circuits connected by a diode *OR* arrangement to one common output circuit as indicated in Fig. 2. As will be shown by the following description of operation, this circuit is sensitive to the average value of one half-cycle of the voltage being measured. It, therefore, approximately measures the rms value of the applied voltage; its performance is not appreciably affected by distorted voltage wave shapes as are peak sensing circuits. Under transient overvoltage conditions, it will integrate the transient overvoltage envelope and trip only if the total energy in that envelope is liable to cause damage to equipment.

The alternating voltage whose excess magnitude is to be detected is connected to the autotransformer *T1*. Through means of taps and an adjustable voltage-dividing resistor *R1*, it is reduced to the proper level for the remainder of the circuit. When the upper a-c terminal is positive (hereafter referred to as the positive half-cycle) diodes *CR1* and *CR2* conduct (all others block), and a voltage reduced by the divider formed of resistors *R2* plus *R3* and *R5* is applied to winding no. *1* of the saturable reactor *AR* in series with resistor *R6*. The voltage across *R6* is small (0.5 volt) during this half-cycle. The core of *AR* is made of square hysteresis-loop magnetic material. Assuming that the flux in the core of *AR* is at negative saturation at the start of the positive half-cycle (how this is accomplished will be shown), it will be driven a fraction of the total flux difference between positive and negative saturation toward positive saturation during the positive half-cycle as *AR* absorbs the volt-seconds contained in the half-wave of phase voltage. During the negative half-cycle, diode *CR3* and *CR4* conduct; *CR1* and *CR2* block. Voltage increases across winding no. 2 until the Zener voltage of diode *CR5* is reached. For the remainder of the negative half-cycle the voltage across winding no. *2* is limited to the Zener voltage of *CR5* plus the forward drop of *CR3*. Excess voltage is dropped across resistor *R4*. In effect, a voltage of essentially constant magnitude is applied to winding no. 2 during the negative half-cycle, and the flux is driven toward negative saturation. The saturable reactor is designed so that under any applied voltage less than the

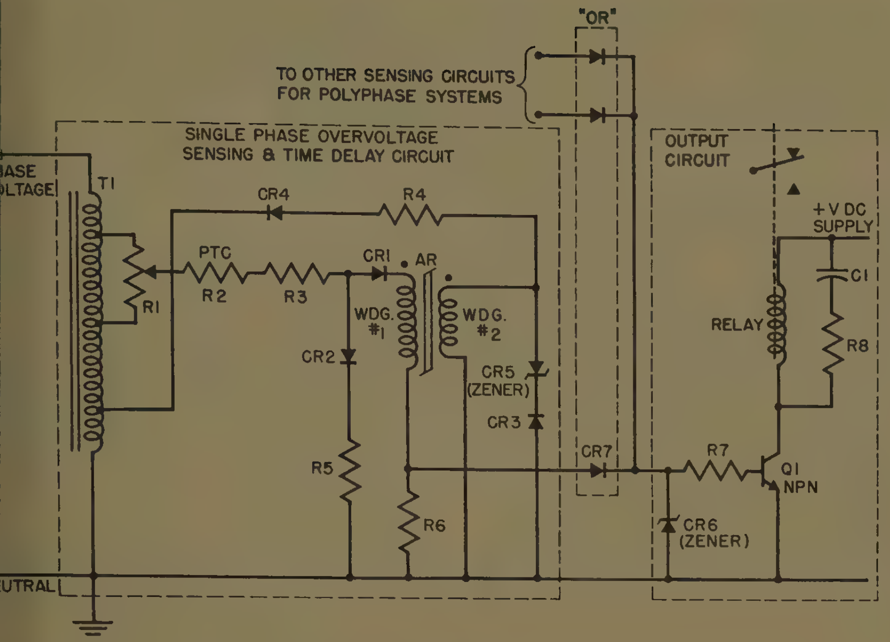

Fig. 2. Overvoltage circuit with single-transistor output

inimum trip level, the change in flux the negative direction tends to be eater than in the positive direction, and e core of AR is therefore at negative turation at the start of each positive alf-cycle.

When the alternating input voltage ceeds the minimum trip level, the flux ill be driven toward positive saturation uring the positive half-cycle to a greater tent than it is driven toward negative turation during the negative half-cycle ereby causing a net gain toward posive saturation. After a sufficient number f cycles has passed, the flux in the core ill reach positive saturation. When this ccurs, a large current will flow in winding o. *1* during the last few degrees of the ositive half-cycles which will produce oltage peaks across $R6$. These peaks of oltage are sufficient to turn on transistor $Q1$ for a length of time necessary to pick p the relay. The voltage developed cross $R6$ by the magnetizing current of inding no. *1* is low and will not be suffiient to overcome the forward voltage drop of the base-emitter junction of $Q1$. Therefore, the transistor will remain off until the peaks of voltage occur. Voltage across relay is filtered by means of capacitor $C1$. Zener diode $CR6$ and resistor $R7$ are used to limit transistor base current to a safe level. Similarly, resistor $R8$ in series with $C1$ limits the transistor collector current.

This action of "step flux change" produces a time delay between the occurrence of overvoltage and the time at which the relay will be energized. The time delay is inversely proportional to the overvoltage magnitude. This inverse time delay is a result of the fact that a large overvoltage will produce a greater net gain in the core flux toward positive saturation during each cycle than will a small overvoltage. The equation for this action is given approximately by the following:

$$TD = \frac{K-(E_0)(1/f)}{E-E_0} \qquad (1)$$

where

$$K >> (1/f)(E_0)$$

where TD is the time delay for a given overvoltage; K is a constant determined by the design of the reactor AR; E_0 is the average value of one half-cycle of voltage at the minimum trip level; E is the average value of one half-cycle of the actual voltage being measured; and f is the frequency. It can be seen that the term involving frequency is small, and therefore operation is relatively insensitive to frequency variations.

This circuit has an inherent ability to avalanche slightly when the reactor AR reaches positive saturation. This avalanche quality exists because more energy is required to drive the magnetic state of the core out of saturation than into saturation, and the circuit supplying energy to winding no. *2* is of comparatively high impedance. When the energy necessary to drive the core flux out of saturation is supplied by that circuit during the negative half-cycles, some of the normally available volt-seconds are lost as a result of the increased voltage drop across the impedance. Consequently, the reactor will be driven further into saturation during the following positive half-cycle. This avalanche action produces sufficiently large peak voltages across $R6$ to cause proper operation of the output circuit at, and above, the minimum trip level.

After an overvoltage has been sensed and proper isolating action taken, the overvoltage circuit will automatically reset when the voltage applied to it is decreased below a value approximately 10 volts less than the minimum trip level. If the voltage is completely removed from the circuit after isolation has taken place, reset will not occur, and a subsequent overvoltage transient during generator build up, would cause a trip without a time delay. This hazard is avoided by connecting the overvoltage circuit permanently to the a-c terminals of the generator, and allowing residual voltage of above 25 volts to cause reset. If residual volt-

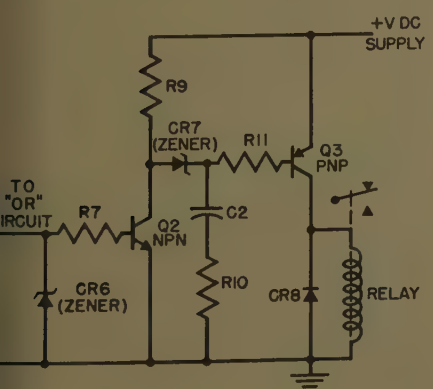

Fig. 3 (left). High-gain two-transistor output circuit for use with overvoltage circuit

Fig. 4. (right) Control and protection panel incorporating semiconductor-magnetic overvoltage and underfrequency circuits

age is not above 25 volts, proper relay circuitry will allow the circuit to reset.

Compensation for changes of the Zener voltage of *CR5* and other circuit values as a result of temperature change is accomplished through the use of a positive temperature coefficient resistor, *R2*. This circuit will operate satisfactorily from −55 C to +120 C if silicon semiconductor components are used. The minimum trip can be adjusted over a range of approximately 15 volts by means of the potentiometer, *R1*. For any one circuit design the time delay characteristic is fixed, and major changes can be made through modification of the circuit component values. The circuit for which results are presented in this paper was designed to replace an existing overvoltage device for a commercial aircraft, and does not meet any particular military specification. For reference, the overvoltage operating range of *MIL-E-7894A* is also shown in Fig. 1.

Experimental Results

A circuit identical to that of Fig. 2 using silicon semiconductors was tested as a breadboard, and found to provide trip and time delay characteristics within the limits of Fig. 1 from −55 C to +71 C. The results of the tests on this circuit are plotted on Fig. 1. The time delay characteristics were obtained by suddenly raising the voltage applied to the overvoltage circuit to the overvoltage value desired. This method of measuring time delay is in accordance with accepted practice in the aircraft industry. A circuit essentially identical to that of Fig. 2, but with different temperature compensation was tested and found to provide operation within the limits of Fig. 1 from −55 C to +120 C.

The circuit of Fig. 2, of necessity uses a saturable reactor approximately 2 inches in diameter. The size of this reactor can be reduced if more amplification is provided by the output portion of the circuit. This can be accomplished by using the output circuit shown in Fig. 3. With the output circuit of Fig. 3 and a saturable reactor of approximately 1.25-inches diameter, satisfactory operation from −55 C to +120 C within limits similar to those of Fig. 1 is being obtained in actual control panels. One of the control panels using this circuit is shown in Fig. 4.

The d-c supply for the output circuits of Figs. 2 and 3 can be from 10 volts to 60 volts if proper precautions are taken in the selection of components. In tests, the circuit of Fig. 2 was supplied with direct voltages of 15 to 40 volts, and the circuit of Fig. 3 with 10 to 15 volts. Operation was satisfactory, and within limits under all conditions of temperature and voltage.

Underfrequency Circuit

In an a-c power system there exists a need for protection from the effects of an underfrequency condition. When, as a result of loss of frequency control, an underfrequency condition does develop in a system, action should be taken to isolate the affected equipment and thereby prevent damage. A device which senses an underfrequency condition can also be used as part of an automatic start-up and shutdown scheme. By sensing whether the frequency is above or below a specified minimum level, it will either allow or prevent the load from being connected to the generator if all other conditions are correct. Typically, the frequency below which proper isolation must be accomplished is 370 cps in a 400-cps aircraft system. Hereafter, this frequency will be referred to as "trip frequency." The tolerance allowed for this trip frequency is typically ±10 cps or 2.7%. Some applications require the underfrequency circuit to provide trip frequency accuracy of ±5 cps or 1.3%. For advanced aircraft, operation within these tolerances must be maintained over a temperature range of −55 C to +120 C and a phase voltage range of 80 to 130 volts.

Circuit Description

In order to meet the need previously mentioned, an underfrequency circuit has been developed which meets the requirements which are most important. This circuit is shown in Fig. 5. For operation, this circuit must be supplied with 3-phase a-c power in the 115-volt phase voltage, 400-cps range. Although only one phase is used for frequency monitoring, the 3-phase power is necessary as a supply for the reference and output circuits. Basically, the circuit functions as a time comparison device. It continuously compares the time of half periods of the frequency being monitored with a reference time composed of a fixed direct voltage and the volt-second capacity of a toroidal saturable reactor wound on a core of square hysteresis-loop material. The time comparison function can be shown by the following derivation:

$$e = N\frac{d\phi}{dt} \text{ fundamental equation} \tag{2}$$

e = induced voltage
N = number of turns in coil
$\frac{d\phi}{dt}$ = rate of change of magnetic flux with respect to time

$$edt = Nd\phi$$

Let e = fixed applied direct voltage E

$$E\int_0^T dt = N\int_{-\phi m}^{+\phi m} d\phi$$

$$\Delta T = \frac{2N\phi m}{E} \tag{3}$$

$\Delta T = (T-0) = 1/2$ period
$+\phi m$, $-\phi m$ = positive and negative saturated flux capacity of core

If ΔT is one-half period of the frequency at the underfrequency trip point of the system being monitored, and ϕm is known for a given core, the E can be arbitrarily chosen, and N computed. The resulting reactor will change from negative to positive saturation in ΔT time with E volts applied to the N turns winding no. *1*, if the core is at negative saturation at the time E volts are applied. The requirement of having the core at negative saturation at the start of each comparing, or sensing, half-cycle is met with an auxiliary reset winding no. *2* which is supplied with current during the half-cycles that the no. *1*, or sensing, winding is not in operation. Since the core material has a square hysteresis loop such as orthonol, deltamax, etc., change in coercive force during the sensing portion of the cycle is negligible, and saturation can be easily detected. In this circuit, by means of a transistor switch, a fixed direct voltage is applied to a saturable reactor, designed as indicated, for the duration of the negative half-cycles of the frequency being monitored. If the frequency is above the trip value, the reactor will not change from negative to positive saturation during this period and no fault will be indicated. If the frequency is below the trip value, the reactor will change completely from negative saturation to positive saturation and a fault will be indicated.

In order to make use of the time comparison function described previously, the circuit shown in Fig. 5 is used. It is composed of 5 distinct sections, each of which will be explained separately. The first is the Regulated Power Supply and Adjustment Section. Three-phase power is half-wave rectified by diodes *CR1*, *2*, and *3*, and then regulated by a 5% tolerance two-stage regulator consisting of resistors *R1* and *R2* and Zener diodes *CR13*, *14*, and *15*. Voltage adjustment to compensate for production parts tolerances is provided by potentiometer *R3*. In operation, this section of the circuit dissipates more power than all of the other sections combined because of the close voltage regulation required to maintain accuracy of trip frequency from 80-

to 130-volt phase voltage within ±5-cps tolerance.

The second, or Temperature Compensation Section, is necessary to provide operation within tolerance over the specified temperature range. The temperature compensation consists of a network composed of *R4*, a positive temperature coefficient resistor and low temperature coefficient resistors *R5* and *R6*. This network provides a voltage at point *A* which decreases as the temperature increases, and thereby compensates for temperature variation of the regulated voltage supply and the critical saturable reactor characteristics.

The next two sections, Cyclic Frequency Sensing and Cyclic Reset, will be described together. Through diode *CR12*, voltage is applied to winding no. *2* of the saturable reactor *AR* during the positive half-cycles of phase *1*, driving it to negative saturation once each cycle. This voltage is limited by resistor *R9* and low voltage reference diodes *CR6* and *CR7* so that reset can be accomplished very rapidly through the use of comparatively few turns in the winding, and so that high voltage peaks will not be induced in winding no. *1* during this operation Any such high voltages would cause damage to transistor *Q1*. Also, during the positive half-cycles, diode *CR4* conducts through resistor *R7* diode *CR5* blocks, and no base current can exist in *Q1*. Therefore, *Q1* is cut off, and no voltage is applied to winding no. *1* of the reactor. Voltage at *B* is zero during the positive half-cycles.

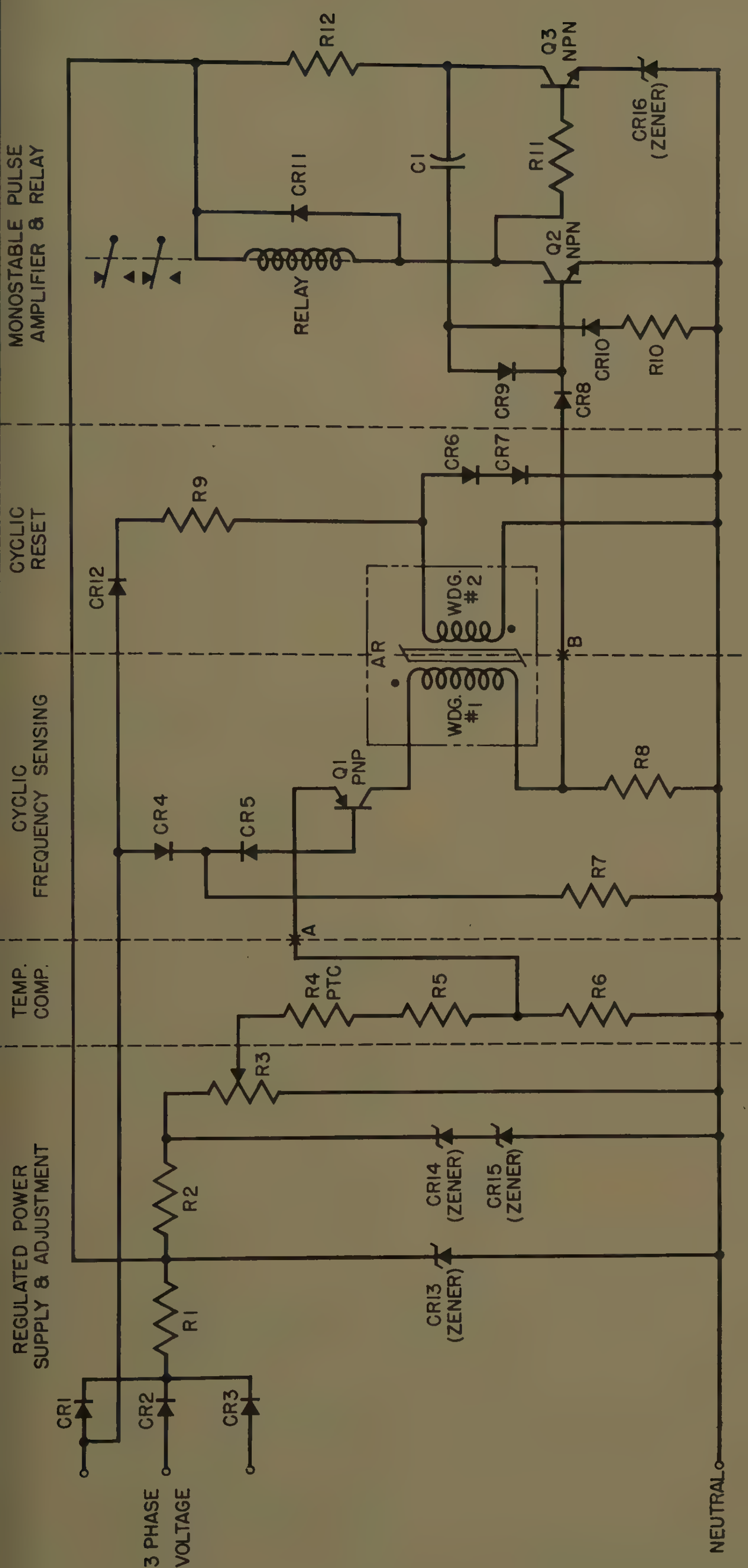

Fig. 5. Underfrequency circuit

During the negative half-cycles of phase *1* *CR4* blocks, *CR5* conducts, and base current flows in *Q1* from *A* through the emitter-base, *CR5*, and *R7*. *R7* is used for base current limiting. With base current flowing, *Q1* turns on to saturation and a voltage approximtely equal to that at *A* is applied to winding no. *1* and resistor *R8*. Voltage drop across *Q1* is very low because of its low saturation resistance. The resistance of *R8* is very low, relative to the impedance of winding no. *1* before saturation, and therefore, the voltage developed across it is approximately 0.5 volts which is caused by the magnetizing current of winding no. *1* of the saturable reactor. These fixed voltage drops do not affect circuit operation, and allowance is made for such fixed drops by the single adjustment *R3*. *Q1* is turned on when phase *1* voltage drops below the voltage at *A* and off when it rises above that voltage. Since the voltage at *A* is very small compared to the voltage of phase *1*, assume that *Q1* is turned on as the phase *1* voltage passes through zero going negative and off when

it passes through zero going positive. When the frequency of phase *1* is above the trip value, the saturable reactor will not change from negative to positive saturation during the time interval that *Q1* is on. Therefore, during the negative half-cycles of phase *1*, the voltage across *R8* will be pulses of approximately 0.5 volts and a duration of one-half period. When the frequency of phase *1* is below the trip value, *Q1* will remain on long enough to allow the saturable reactor to change from negative to positive saturation during a negative half-cycle. Under this condition, a comparatively large voltage peak will occur across *R8* at the end of each negative half-cycle due to the large current which will flow in winding no. *1* when the reactor reaches positive saturation. This voltage peak will be 1 to 5 volts. Since magnetic cores cannot yet be obtained with a hysteresis loop which very closely approaches the ideal used for calculations, the large voltage pulses mentioned do not occur instantaneously, but rise gradually over a range of phase *1* frequency. The rate of peak voltage build up is great enough to allow a proper output circuit to trip at the same frequency, ±1 cps consistently, at any one temperature.

The last section of this underfrequency circuit is the Monostable Pulse Amplifier and Relay Circuit. This is the output section, and, as such, senses the presence of the voltage peaks which occur across *R8* when an underfrequency condition exists, and amplifies them enough to operate a relay. Since these peaks are of such short duration, a conventional amplifier would be unsuitable, and a monostable "flip-flop" circuit is used. This circuit not only amplifies the magnitude of the pulses, but also extends their time duration. The transistors in the flip-flop circuit are used as switches, and are driven to saturation when on and are cut off when off.

When the frequency being monitored is above the trip level, pulses of approximately 0.5 volts appear at point *B*. The bias on the output circuit is such that these pulses will not cause it to function. However, when an underfrequency condition exists, the larger peaks of voltage at *B* will cause transistor *Q2* to conduct, or turn on, because the bias made up of the forward voltage drops of diode *CR8* and the base-emitter junction of *Q2* is exceeded, and base current flows. When *Q2* is turned on, direct voltage from the first stage of regulation in the Power Supply section is applied to the relay and transistor *Q3*, which had been on due to base current flowing through the relay and *R11*, is turned off because its base voltage becomes less than the Zener voltage of *CR16*. When *Q3* is turned off, its collector voltage tends to rise and charge capacitor *C1* through resistor *R12*. The charging current also flows through diode *CR9*, is blocked by *CR8*, and into the base of *Q2* thereby maintaining *Q2* on and *Q3* off, even though the original initiating pulse has disappeared. When the charging current of *C1* decays below the value necessary to maintain *Q2* on, *Q2* will turn off and *Q3* turn on. When *Q3* turns on, the circuit resets by allowing *C1* to discharge rapidly through *Q3*, resistor *R10*, and diode *CR10*. Once reset, the output circuit is ready to function when the next large voltage peak appears at point *B*. Diodes *CR8*, *CR9*, and *CR10* are used for blocking functions during the modes of flip-flop operation. Diode *CR11* is used to protect the tansistors from inductive voltage peaks produced when the coil of the relay is supplied with pulsating direct voltage. The monostable output circuit is designed so that when the peaks of voltage across *R8* are large enough to turn on *Q2* each such peak will cause the output circuit to apply a direct voltage to the relay for a duration of approximately one-half period of the frequency at which the monitored system is required to trip, and of a magnitude large enough to cause the relay to be energized or picked up.

Experimental Results

A circuit identical to that of Fig. 5 using silicon semiconductors was tested as a breadboard and as a portion of production control panels and found to provide satisfactory operation. Its trip frequency is adjustable with a single potentiometer from 390 to 350 cps and, once adjusted, can be held to ±5 cps over a temperature range of −55 C to +120 C and a phase voltage range of 80 to 130 volts. Phase voltages of 180 volts will not damage components provided the duration of such a voltage is limited to a few seconds under high temperature conditions. Accuracy of trip point above 130 volts or below 80 volts is not critical when overvoltage and undervoltage devices are also used. The usual time delay used to prevent transient under-frequency conditions from isolating system components is longer than the overvoltage time delays, and therefore the overvoltage circuit would govern the isolating action. When an undervoltage is present (below 90 volts), the undervoltage circuit will function after approximately the same time delay used for an underfrequency condition.

In general, the frequency at which this circuit trips will decrease as the average phase voltage is decreased, and increased as the average phase voltage is increased. This circuit draws all of the necessary power from the three-phase system being monitored, although only one phase is used to provide the frequency signal. Due to this type of operation, the trip point will not be within ±5 cps of the desired frequency over the temperature range when the phase voltage is above 130 volts or below 80 volts. The circuit itself is inoperative below approximately 45 volts, average phase voltage, and will not indicate a fault condition. Also, if the voltage of the phase which is being used to provide the frequency signal is allowed to drop below 45 volts while the other phase voltages remain normal, the circuit will indicate an underfrequency condition even though the system frequency is normal. The variations in trip frequency for the breadboard of this circuit over the voltage and temperature ranges are shown on Fig. 6.

To obtain the required ±5 cps ac-

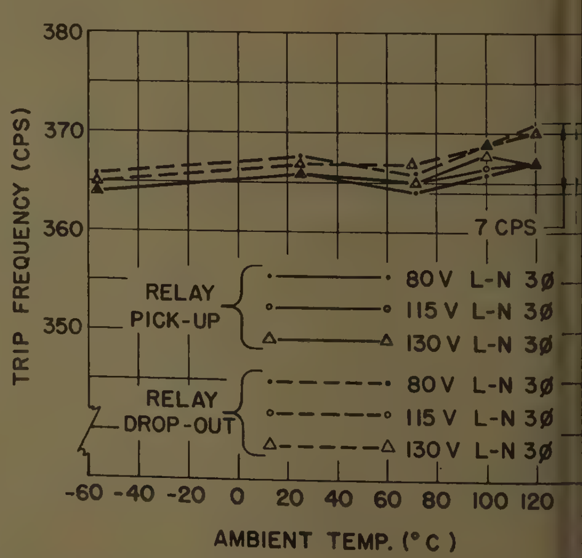

Fig. 6 (right). Performance of underfrequency circuit (Relay picked up indicates underfrequency conditions. Relay dropped out indicates normal frequency)

curacy of trip frequency over the operating conditions of an advanced aircraft electric power system, this circuit incorporates some features which would be undesirable and unnecessary in less critical systems. As can be seen from Fig. 5, this circuit is complex. It requires approximately 15 watts of power from the 3-phase a-c system and therefore provision must be made to dissipate the heat generated in the components, and to maintain an ambient temperature around the circuit not to exceed 120 C. In less critical applications the circuit could be modified to use fewer parts and require less power to operate.

Conclusions

The two circuits described in this paper were developed to meet the need for accurate fault sensing over extreme temperature, altitude, and vibrational conditions. They have been proved to operate satisfactorily from minus 55 C to plus 120 C, and up to 50,000 feet altitude. It is felt that these two protection circuits have the following advantages over presently used gas tube, dash pot relay, tuned circuit, and sensitive relay fault sensing and timing schemes:

1. They maintain stability of trip point throughout the environmental range.

2. Since they are primarily solid state circuits and use hermetically sealed components, altitude and vibration will have little or no effect on performance.

3. The size and weight of these circuits when assembled will be less than that necessary for more conventional protection schemes which are modified to provide the same performance.

As characteristics of square hysteresis-loop magnetic cores and semiconductors become more definite and easier to control, design of circuits such as the ones presented in this paper will become relatively simple and more exact.

Simplified versions of these circuits (without voltage supply, amplifiers, and relays) can be used in integrated static, or solid-state protection schemes. Use of these circuits in such a manner would enhance the advantages listed.

These circuits as they are described in this paper are somewhat complex. Their use is determined by consideration of the suitability of less complex schemes and the accuracy and environmental limits required. Most of the recently issued specifications eliminate the use of the more commonly known protection schemes without deviation, but the circuits of this paper will provide performance in compliance with the specification requirements with minor, or no, deviations.

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Photovoltaic Solar Energy Converters for Space Vehicles—Present Capabilities and Objectives

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THE PRESENT accelerated program for space vehicles demands development of auxiliary power supplies which conserve weight and outlast conventional batteries. The solar radiation flux outside the earth's atmosphere is about 130 watts/ft^2 (watts per square foot) at normal incidence. Utilization of this energy is within reach of present technology based on the following facts:

Silicon solar cells of 10% efficiency are now available commercially so that an output power up to 13 watts/ft^2 is attainable.

Transistorization of instrumentation, and data-recording and transmission equipment reduces power requirements of many space vehicles to levels below 1 kw.

Large-scale production of solar cells should reduce the price of solar power delivered to below \$300/watt.

The lifetime of solar batteries is estimated to exceed that of conventional batteries by a factor of 100 and may even exceed the lifetime of the vehicle.

The weight of silicon solar cells is negligible, compared with mechanical supports, resulting in a weight far less than one pound/watt delivered.

The objective of this paper is a critical analysis of:

Experimental data of silicon solar cells under various operating conditions and environment.

Methods of overcoming present limitations of silicon solar cells, and preliminary results on potential new materials and device designs.

Problems arising from the unique properties of the silicon solar cells as a circuit element.

Some Properties of Silicon Solar Cells

Fig. 1 presents pictorially a silicon solar cell; typical dimensions are 1 inch by 1/2 inch by 40 mils. A p-region and an n-region form a p-n junction parallel to and about 0.1 mil below the top surface. While the entire back surface is nickel-plated and soldered to serve as an ohmic contact, a narrow strip of solder on the top surface makes contact to the p-region. Light impinging on the top surface causes this 2-terminal device to act approximately as a constant-voltage generator, the current being proportional to the incident light intensity. For convenience in data presentation, we shall assume sunlight outside the earth's atmosphere normally incident on a typical cell of 10% efficiency.

Temperature Characteristics

Prince[1] reports the calculated variation of the open-circuit voltage V_{oc}, with temperature as $-dV_{oc}/dT=2.88$ mv/C (where mv=millivolts, and C=degrees centigrade). This is in agreement with the experimental data of Fig. 2. Since V_{oc} is determined by the characteristics of the p-n junction, it is therefore independent of cell area.

Above zero C the short-circuit current I_{sc} remains constant with the temperature

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This work was done at the Lockheed Missiles and Space Division Research Laboratories, Palo Alto, Calif.

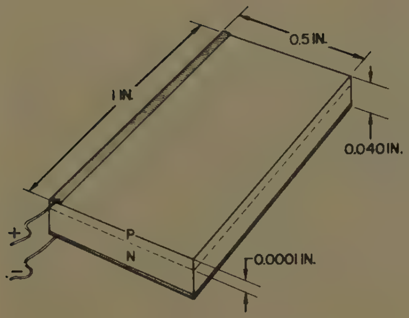


Fig. 1. Silicon solar cell

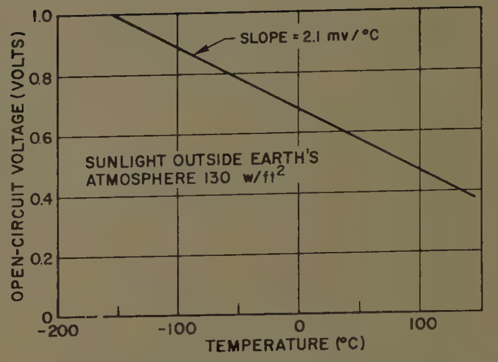


Fig. 2. Open-circuit voltage versus temperature

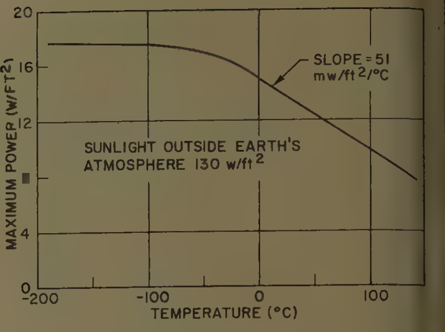


Fig. 4. Maximum power versus temperature

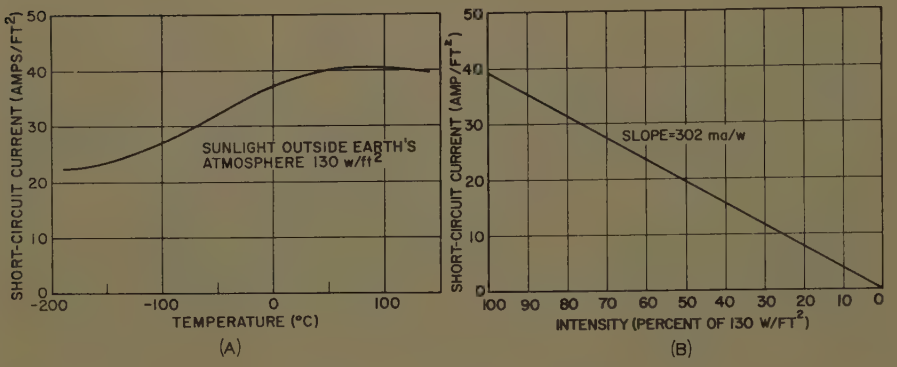


Fig. 3. Short-circuit current

A—Versus temperature

B—Versus intensity

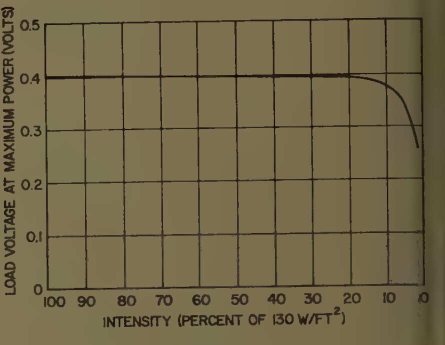


Fig. 5. Load voltage at maximum power versus intensity

as shown in Fig. 3(A). Clearly, then, the maximum power output P_{max}, shown in Fig. 4, varies linearly with temperature as

$$dP_{max}/dT = 51 \text{ mw/ft}^2\text{/C}$$

Although the low-temperature behavior is not discussed here, effects of thermal strains have been observed which lower the output power frequently below that shown in Fig. 4. Since the efficiency is strongly temperature-dependent, proper thermal design of the cell and its mounting is required to assure optimum utilization.

It has been found that I_{sc} and P_{max} are proportional to the active area of the cell.

Intensity Dependence

The short-circuit current I_{sc} varies linearly with light intensity I_0 as

$$dI_{sc}/dI_0 = 302 \text{ milliamperes/watt}$$

because each quantum of sunlight of the proper wavelength produces one hole-electron pair; see Fig. 3(B). The open-circuit voltage is essentially constant with intensity; hence the output power follows the I_{sc} curve. Since the load voltage (Fig. 5) is constant with intensity, the silicon solar cell is ideal for charging storage batteries.

Measurement of cell response as a function of angle of incidence revealed no marked deviation from the expected cosine law variation.

Reflectivity

Fig. 6 shows the reflectivity of the silicon solar cell in the region 0.4 to 1.8 microns. The region from 0.4 to 1.0 micron can be readily interpreted on the basis of the reflectivity of the boron-silicon top surface of the cell. The reflectivity at longer wavelengths, shown as a dashed curve, is influenced by the reflection from the metallic back electrode, since the silicon becomes transparent in this range. The effective absorptivity of the cell may be obtained from this curve by subtracting the reflectivity from unity. Some manufacturers are currently coating their cells to reduce the reflectivity at the surface, thus increasing the output power significantly.

Spectral Response

Experimental data given in Fig. 7 show the relative response of a silicon solar cell as a function of wavelength (solid curve) and, for comparison, the solar energy spectrum outside the earth's atmosphere (dashed curve) is also given. Note that the cell responds only to the radiation between 0.5 and 1.0 micron; this range contains only about 50% of the energy of the solar spectrum. Utilization of the other half of the spectrum is a goal of present development efforts.

Radiation Damage

Photovoltaic devices are more sensitive to radiation damage than almost any other type of energy converters. Fig. 8 shows percentile curves of failure and survival under 1 megelectron-volt gamma radiation for preassigned tolerances of derating in the short-circuit current. The resultant deterioration is primarily dependent on the radiation dosage, but may also be a function of radiation flux as indicated in Fig. 9. At low fluxes, radiation damage is partially compensated by annealing effects, while calculations by Enslow, et al.,[2] indicate that, due to carrier production (photovoltaic effect) by gamma radiation, an increase in tolerance dose is anticipated. Junga and Enslow[3] have examined the effects of annealing, and the radiation damage due to fast and thermal neutron irradiation.

Toward Kw Generators

Many schemes have been proposed to obtain large-area low-cost photovoltaic generators; in particular, vapor deposition of thin silicon junctions has been repeatedly and unsuccessfully attempted. Presently it is most economical to build arrays consisting of hundreds or even

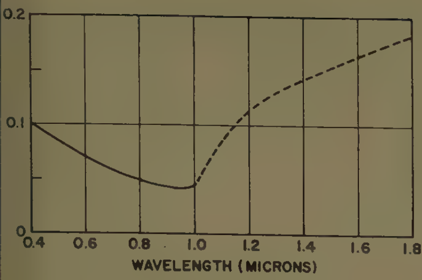
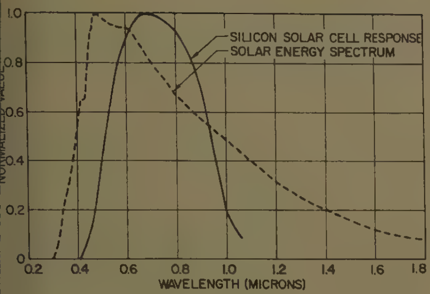


Fig. 6. Reflectivity versus wavelength



ig. 7. Relative response versus wavelength

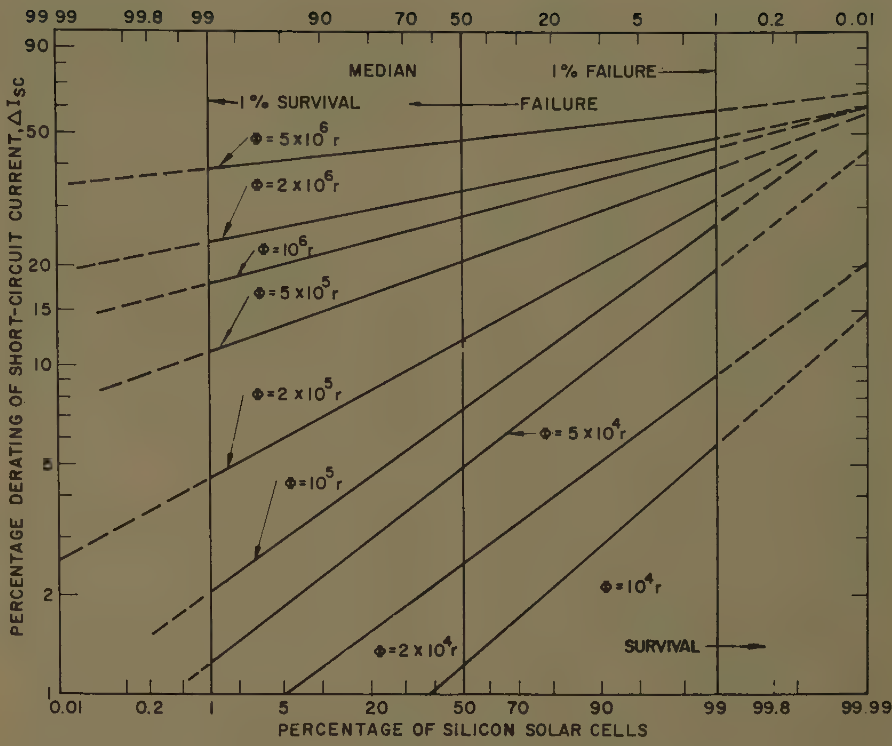


Fig. 8. Percentage of silicon solar cells that have short-circuit current derating ΔI_{sc} at a dose Φ

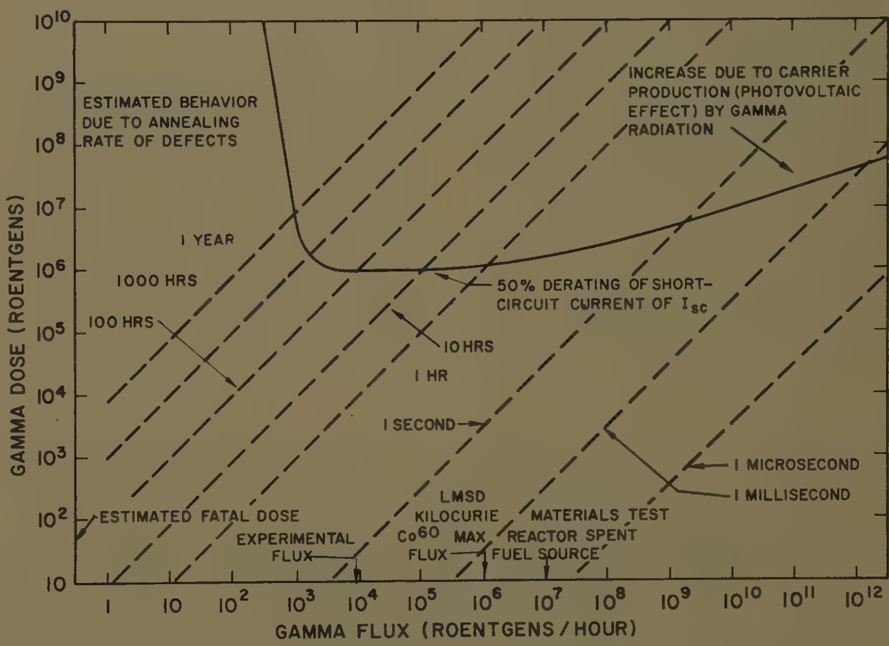


Fig. 9. Flux-dose plot for silicon solar cells under gamma radiation

housands of individual small cells. Although it is feasible to make solar cells of 1 or even 2 inches in diameter, the brittleness of the material, the need for reliability, and the cost of handling during manufacture and assembly determine the optimum area as a fraction of a square inch. In assembling large numbers of cells the following facts are important:

Each cell delivers at optimum efficiency only at one operating point, a current $I_m(<I_{sc})$ and a corresponding voltage $V_m(<V_{oc})$.

A parallel combination of cells operates at the voltage of the cell producing the least voltage.

A series combination of cells operates at the current of the cell producing the least current.

To assemble an array of maximum efficiency, each cell should be tested individually; parallel combinations are matched according to V_m, and series combinations are matched according to I_m.

Heeger and Nisbet[4] have developed techniques of testing and matching cells quickly and economically.

Limitations of Silicon Solar Cells

Novel fabrication and processing techniques are constantly improving the efficiency, reliability, and reproducibility of silicon solar cells, and it is optimistically hoped that the theoretical limit of 19.6%, calculated by Loferski,[5] may be approached. Since the efficiency is reduced by the contact resistance, surface reflection, and recombination centers, to mention only a few factors, it is estimated that a cell which is 15% efficient constitutes a practical upper limit.

Possible Improved Photovoltaic Devices

Cadmium sulfide (CdS) is a particularly promising material since it exhibits a strong voltaic effect[6] and the mechanism is somewhat different from that of silicon. CdS exhibits a photovoltaic response not only at wavelengths shorter than its absorption edge as usually expected, but also at wavelengths for which the crystal is usually transparent. For a detailed discussion of the photovoltaic mechanism in CdS, see reference 7. At present, CdS photovoltaic cells have been made with an efficiency up to 7%, or about half that of silicon. The advantage of CdS arises from the fact that the decrease in efficiency with increase in temperature in CdS is so small that CdS and silicon are equally

efficient at about 150 C. This is due to the larger energy band gap (2.4 electron-volts) in CdS.

An increase in the efficiency of a photovoltaic device may also result from ingenious device designs. For example, it may be advantageous to stack up materials with appropriately selected energy band gaps, in such a way that the largest band gap is nearest the incident light. Each layer would respond to a certain portion of the incident spectrum and transmit longer wavelengths which in turn would be utilized by subsequent layers. This improved method would utilize the over-all solar spectrum far more efficiently than silicon alone; thus efficiencies in excess of 20% might be achieved.

Recently Goldstein and Pensak[8] observed a high-voltage photovoltaic effect in vapor-deposited cadmium telluride, and attributed this effect to a series combination of minute individual cells. A high-voltage device is presently not available, and high voltages are obtained by cascading a large number of individual cells or by using circuit techniques for voltage conversion. In summary, then, commercial silicon solar cells of up to 15% efficiency may in the near future become available in large quantities and, in the more distant future, novel materials and devices are likely to replace them.

Long-Range Objectives

Photovoltaic energy converters are presently used for space vehicles because of weight and reliability considerations; in many respects they compare favorably with the conventional batteries and thermoelectric energy converters. Large-area power stations consisting of acres of solar cells have been proposed as energy sources for space laboratories on the moon and other planets. To be competitive with other energy converters we must look primarily toward improved fabrication methods resulting in lower cost and higher reliability, in addition to new materials and devices leading to higher conversion efficiencies.

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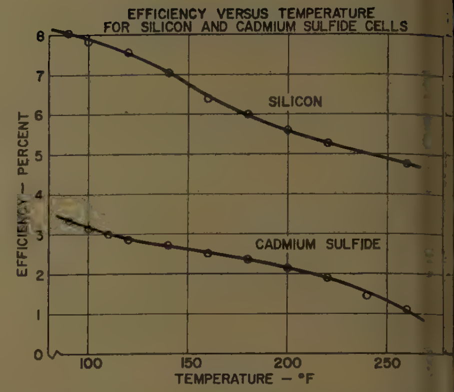

Fig. 10. Comparison of silicon and cadmium sulfde solar cells

Discussion

Henry Oman (Boeing Airplane Company, Seattle, Wash.): The authors indicate that theoretically cadmium sulfide solar cells should be superior to silicon solar cells with respect to loss of efficiency at higher temperatures. We have measured the efficiency of both silicon and experimental cadmium sulfide solar cells at temperatures up to 260 degrees Fahrenheit. One of the resulting comparisons is shown in Fig. 10. Three other cadmium sulfide cells that we tested performed in essentially the same manner. It is apparent that the experimental cadmium sulfide cells that we tested had considerable loss in efficiency at elevated temperatures.

W. W. Happ: Mr. Oman's contribution is most interesting and should stimulate investigation of the mechanism involved, which will deviate from the simple mechanisms on which our theoretical predictions were based.

Electrical System Transients and Sensitive Circuit Control for Missiles and Space Vehicles

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MEMBER AIEE

At one time a fairly extensive literature was compiled on the phenomenon then known as "radio noise."[1–3] At first this literature was almost exclusively concerned with what could be heard coming out of the head phones of a radio set that was not signal, and later on with the "grass" that grew on the radar picture. There was an intense flurry of interest in the problem at the latter part of World War II, because of the general introduction of very-high-frequency equipment. But once the problem had been brought down to its proper proportions and the interference eliminated, the whole question was dropped. Now, with the advent of solid-state devices and their susceptibility to destruction with transient impulses, and the increasing use of digital-type computers to which may transients look exactly like a "bit" of information, there is a revival of interest in the subject. It is hoped that there will be, at last, enough continuing effort to obtain interference suppression by design, instead of last-minute frenzied efforts with a pocketful of filters and capacitors. Perhaps the present changes of the earlier names to "radio-frequency interference" or "electromagnetic interference" will help.[4–8]

Inasmuch as the information on the problem is not much greater now than

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was in World War II, with a consequent ack of appreciation for the requisites for n engineering solution thereof, this aper sets forth an outline of the problem n engineering terms. With this outline, each individual case can be examined n its own merits. Above all, it is the uthor's desire to take the problem out of he realm of the mysterious. Transients lo not just happen; to cause trouble, hey must be coupled to a "receiver" cir-uit, and they do not flit mysteriously nither and yon with no visible means of upport.

General Considerations

Before proceeding further, the main outlines of the problem must be considered: First, there must be a source creating an interference voltage, second, there must be a receiver circuit which would be affected by the interference voltage, and lastly, coupling of some sort must exist between receiver and source. If any of the three elements is missing, interference will not result. For example, a motor may have violently sparking brushes creating a series of sharp transient pulses on the line, and there may be a very sensitive digital computer fed from the same line. However, if the coupling between the motor and the computer has been destroyed by filters, shielding or any other means, interference will not result. All elements must be present; there can be no exception to this rule.

Interference Sources

Roughly speaking, any piece of electric apparatus is a potential interference source. However, the interference producing apparatus is not so important as the type of interference produced. Next in importance is the voltage/current/frequency relationships of the interference.

Transient Generation by Load or Regulation Change

In almost every case, transients generated by this method are characterized by fairly steep initial wavefronts, going back to a steady-state value at some later time, depending upon the regulation method, internal impedance, etc. The transients produced by this method are not more than 25%, except where the loads are at least 100% of full load, and the generator has a high impedance.

Transient Generation by Discharge of Storage Elements

1. *Capacitive Discharges.* This is very infrequent. First a capacitive element must be charged to a voltage above the line to which it is to be connected, and then a connection must be made to the line, so that the stored energy will be discharged. This transient is characterized by a high peak voltage followed by an exponential decay to normal line voltage. It may be oscillatory, depending upon the circuit parameters.

2. *Inductive Discharges.* This is a very common form of transient generator. Motors, solenoids, relays, or the interruption of any inductive circuit will cause this transient. In the inductance L, a current I flows, resulting in the energy storage

$$J=\frac{1}{2}LI^2 \text{ joules} \tag{1}$$

When the circuit is opened, this energy must be discharged and/or dissipated. This may be accomplished by an arc at the switch contacts, or it may be transferred to the distributed capacitance of the inductor and its circuit up to and including the switch. The energy stored in the capacitance, the capacitance C, and the voltage E are related as follows

$$J=\frac{1}{2}CE^2 \text{ joules} \tag{2}$$

Thus, if all the energy in the inductor is transferred to the capacitance, the maximum instantaneous voltage across the capacitance will be

$$E=\sqrt{\frac{L}{C}}\,I \text{ volts} \tag{3}$$

This may be an oscillatory discharge; if the resistance is R, the frequency f will be

$$f=\frac{1}{2\pi}\sqrt{\frac{1}{LC}-\frac{R^2}{4L^2}} \text{ cycles/second} \tag{4}$$

Thus, depending upon the circuit constants, an oscillatory discharge may appear across the inductor/circuit-capacitance combination. A transient has been created, the energy, voltage, and frequency of which may be readily calculated from a knowledge of the circuit parameters. So far, however, it must be remembered that this transient exists only in the L, C, and R combination of the inductor and distributed capacitance up to the switch that opened the circuit in the first place.

3. *Transient Generation by Diode Switching.* The introduction of the high-efficiency silicon diode has brought about a whole new class of interference producing devices. A relay or solenoid produces a transient only when it is switched—when diodes are introduced to make the d-c device operate as an a-c device, transients occur at or near the zero voltage points of the cycle. Here, the diode changes abruptly from a closed circuit to an open circuit. Unless this switching takes place at exactly current zero, some stored energy remains in the solenoid or relay, and must be discharged. It has been shown[9] that there is a large reverse current spike flowing through the diode at this time, producing an interference pulse. On 400-cps (cycles per second) circuits where half-wave rectification is used, the interference will be 400 pulses/second; if full-wave, 800 pulses/second. The reason that this interference has arisen with the introduction of the silicon diode is two-fold: first, the diode was introduced at the same time that a-c systems on aircraft became popular, so that the applications for diodes as rectifiers multiplied; second, this diode is a much more efficient switch than others previously used. Germanium diodes, etc., also produce switching pulses, but of much less intensity than the silicon.

Lamp Load Swithcing Transients

The "cold" resistance of the ordinary incandescent lamp is approximately 1/10 the "hot" resistance. Therefore, lamp circuits, when switched to provide displays of one kind or another must be carefully treated. Any lamp will have a steep-fronted current input, and a large filamented lamp, heating relatively slowly, will have an input current of 8 to 10 times the steady-state current. Such sharp wavefronts are capable of producing interference far beyond that which one would normally expect, and must always be treated with caution.

Receiver Circuits

The receiver may generally be anything which is capable of a misoperation (not necessarily a malfunction) by the introduction of an unwanted signal. It may be an ordinary sensitive relay, or it may be some sort of amplifier-actuated device, such as a transistor or vacuum-tube amplifier, integrating and/or storing input signals. In any event, receivers may be put into two general classes: high input impedance and low input impedance. Generally, the main difference between these classes is the energy required to actuate them. For example, if the input impedances of two amplifiers are 1,000 and 100,000 ohms, respectively, and a 500-volt pulse lasting for one millisecond is required to actuate each of them, the watt-seconds of energy in both cases will be

$$J=\frac{500^2}{1{,}000}\ 10^{-3}=0.25 \text{ watt-second for the 1,000-ohm input}$$

$$J=\frac{500^2}{100{,}000}\ 10^{-3}=0.0025 \text{ watt-second for the 100,000-ohm input}$$

This is equivalent to saying that the high-impedance receiver is more "sensitive" than the low-impedance device. Transients with 0.25 watt-second of energy are quite rare, those of 0.0025 watt-second may be more numerous, for this is equivalent to the discharge of an inductor of 0.1 henry with a current flow of 0.7 ampere.

It is very clear that one method of desensitizing a receiver circuit is to change it from high- to low-impedance input. Essentially, this is what is done when an input filter is installed. The receiver may look into a high-impedance load, but to the rest of the circuit, the receiver may look like a very low impedance. This must not be indiscriminately done, for one undesirable effect may be to convert a high-impedance line with a high-voltage pulse on it to a low-impedance line with a high current pulse on it. In coupling to other circuits, the high current pulse is much more effective, as will be seen later. It must suffice to say here that while low-impedance devices are very desirable at transient frequencies, great care must be exercised to insure that the conversion to low impedance does not bring up other troubles.

Coupling Circuits

Circuit coupling is much misunderstood. It is quite common to speak of a solenoid, relay or other device radiating energy to receivers, whereas this is only very exceptionally the case. Thus, some time will be taken to show how this comes about. It is very important that we use correct terminology.

Radiation Coupling

In reference 10, the following is given as the field strength in electromagnetic units, at a distance of r centimeters from and normal to a dipole of length l centimeters, and a current of I amperes flowing in the dipole.

$$h=lI\left[\frac{\omega}{rc}\sin\left(\omega t-\frac{r\omega}{c}\right)-\frac{\cos\left(\omega t-\frac{r\omega}{c}\right)}{r^2}\right] \quad (5)$$

From an examination of this equation, the following is evident:

1. The first term varies inversely with distance, the second inversely as the square of the distance.

2. The first term is smaller than the second in the ratio ω/c at zero distance. Numerically, they become equal at $r=c/\omega$, at one megacycle this is 478 meters. At lesser distances the second term predominates.

3. As ω approaches zero, or the d-c condition the first term vanishes, and the second term only is left. It is now identical with the familiar equation for magnetic induction at d-c.

4. As a consequence, at the distances commonly involved in aircraft and missiles which is a few meters at most, the radiation field (the first term) is very small and is not effective in producing an interference field; the induction field (the second term) decreases rapidly with distance but is the actual interference field, if one is effective. At all except the kilomegacycle range and above, the radiation field may be neglected for all practical purposes.

Induction Field Coupling

From what has been shown in the previous section on Radiation Coupling, any field set up by current flow can be calculated from ordinary induction field theory. Then, if the length of wire composing the receiver circuit that is subjected to a certain field is known, the total field may be found, and the voltage induced in the receiver wire found by a knowledge of the rate-of-change of flux in the field.

$$E=-N\frac{d\phi}{dt} \text{ volts} \quad (6)$$

This is the only method by which induction coupling may exist, e.g., 1. an induction field is set up by a current flow, 2. a receiver wire is in this field, and 3. the induction field is changing as a function of time. An unchanging field, as one set up by a direct current, will not couple into an adjacent circuit, but the ripple, hash or transients on the d-c line may be coupled since they represent a field flux change.

1. *Induction Field Extent.* A current flow in a wire implies a return flow, and the induction field thus is thought of as a field between two conductors. Now, the inductance of such a loop of wire depends directly upon the distance between the two wires and their diameter. The total impedance Z, is

$$Z=R_{ac}+j\left(0.0529\,\frac{f}{60}\log_{10}\frac{2s}{d}+X_i\right) \text{ ohms/1,000 feet} \quad (7)$$

where

f = frequency, cps
s = distance between the two wires
d = wire diameter
X_i = internal reactance

Thus, at all except direct current, spacing the wires close together decreases the circuit impedance. Obviously, it also reduces the extent of the field, although theoretically the field extends out to infinity. Let us examine this concept. If we assume two conductors, going and return, the impedance of a field element directly between the two will be less than for a field element with a path twice as long. Thus, a field set up by alternating current will tend to concentrate in as small an area as possible between the two conductors. Where there is a single conductor with return through a flat ground plane, as in aircraft or missile structure, the field will concentrate in a small area between the wire and the structure, with the return current in structure confined to a small area directly opposite the wire. The higher the frequency, the more confined the field will be, and with very close spacings where wiring is laid directly on the ground plane, or where twisted pairs are used, the field may be quite small.

2. *Induction Field Shielding.* The permeability of copper is the same as that of air or vacuum, so that copper braid shield has no effect on a field other than the voltage induced in it as a function of field change. The only material which is effective as an induction shield at low frequencies is a ferromagnetic material which offers a lower reluctance path than air or vacuum. At high frequencies, a solid copper shield is effective if correctly installed. The installation of shield braid to cut down induction field coupling is usually a waste of materials, it is effective as an electric field shield. This is why military specifications require that power cables not be shielded—it gives a sense of security which is not there.

3. *Induction Field Control.* Where power cables are used, the induction field may be minimized, not eliminated, by making the current return path as close as possible to the supply wire. This means running wires as close to the ground plane as possible, or possibly running twisted pairs as an extreme measure. Further, power cables should never be run in bundles with other wiring. Since the field decreases inversely as the square of the distance, a small separation will decrease coupling by decreasing the interference field.

Electric Field Coupling

Coupling may occur as a result of the capacitance between two objects. If a voltage appears on one of two objects, a voltage will also appear on the other by

ue of capacitive coupling. Current is involved, all that is necessary is that object be charged and that there be acitive coupling to the other. Shielding here is very simple, and may consist of cing the two objects so that there is no acitance between them, or alternatively, interposing a metallic shield between them which is at ground potential, that there is capacitance between th objects and ground but not between objects.

It is unfortunate that the difference tween electrostatic and electromagnetic elding is not understood; one finds at electrostatic shielding is frequently plied in the mistaken notion that it will o serve as an electromagnetic shield. is probably these instances which have en rise to the idea that shielding is inective, or in a given case does not work all. Electrostatic shielding is very sily applied and understood, and is enely effective if the coupling is capacie. It is purely fortuitous when elecostatic shielding entirely cures a case of terference coupling, in most cases the upling path is through both the electric d the magnetic fields.

terference Control

To recapitulate what was stated under General Considerations," in order to ve interference of any kind all three of e following elements must be present:

A source must be actively generating terference voltages and currents.

A receiver circuit must be present ich may be affected by the interference ltages or currents.

A coupling means must be present beeen the source and the receiver.

If any one of the foregoing elements is issing, regardless of the presence or tensity of the others, interference will t result. Clearly, then, interference ay be eliminated by controlling any one the three elements. Unfortunately, in e past more attention has been paid to urce suppression than to other methods, obably because the device itself is so adily identifiable as an interference urce. The result very often has been to ave the designer of the receiver virtually free hand; no restrictions are placed on s activities. The result is obvious, the r-frame manufacturer is required to install an article to which the designer has tached a list of almost impossible deands in the way of voltage, frequency, d transient tolerances. Meeting these mands has involved greatly increased sts in weight and performance of the hicle, which in most instances would have been avoided if the equipment designer had thought of his article as a part of a system, not as an isolated unit.

Interference Control by Source Suppression

Many cases where this is the most practical method immediately come to mind. A motor may have sparking brushes; the contact on a relay or switch may spark violently. In such cases there is no doubt as to the remedy, since suppression of the interference also improves the operation of the circuit or device. The case is not so clear in other areas.

1. *Relay or Solenoid Slugging.* If interference is caused by the discharge of the stored energy of an inductor it may be eliminated by providing a low-impedance path in which the energy may be dissipated. Such is often done by providing bypass diodes, etc. However, this is not always clear gain, for a relay or solenoid may be "slugged", or its time of operation changed, by the application of diodes. In some cases, this may delay the relay drop-outs to a half-second or so, giving rise to unco-ordinated circuit operation. Slugging, either by diodes, resistors, or capacitors, may certainly be used to suppress a source transient, but the effect upon the operation of the component in the system must also be taken into account.

2. *Source Filtering.* This is a very popular method of source suppression, and may be very effective, especially on small motors. Commonly, a pi-type filter is used, capacitor-inductor-capacitor, so that the motor looks into a low impedance, and the impedance looking toward the motor is also low. This will have two effects which must be taken into account.

a. The interference generator in the motor may be considered as one operating at many frequencies, with resonant and antiresonant internal impedances. That is, at some frequencies, it is a voltage generator, at others it is a current generator. The effect of terminating such a generator in a low impedance capacitor is to make the interference currents high at all frequencies. The capacitor must be physically located directly on the device, so that the interference currents do not flow outside it.

Further, it is essential that the high r-f currents inside the device be confined there, all openings, etc., must be carefully shielded by construction or by placing a heavy copper screen over them. In any event, the r-f interference currents and the heavy associated field must be contained within it. If the currents flow outside at any point, a field is set up which may couple to adjacent wiring.

b. The filter looks like a low impedance to the rest of the circuit, and may cause the flow of heavy transient currents instead of the transients appearing as voltages on the line. Many cases of interference have been "cleared" in this manner, the interference voltage disappears but the interference is still present, possibly in another receiver previously unaffected.

Interference Control by Receiver Design

This is a most controversial area, and no hard-and-fast rules can be set down. However, it is certainly the responsibility of the designer to make his equipment as inherently good as he can. That is to say, the design should not be such as to unnecessarily increase the susceptibility to coupling to an adjacent field; inputs should be isolated as much as possible, and certainly there should be no coupling between input and output.

As a rough criterion, it would seem that a device should be at least one order more sensitive to signals on a designated input terminal than to signals of the same frequency upon power supply, control or output wiring. This might not seem to be much of a requirement, but the author has had experiences with equipment more sensitive to signals on the power supply than to signals on the input. It is certainly no reflection upon the man installing such equipment that he has interference troubles, although this is generally where the blame falls.

There is no point in taking this facet of the problem any farther. The equipment designer should certainly know the part his design will play in the system, and not make severe demands upon others merely because it lightens his burden.

Interference Control by Coupling Control

1. *Induction and Electric Field Coupling.* There is nothing quite so effective as physically separating the receiver from the source, the induction field decreases as the square of the distance, the electric field directly as the distance. The applicable methods here may be summarized briefly as follows:

a.. Provide physical separation between the two circuits.

b. Provide electrostatic shielding by design if possible i.e., run sensitive circuits close to the ground plane, with metal between them and source circuits.

c. Minimize the field area of both receiver and source by running the circuit wire and its return path as close together as possible. This may mean a twisted pair or a wire close to the ground plane.

d. If a receiver wire must be exposed to a field, provide for cancellation effects by running it as a twisted pair with proper treatment at the receiver end so that the two induced voltages cancel.

2. *Source Wiring Impedance Control.* If such wiring is made low impedance by the installation of capacitors, bleed resistors, batteries, etc., high interference voltages cannot occur on account of the energy requirements. However, such devices may convert a high-voltage transient to a high current one and unexpected coupling occurs, because of induction fields. Properly used, these are excellent devices, and their usages may be summarized as shown:

a. Install capacitors, resistances, batteries, so that the induction field set up by high interference currents do not couple into wiring farther down stream.

b. A storage battery has an effective capacitance of 17,000 microfarads/ampere hour,[11] at least in the smaller sizes. It is much better than a capacitor for making a line low impedance on d-c circuits.

c. A capacitor must be the noninductively wound type. This is a special type, and must be so specified and purchased. Electrolytics, tantalytics and metallized capacitors are generally unsuitable for this service. Only the highest quality oil-filled capacitors should be used. A further restriction is where capacitors are used on a-c lines. Only certain types are suitable; these are so designated in the manufacturers' literature.

Conclusion

The foregoing was not intended to introduce anything new or original. Certainly, there is little of this in most engineering fields. The intent has been to set forth well-known, tested, tried-and-true methods of controlling the coupling of transients into sensitive circuits. All of the methods set forth are well known to those who have been chasing "radio noise" for many years. It is the devices affected by interference voltages and currents that have changed. The principles are the same as those used by designers over 15 years ago. To summarize, these principles are:

1. For interference to be a problem, there must be a source, a receiver and a coupling means.

2. Methods of reducing or eliminating the interference produced by most devices are well known, and when applied within their limitations are successful.

3. Design of the receiver for decreased sensitivity to interference signals on other than its input is a fundamental responsibility of the equipment designer.

4. Interference fields may be reduced in intensity and volume by control of the wiring to the device producing the interference.

5. Coupling between an interference field and a sensitive circuit may be decreased by control of source and receiver wiring to the point that it is not significant.

6. Interference field control may be ob- tained by decreasing the impedance of th lines carrying the interference signals. Unless care is used, this may decrease interference voltages at the expense of inter- ference currents.

7. No one method of control is to be pre- ferred above all others, judicious use of th various control devices is preferred. Never- theless, in original design it is well to exercis control over all areas if possible, so that a interference-free installation will result.

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Statistics Applied to Electrical Laboratory Investigations

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RESULTS OF INVESTIGATIONS are generally expressed in one of two ways. The first is the "yes-no" type, i.e., a statement that a certain event did or did not occur. The second type expresses the results in numerical values.

The yes–no type of operation is a generalization of which the commonly known "Go–No–Go" principle of testing is an example. By definition, in this operation, the causative agent has only a single value and the operation of this agent causes one of two possible reactions. Thus, when the current passed through a circuit breaker is held constant, the results can be expressed in only one attribute of the two possible attributes: "breaker closed" or "breaker open." This is commonly referred to as "testing as an attribute."

In contradistinction, this simple statement can be elaborated upon by making two changes: First, the current is made to vary, and second, only one attribute is considered. For instance, the current is increased from below a value considered critical until the breaker "pops," which is the single attribute considered. A group of operations most likely wi demonstrate that there are as man distinct measurable current values a there were trials performed to operate th breaker. The variable measurements ar then related to the single attribut This is then referred to as "testing as a variable" and furnishes "measuremen data" to use the commonly used terms.

As a variant of the foregoing, the tw preceding parameters of variable curren and fixed attribute are considered to gether with another variable. For ex ample, there is the case of determinin the value of current necessary to make breaker operate and the time consume in opening. As a result of a number

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erations, this furnishes information on
e variations in current and the opening
ne interval at the various current levels.
ie aim may be to determine a "cause
d effect" relationship, such as between
rrent value and opening time at the
ed attribute of "open breaker." This
a case of correlation.

The foregoing sequence is developed the paper, except for the "correlation" ample. The data from the circuit eaker test are not suitable for the rrelation demonstration.

The following example will illustrate e application of "confidence limits" to tributes. It is common practice to test sample and express the results in perntage of failure and success. It is inequent that thought is given to the ze of the sample n or to the relation of ie sample size to the results. Likewise, ie sample attributes are tacitly assumed be representative of the population om which the sample is taken and of hich the sample represents only a small action. This amounts to jumping from n unwarranted assumption to a wrong onclusion. In contradistinction, the folwing illustrates the statistical approach: n instantaneous circuit breaker is to e tested for the accuracy of its electrical peration. At a certain value—3.9 amp amperes) or higher in this instance— he mechanism is expected to open the ircuit carrying the current. Operation nust be obtained at 4.7 amp to avoid amage to critical components. To test he accuracy with which this is accomlished is the purpose of this experinent. To express the results and the mplications in numerical terms is the unction of the statistical methods emloyed here.

In laboratory tests of this device, it is tandard procedure to choose a round umber, say 20, as the number of trials o be performed. Then when the 20 rials are completed and no failure was bserved, complete success is announced. But such embarrassing questions remain nanswered as: What would have hapened at the 21st trial? and What may appen in 1,000 trials? Perhaps these loubts are manifestations of the engineerng conscience.

To resolve these speculations, one will ave to answer the question: Is there a ossibility of failure at all? If the answer s negative, there is no problem. However, if there is any chance of failure, then tatistics enter into the picture. Suppose an answer relating the foregoing est to 1,000 operations is desired. The operations answer is to be in numerical, quantitative form. Also, we should like to have an appraisal of the degree of confidence we may place in the answer.

Without going into derivations, which may be found in references 1 and 2, we resort to a table of "0.99 Confidence Intervals" as prepared in reference 3. There one finds the "Numbers Observed," column which here refers to the number of failures, or zero. The "Size of Sample" is 20. Then, below, comes the answer "0–23." This indicates that if the present process of sampling by 20's were continued, one might obtain samples containing failures, and that these defectives are likely to be as high as 23% of a sample.

This expresses the idea that in a sample of 20 trials a "no failure" condition is as likely to arise as 5 failures. The implication is that when one sample is drawn from a "Universe" constituted of successes and failures, the distribution may be estimated only by this single, small sample; see Fig. 1. This definition of likely success and failure, or this small degree of definition, is of little use in most cases.

So the elation over a failureless test turns into uncertainty and a desire to know the worst. The universe, which as shown in Fig. 1 may contain 12% defectives, is useless. It is now necessary to change the qualitative statement of failure and success into a quantitative one. One must settle upon some figure as a level of acceptability. Say that 3% failures are permissible. The task is to find a level of testing that will clearly define this 3% as the limit of failures.

Therefore the "Confidence Interval" table is used, in reverse this time, in search of a pair of numbers of which the higher is 3% or less. This may be found under "Size of Sample" 1,000, with "Fractions Observed" of 0.02. This information—that in a test of 1,000 trials a maximum of 2% failures are permitted —is to assure ourselves that the universe from which this sample is drawn does not contain 3% or more failures.

A discomforting proviso, however, now appears: The foregoing tabulation is for a 99% confidence level, i.e., nothing is stated about 1% of a large number of operations. Whether to assume that this 1% will present itself in operation as a failure or as a success is left to the individual. Generally, the more impetuous consider the chance of failure very unlikely in this 1% of the universe, and ignore it. Yet, others do not. The quandary in which the engineer finds himself points up the reasoning of those who deny to statistics the status of a science.

The foregoing illustrates a universal

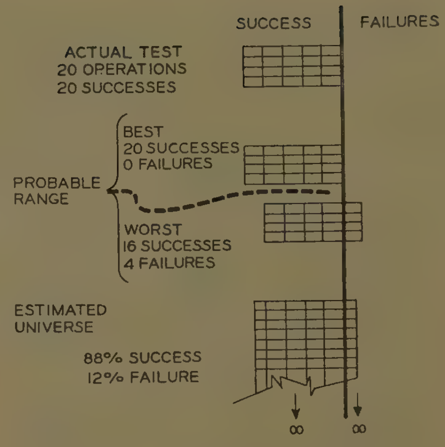
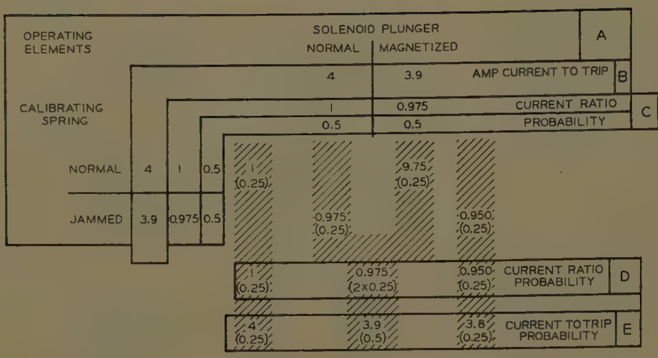


Fig. 1. Attribute distribution

truth that the quality of the input determines the quality of the output. In this case, the quality of the input is determined by the diligence of the search and, in turn, by the quantity of carefully measured statistics. The quality of the answer is expressed by the refinement of definition of the boundaries within which the true value may be expected.

For the time being, let us assume that we wish to improve the quality of the answer by increasing the quantity of input statistics. Then, a very large sample results, say, one calling for 1,000 trials. Generally, a 1,000-trial sample is too onerous a burden on the budget. "Having no budget, we must think" to paraphrase a famous scientist. This leads us into another aspect of statistics, connected with careful measurements. We graduate from the "yes-no" sort of operation into one using gradations expressing the many possibilities between the yes-no limits. The appeal of novelty and the impressive statistical jargon tempts many to process data indiscriminately by statistical methods. In any event, it is mandatory that the measurement data, i.e., current values in the operation at hand, be made to vary only by changing at random conditions. Where a single influence varies the operation of a device, and this influence has only two values, only two variates are generated. Such a condition could be caused by the random magnetization of a relay pole as a function of random transients at the instant of disconnection. Assuming that the relay is otherwise perfect, only two values of critical current, two variates, would be found in a test. This is the "yes-no" type again. Usually more than one randomly changing condition affects an operation. Consider the simplest case: In addition to the forementioned magnetization influence on the operation, there is a variation caused by a mechanical



imperfection, the frequent jamming of a calibration spring. Thus, without the former and the latter influences we would have a device which normally would operate at 4 amp. This current is decreased by 0.1 amp under adverse mechanical conditions, and a similar amount under adverse magnetic conditions. Further, it is stipulated that any operating condition has as much chance of occurring as any other, so that the probability of each condition is $p=0.5$. The question to be answered now is: What values of trip current will occur and with what frequency?

Referring to Fig. 2, block *A* shows the two operating elements and the two conditions for each. Block *B* shows the magnitude of the trip current of each condition. Block *C* converts the current values into relative units, where 4 amp = 1. It also gives the probability of occurrence of each condition. The interplay between the various operating conditions gives rise to the four combinations shown in the field of the figure, the combinations being expressed in terms of the trip current and the probability of occurrence. The combinations are "added" in block *D* in a fashion which indicates the probability of the occurrence of a certain value. This is then converted into absolute units in block *E*. Inspection of this latter block reveals that the trip current may have three values, i.e., 4, 3.9, and 3.8 amp at probabilities of 0.25, 0.5 and 0.25, respectively. This probability sequence may also be expressed as $(0.5+0.5)^2$. This is the inception of a development tending to express the distribution of chance-generated values. It is illustrated in Fig. 3. The foregoing is recognized as an oversimplification of a complex process. It is intended as an illustration only and is not for analytical dissection by purists. To satisfy the latter and to advance toward the true state of affairs, let each effect have three values instead of two. This creates another and smoother distribution, as shown in Fig. 3, approximated by the $(a+b)^3$ expression. As this concept is carried

Fig. 2 (above). Distribution development

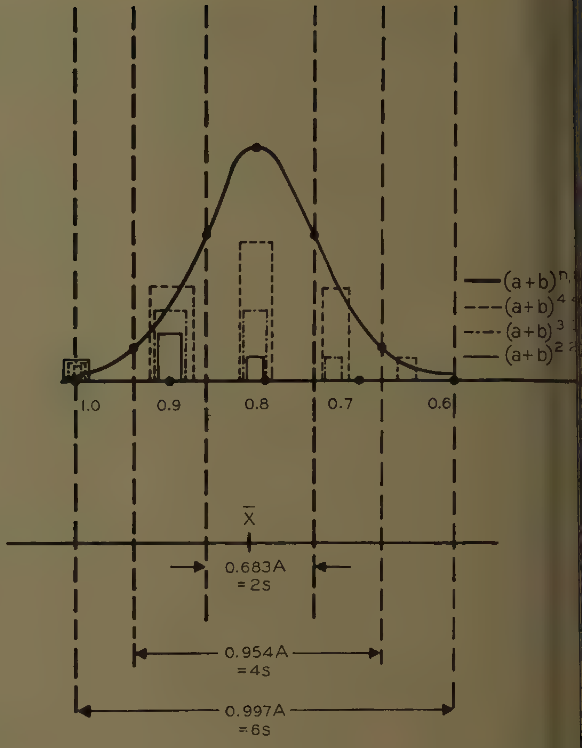


Fig. 3 (right). Development of normal curve

Note: A = area under the normal

further, the influences become larger in number and the differences between "cells" which created the steps in the histogram become increasingly smaller. Finally, arrriving at reality where each effect manifests itself in almost limitless numbers of values, the discrete steps disappear and the distribution assumes the bell-shape curve.

This is called the "normal curve of error" or the "normal." In this curve, the probability of an occurrence p of a certain current value is plotted on the Y axis. The current values are plotted on the X axis. Each value p has a complementary value q, the probability of nonoccurrence of a certain value, where $p+q=1$. A curve so normalized is said to exhibit probability density, indicative of the degree of probability that each current value has of occurring. This concept is linked to practical applications by considering the sample n. The probability that a certain current value will occur is

$$P_x = pn$$

Obviously, the probability of all values to appear is 1. Thus the area under the curve represents probability.

The shape of the normal is fixed by mathematical expression.[6] It approaches symmetry in large samples. The axis of symmetry intercepts the X axis at $\bar{X}$, the arithmetic average of the individual sample values. The unit which expresses the distance a value X is distant from $\bar{X}$ is called the "standard deviation" s, where

$$s=\sqrt{\frac{(\bar{X}-X_1)^2+(\bar{X}-X_2)^2+\ldots(\bar{X}-X_n)^2}{n}}$$

Let $\bar{X}-X=x$, the "variance" of individuals, then

$$s=\sqrt{Sx^2/n}$$

where S = sign of summation.

Thus, taking s as a unit measure, any variance may be expressed as x/s. This is valid from one distribution to another, regardless of differences in the magnitude of $\bar{X}$ or s, the only relating link being the adherence of the individual distributions to the normal.

The areas under the normal extending from $\bar{X}$ to any chosen value of X, equal to x, are expressed in tables in the various references as x/s, as a percentage of the total area. Thus, having x/s, we may readily ascertain the probability which any value in a sample may have of occurring in $\bar{X}-X$ interval. The practical application is based on the assumption that the sample is representative of a larger group. This larger group is generally called the population, or universe. Therefore, within certain limits, any quality of the sample may be inferred to the population. The inference is made that if X has a chance p of occurring in the x/s interval in the sample, any value similar to X in the population has a chance p of occurring.

Before proceeding, it must be realized that all the foregoing has been carefully stipulated as based on "random" occur

es. Thus, jamming of the spring and gnetization must have equally as much nce of occurring as of not occurring. l, it is important that there is no as," i.e., a shift in the operation pa-neter by a constant, extraneous force. illustrate, consider the influence of easing temperature caused by solenoid ting during a test. As a result of ting, electrical sensitivity shifts grad-ly, and the relay mechanism may nge friction constants. Thus, the a no longer conform to the mathe-tical basis of simple statistics. Fig.) illustrates the operation of the relay, h the individual operating currents tted about a central (time) axis and n grouped to be presented in a histo-m at the end of the test. This run s made with the relay permitted to cool between successive operations. In ntradistinction, when the total action a series of operations in quick suc-ssion, by heating, inclines the central is, a distribution is generated which s only a remote relationship to a bi-mial distribution; see Fig. 4(B). In neral, relatively large samples of the nd discussed here will approximate the rmal curve. The distribution illus-ated in Fig. 4(B) is said to be "skewed." any skewed distributions appear, they ould serve as "red flags." Before ocessing apparently skewed distribu-ons, a mathematical routine should be nployed to define their divergence from e normal. However, it should be finitely ascertained that this skew-ss is the result of the normal operation the test specimen and not introduced error.

Assuming that the data of Table I are cepted as statistically normal and are ocessed as shown, it is necessary to tain the elements which correlate with e normal distribution. The first ele-ent is the arithmetic average $\bar{X}$ and the dividual variances x. The root-mean-uare value of these variances yields s he σ, or standard deviation). The area der that portion of the curve which is us and minus s from $\bar{X}$ is equal to 0.683 the total area under the curve (Fig. 5). his amounts to saying that approximately 68% of all values should be found equal to $\bar{X}+s$. Thus, in the relay under test, where the average is 4.35 amp and the standard deviation is 0.12 amp, 68% of 20 values should lie between 4.35 amp $\pm$0.12 amp, or 4.23 and 4.47 amp. Actually, 75% fall into this interval. Similarly, 95% of the data should be in the interval of $\bar{X}\pm2s$, or 4.11 to 4.59 amp. This holds true in the sample. Finally, all values should be within the range of $\bar{X}$ plus and minus $3s$, or 3.99 to 4.71 amp. This also is borne out in the sample. The discrepancy at the "one sigma level" may be due to rounding off, as several values are very close to the critical value and could be "persuaded" to join the other group if closer reading and less rounding off were practiced. This degree of accuracy is commonplace, however, and need not concern us.

Had there been a value greater than $\bar{X}$ plus or minus $3s$ it would have been viewed with suspicion. Values beyond the "three sigma limit," to name the common term, may be generated by test error, computation error, or a distribution which is not normal. The term "maverick" is usually tacked on to such a value.

Under this term, statisticians group all values which are unexplainable and are tempted to neglect the embarrassing intruder. In a science where everything is explained methodically and along purely mathematical lines, the presence of a maverick requires deliberate action. To grant the maverick status as a true relative would upset the statistical soundness of the data collected. To deny the maverick its position requires an explanation, involving the observational and computational competence of the statistician. The disposition of mavericks is not a statistical exercise, but a task requiring knowledge of the specimen and the test. There is no maverick present in the example, so we will proceed.

Table I. Observations and Computations

Observation	Computations	
X, Amp	$X\times10^{-2}$	$(\times10^{-2})^2$
4.50	15	225
0.30	5	25
0.35	0	0
0.45	10	100
0.45	10	100
0.30	5	25
0.40	5	25
0.40	10	100
0.20	15	225
0.25	10	100
0.40	5	25
0.35	0	0
0.40	5	25
0.25	10	100
0.10	25	625
0.35	0	0
0.45	10	100
0.35	0	0
0.15	20	400
0.60	25	625
SX 87.00		$Sx^2=2{,}825\times10^{-4}$

$n=20$ $\quad s=\sqrt{\frac{Sx^2}{n}}=\sqrt{\frac{0.2825}{20}}=0.12$ amp

$\bar{X}=4.35$ amp $\quad s'=\sqrt{\frac{Sx^2}{n-1}}=\sqrt{\frac{0.2825}{29}}=0.122$ amp

$s_{\bar{X}}=\frac{s'}{n}=\frac{0.122}{20}=0.027$ amp

While this sample is composed of 20 operations, it cannot be offered as representative of the conditions which would prevail if an infinite number of 20-operation samples were drawn. Results for an infinite number of samples would resemble results for this sample, but the likelihood exists of acquiring individuals with extreme readings. This likelihood is obviously equally as great at the upper as at the lower limit. Hence, as compared with the limited sample distribution, a universe distribution will broaden out. There is very little one may do now to ascertain the relation of the sample

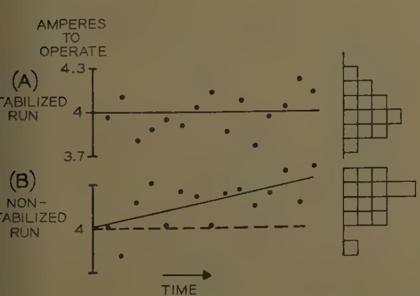


Fig. 4 (left). Bias operation

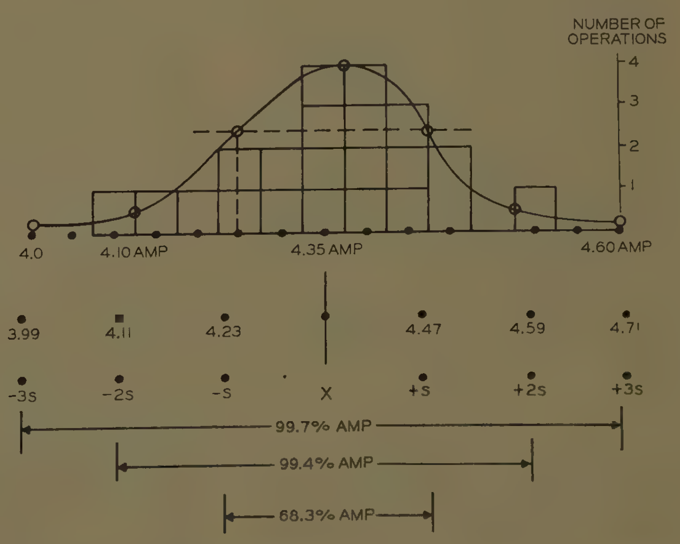


Fig. 5 (right). Typical observation, histogram

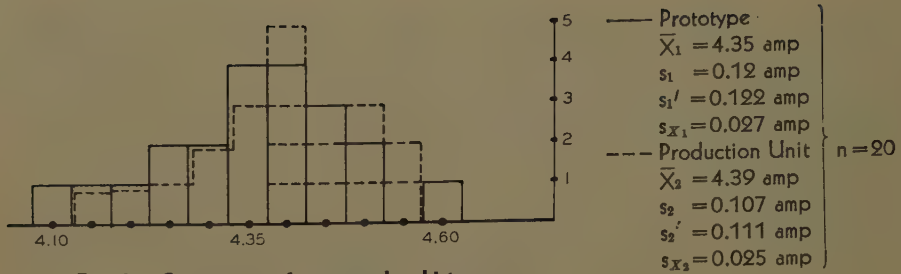


Fig. 6. Comparison of two samples, histogram

average to the universe average. We are limited to the statement that the sample average is the best estimate obtainable.

The sample deviation and the sample size are the determinants of the universe deviation s'. An estimate of s' may be made by considering that the likelihood of larger values in the universe than in the sample is a function of the magnitude of the variations in the sample and an inverse function of the sample size, or more correctly of the degrees of freedom existing in the sample. Thus

$$s' = \sqrt{\frac{Sx^2}{n-1}}$$

where S is the sign of summation and x denotes the individual variances. This yields an estimate of 0.122 amp for s' for the sample considered. Where the sample is expected to range from 3.99 to 4.71 amp, the universe may range from 3.98 to 4.72 amp. The reason for the small difference lies in the size of the sample and the good statistical normalcy. With a sample of smaller size, the multiplier exerts a much more pronounced effect, and with greater values of x a similar result occurs. In general, a sample size of 20 is rarely corrected for universe value.

Further confusion is precipitated by taking more samples, and particularly samples from a universe which may be somewhat different from that of the sample already taken. What then would be the nature of these new samples? Naturally there will be a variation in sample averages and ranges. The degree of variation between preceding and new samples is a function of the sample size both of the original and the new one, and of the variation noted in both. The degree to which samples will vary can be computed with two dimensions defining a normal sample.

First, there is the variation of average value of samples. This variation is the standard error of sample averages $s_{\bar{X}}$ where

$$s_{\bar{X}} = \frac{s'}{\sqrt{n}}$$

or a sample of 20 operations and a universe standard deviation of 0.122 amp, $s_{\bar{X}}$ = 0.0272 amp. Second, the standard deviation of the sample standard deviation s_s also will vary from sample to sample, where

$$s_s = \frac{s'}{\sqrt{2n}}$$

For a sample of 20 and a universe deviation of 0.122 amp, $s_s = 0.019$ ampere. Thus a definite dimensional limitation has been set to an uncertainty: Sample averages will vary by plus and minus $3s_{\bar{X}}$ and standard deviations by plus and minus $3s_s$. In numerical terms, the sample average of 4.35 amp ranges from 4.43–4.26 amperes. Simultaneously and independently, the standard deviation of 0.12 amp ranges from 0.177–0.063 amp. However, the important dimension of probability is still missing: The probability of occurrence is greater for the intermediate values than for the extremes of the range. This again introduces the concept of distribution.

An example of the practical use follows. Assume that the previously described relay were accepted as a prototype. A subsequent unit is then submitted as representative of production. It would be the laboratory's responsibility to determine relative quality. Basically then, it must be determined whether the prototype and the production unit are from the same universe. A universe may be defined in terms of average value, the type of distribution and the spread of values. Once it is assumed that the distribution of the test values is normal, then the value $\bar{X}'$ and the deviation s' uniquely define the universe. If two samples are taken from the universe, their individual averages $\bar{X}_1$ and $\bar{X}_2$ as well as the deviations s_1 and s_2 should bear a certain relationship.

This relationship is rarely clear-out, but resides in a grey area between total and zero affinity. The exact degree of relationship is estimated on the basis of the sample statistics. In the present case, the significance of a difference between averages of two samples is inversely proportional to the magnitude of the deviations existing in the samples. Similarly, the difference between individual deviations has significance only in terms of the magnitude of the universe devia-tion. To illustrate, note Fig. 6, in which the basic difference between the two samples is plotted, and numerical values are indicated. On the surface there seems to be little difference between the two items. It is the mission of statistics to reduce this superficial similarity and substitute for it an estimate couched in definitive numerical terms.

This is accomplished as follows: The difference between such sample averages is expressed in terms of the degree to which sample averages vary normally when picked at random, i.e., in terms of the standard error of sample averages. The interaction of the two individual standard errors (Fig. 6) gives rise to the standard error of sample differences $s_{\bar{X}_1-\bar{X}_2}$ where

$$s_{\bar{X}_1-\bar{X}_2} = \sqrt{s_{\bar{X}_1}^2 + s_{\bar{X}_2}^2} = \sqrt{0.027^2 + 0.025^2} = 0.037 \text{ amp}$$

The value of 0.037 amp is a measure used to assess the difference between two sample averages, where

$$\frac{x}{s} = \frac{\bar{X}_1 - \bar{X}_2}{s_{\bar{X}_1-\bar{X}_2}} = 1.1$$

The ratio x/s expresses the difference between two averages in terms of the standard errors of the sample averages. The probability that a certain value of x/s occurs in a given sample size is expressed in the t distribution (Fig. 7). Note that for a large value of x/s, i.e., one generated by a large difference between sample averages or by a small deviation value in the samples concerned, there is relatively little chance that two samples are derived from the same universe. The converse is also true.

In the present case, where x/s is equal to 1.1, there are 27 chances out of 100 that the difference is accidental. This in general would not be accepted as indicative of a significant difference. It illustrates a misconception which has plagued

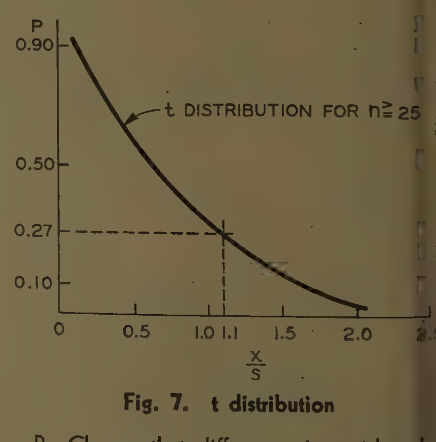


Fig. 7. t distribution

P = Chance that difference is accidental

itisticians for some time, that statistics a substitute for decision-making. Here decision was made for us, and it would em to follow that statistics are useless. he view opposed to this summary disissal holds that statistics furnish clues ly in most cases. The clues given in e present case warrant us to decide that ere might be some change in the mean erating point of the relay, and if so, at the change is not of significant agnitude.

There still remains a question with gard to the variations in the trip point. he particular question may be phrased s follows: To what degree is the prototype standard deviation of 0.12 amp lated to the corresponding value of 0.11 mp of the production unit?

The probability of the occurrence of a ertain numerical ratio between two ample deviations is predicted by the F istribution. Essentially, this is a family f curves, with one curve to each sample ize. The individual curve correlates he numerical value of the ratio between he sample deviations, the independent ariable, with the dependent variable, the robability that such a difference occurs ccidentally. The value of F together vith the degree of freedom existing in the amples is symbolized by $F_{n'_1 n'_2}$ and xpresses the affinity between the universe deviations of the two relays under onsideration as

$$F_{n'_1 n'_2}=\left(\frac{s'_1}{s'_2}\right)^2=1.21$$

Fig. 8 shows the location of the F value generated by the prototype deviation and the production type deviation as 1.21. A value of $F=2$ would have given a chance of only 5 in 100 that the value is accidental, according to published tables, which do not extend below this value.[2] Therefore, estimating from an extrapolation, there are about ten chances in one hundred that the values of deviation have a common universe. Conversely, then, there is a difference! Since the deviation is smaller for the production unit, the latter can be said to be an improvement over the prototype unit.

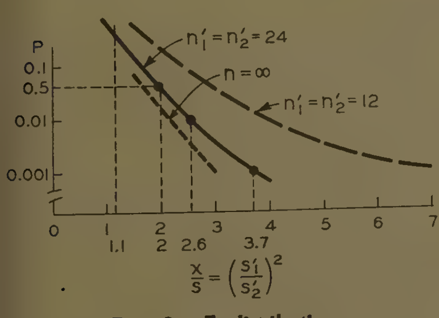


Fig. 8. F distribution

P = Chance that difference is accidental

Sampling Plans

We have demonstrated the relationship between the universe and a sample, or the relationship between two samples. This relationship furnishes the basis for the various sampling plans. A sampling plan is a procedure designed with a specific product and quality level in mind. The procedure is set up with the aim to employ a minimum of computation at the point of sampling, to reduce the number of samples to a minimum and to permit sampling at a relatively low level of skill.

The result of the sampling plan is a quantitative comparison between an unknown specimen and the quality level demanded of a particular product. Sampling usually proceeds by establishing whether a specimen does, or does not, operate within acceptable statistical limits. The result of the sampling, then, is a classification of the specimen as either acceptable or not acceptable.

The preceding case of the overload breaker will illustrate a sampling plan. To repeat the original specification, the relay is required to operate in the range of from 3.9 to 4.7 amp. To state that no single instance "ever" shall occur where the relay may operate outside this range is to remove the problem from the realm of present technology. The ever implies infinite reliability which simply does not exist. The ever is usually modified to a phase like "within a reasonable number of operations." This is an evasion of responsibility against the hope that a statistician will "pick up the ball" and assign a number to the word "reasonable." But this is not a statistical task! It is, certainly, the task of a systems engineer who knows the degree of reliability required of each part to obtain an overall reliability from his system. After, and only after, this expert has assigned the level of reliability expected, can a sampling plan be executed. An ideal input to the statistician would then be: "This relay is to operate between 3.9 and 4.7 amp in 997 out of 1,000 cases!"

At this juncture, the differentiation between "quality" and "reliability" may be mentioned. For the purpose of this paper, it may be defined in these terms: So far we have dealt with quality, which expresses performance in arbitrary abstract terms. Distinct from this is reliability which uses the abstract quality data and applies them against the level of performance as required of the object in use. The resultant expression is one which predicts the degree to which the quality of the part will allow it to fulfill its mission in the field, or how "reliable" it is. In most cases, the mission is expressed in terms of the probability of failure within a certain number of operations, or failure within a certain time span. This subject is not intended to be dealt within in detail. Reference 7 gives an excellent treatment of this item.

The first step in establishing a sampling plan is to question the ability of the item to meet specifications. In the present case, the prototype obviously cannot meet specifications, because in the sample of 20 operations, with a mean value of 4.35 amp and a deviation of 0.12 amp, the three sigma limits are 3.99 and 4.71 amp. The latter value is above specification. The subsequent production unit appears equally unacceptable. Where time is available, the testing of further units is recommended. However, posing the usual condition that this is a "fire drill" and that better units are expected later, for the time being, the requirement is assumed to be the universe of 3.9–4.7 amp.

The plan is then to set up such limits for the acceptance test that those not conforming to the required universe are rejected. Assuming that we wish to use 10 test operations as the acceptance test, as a compromise between economy and assurance, the test limits are derived as follows: The range permitted is 3.9–4.7 amp, putting the objective average $X_p=$ 4.3 amp. Referring to Fig. 10, the objective average is set up as the center line. The standard error of sample averages $s_{\bar{X}}$, operating on $\bar{X}$, has a normal distribution. Therefore, the extreme limits of high and low between which action occurs, i.e., the

$$\text{Action Limits of Sample Averages} \\ =\bar{X}\pm 3s_{\bar{X}} \tag{1}$$

where

$$s_{\bar{X}}=\frac{s'}{\sqrt{n}}$$

As s' cannot be ascertained, it will be estimated as

$$s'=\bar{s}K$$

where $\bar{s}$ is the average of the deviations of samples of 10, and K is a multiplier to correct for sample size, given in equation 2 as equal to 1.084. Equation 1 by substituting the foregoing becomes

$$\begin{aligned}\text{Action Limit of Sample Averages}& \\ &=\bar{X}\pm 3\left(\frac{1.084\bar{s}}{3.17}\right) \\ &=\bar{X}_p\pm 1.026s \qquad (2)\end{aligned}$$

The extreme limits of averages of equation 2 are of interest. They are created by exceptionally high or low individuals

in the sample of 10 operations. These individuals are three standard deviations from the sample average $\bar{X}$. Thus, the limits in terms of individual values may be approximated as

Approximate Limit of Individuals $= \bar{X} \pm 3s$

where s is the still unknown standard deviation, which is estimated as equal to the sample deviation average $\bar{s}$, or

$$s = \bar{s} \tag{3}$$

The greatest magnitude s may attain by chance variation is determined by its variability in the universe. This variability is the standard error of sample standard deviations s_s and

$$s_s = \frac{s'}{\sqrt{2n}} = \frac{sK}{\sqrt{2n}} = \frac{1.084s}{\sqrt{20}} = 0.242s \tag{4}$$

Thus, the limits to which s may vary can be expressed as

Action Limit of Sample Deviations

$$= s \pm 3s_s$$
$$= s \pm 3(0.242s)$$
$$= 1.726s \text{ and } 0.274s \tag{5}$$

The limits of individuals in a sample may be estimated by combining equations 3 and 5 to yield

$$\text{Limit of Individuals} = \bar{X} \pm 3(1.726s) \tag{6}$$

This represents the extreme value of an observation, when by chance, the maximum values of the sample deviation, the standard deviation of the sample standard deviation and the standard error coincide.

Since equation 6 gives a general expression for determining the limits in a universe, we set $\bar{X}$ to the extreme values of equation 2. Then by combining equations 2 and 6 as

Limits of Individuals

$$= \bar{X}_p \pm 1.026\bar{s} \pm 3(1.726s)$$

Setting, as before, $s = \bar{s}$, then

$$\text{Limits of Individuals} = \bar{X}_p \pm 6.214s \tag{7}$$

Referring to Fig. 9 it may be seen that equation 7 includes the range of from 3.9–4.7 amp. Therefore, so that this range may not be exceeded in general operation, in sampling with 10 operations the standard deviation of any sample must be equal to, or be smaller than

$6.214s = 0.4$ amp

$s = 0.0644$ amp

Therefore, in taking samples of 10 when the average is 4.3 amp, no greater range than $6s = 0.38$ must appear in the sample. The foregoing demonstration has been given, principally, for the purpose of showing the factors involved. The setting up of sampling plans is considerably simplified by using equation 5 or any similar reference.

Nothing yet has been said as to the probability of occurrence of the conditions of equation 6. The probability can be estimated as follows: First, note that three items are involved in arriving at equation 6: the standard error of sample averages, the standard deviation of the sample standard deviation, and the sample deviations. In each case, they were assumed to operate over the entire probability range, from +3 sigma to −3 sigma. In a normal distribution the area of the normal over $\pm 3s$ is set at 0.997. Therefore, the chance that action occurs beyond the three sigma limit is $1 - 0.997 = 0.003$. Thus, three probabilities of magnitude 0.003 result in a total probability of 0.003^3.

This is a very low degree of probability. It would, perhaps, be very fine to have the level of reliability implied by this low order of probability, but it also would be a very great economic burden. It is up to the engineer to set his requirement of reliability correctly, knowing that even as low a factor as 10^{-1} will involved economic penalties, in that fewer units will be found acceptable than in a higher order.

The approach to reducing the probability is through the application of the probability of each item's contribution to

Table II. Potentiometer Test Data

Position In. X	Resistance, Ohms $= Y$: Nominal, Y_p	Actual, Y	Estimated, Y_c	X
0	0	1.5	3.6	+2.1
0.5	75	77.0	78.8	+1.3
1.0	150	152.6	153.9	+1.1
1.5	225	228.0	229.1	+1.1
2.0	300	313.7	304.2	−9.5
2.5	375	379.3	379.4	+0.1
3.0	450	451.8	454.5	+2.7
3.5	525	530.2	529.7	−0.5
4.0	600	606.0	604.8	−1.2
4.5	675	681.4	608.0	−1.4
5.0	750	751.5	755.1	+3.6

$SX = 27.5$ $\quad SY = 4{,}173.0$

$Sx^2 = 96.25$ $\quad Sx^2 = 3.37$

$Y_c = 3.6136 + 150.3X$

$$s = \sqrt{\frac{Sx^2}{n}} = 3.37 \text{ ohms}$$

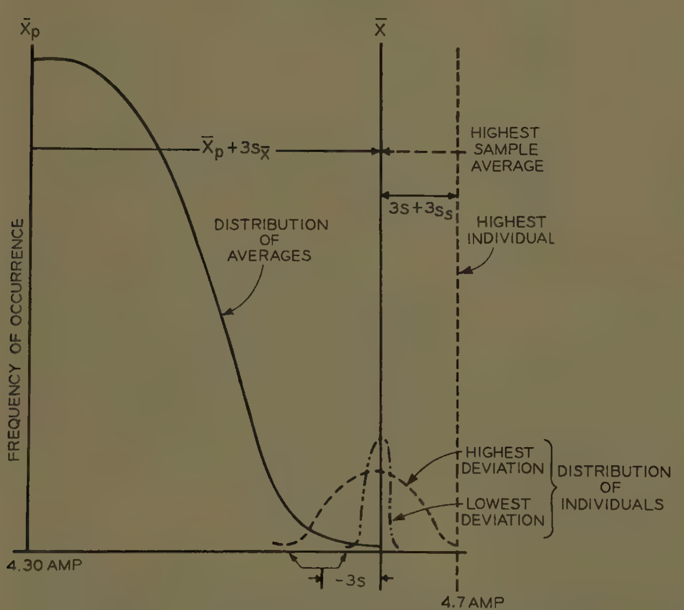


Fig. 9. Action limits

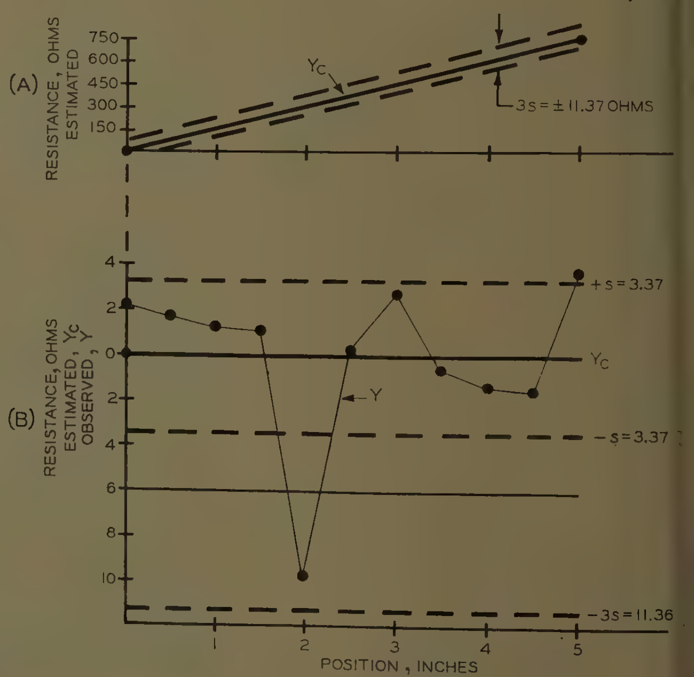


Fig. 10. Estimating equation

he total probability. Thus the operating level is set on a probability requirement 0.003, assumed to conform to the reliability requirement of the part in operation. Since there are three factors involved, then

$$P_1 \cdot P_2 \cdot P_3 = 0.003$$

and since each factor has an equal likelihood of contributing to the total probability

$$P = \sqrt[3]{0.003} = 0.14$$

This indicates that instead of carrying all considerations to the three sigma limit, it would be permissible to operate at the 1.2 sigma limit—which corresponds to an area of 0.14 under one "tail" of the probability curve, i.e., 14% of the total area.

Demonstration of Correlation

The preceding demonstration was concerned with the variability of the dependent variable, where it was held constant, or essentially so. In the following description the independent variable does vary as a normal function of its operation. The requirement is to set up a correlation between the variations of the dependent and the independent variable. In servomechanisms it is becoming increasingly important to ascertain the response of the system. The response, in simplest terms, is the correlation of input to output.

In the case to be discussed, the input consists of the 5-inch linear movement of a linkage element. The mechanism rotates the arm of a potentiometer. The resistance of the potentiometer is the output. Normally, the potentiometer is a 750-ohm total resistance type. The linkage, ideally, should not introduce any error and should be "set" so that at 0.0 inch, zero resistance will be shown in the output. The input X to output Y_P relationship can therefore be expressed as

$$Y_P = 150X \tag{8}$$

However, on testing this device the first run yields the values shown in Table II. There is a definite discrepancy between ideal and actual conditions.

As a first approximation, it is assumed that the output is straight line function of the input; therefore, the line will have the form

$$Y = a + bX \tag{9}$$

Since this sample consists of 11 observations, each of which is believed to be expressable in general terms by equation 9, the average of n observations may be expressed as

$$SY = na + bSX \tag{10}$$

where S is the sign of summation.

By this means, an equation line is described and so situated that positive and negative variations of Y are equal. However, this condition could be satisfied also by a line with a slope not necessarily passing through the center of gyration of all readings. To accomplish this it is necessary to plot the curve in accordance with the Law of Least Squares. This will assure us that with this line equation the smallest sum of squares of deviations of Y will be produced in comparison with any other straight line equation. The expression is, at one value each for a and b

$$SXY = aSX + bSX^2 \tag{11}$$

This condition, of course, also satisfies equation 10. However, to reach a solution, it is necessary to utilize both equations in a simultaneous solution. Therefore, combining equations 10 and 11 and utilizing the values of Table II, the following simultaneous equation is set up

$$\begin{aligned} 4.173 &= 11a + 27.5b \\ 14.567 &= 27.5a + 96.25b \end{aligned} \tag{12}$$

This yields the estimated value Y_c, the "estimating curve"

$$Y_c = 3.6 + 150.3X \tag{13}$$

The significance of equation 13 lies in its unique definition of the most likely operating center line Y_c of this linkage-potentiometer combination. See Fig. 10(A). While this is very valuable, a more important piece of knowledge concerns itself with the degree to which the output of this device may be expected to vary in a large number of operations.

To obtain the most likely operating point of the device for any one input, equation 13 is used. This, note carefully, is the most likely point only. The likelihood of another value occurring depends upon the degree of dispersion which existed, i.e., the scatter of the points about the estimating line. To begin with, there is an appreciable difference between Y_c and Y_p, the objective line of operation. However, this is of much less importance than the scatter of the 11 measured values of Y_c. See Fig. 10(B). The extent to which they differ from Y_c is an indication of the dispersion of this sample. This dispersion determines the probability of occurrence of larger values than the "most likely" value.

Solving for Y_c for each value of X, by means of equation 13, we obtain the best estimate of Y. The difference between the estimated and the actual values, therefore, is a measure of the dispersion. This is treated in a manner analogous to that previously discussed under sampling techniques—in terms of the standard deviations, standard errors, etc. The analogous operations are:

$$\text{General Form} \left\{ \begin{aligned} s &= \sqrt{Sx^2/n} \\ s &= \sqrt{S(\bar{X}-X)^2/n} \end{aligned} \right.$$

$$\begin{matrix}\text{Corresponding} \\ \text{Form for} \\ \text{Correlations}\end{matrix} \left\{ \begin{aligned} s &= \sqrt{S(Y_c-Y)^2/n} \\ s &= \sqrt{(a+bX-Y_c)^2/n} \\ &\quad \text{where } Y=(f)X \end{aligned} \right.$$

Referring again to Fig. 10(A), the three sigma limits are lines situated at a distance of $3s$ above and below the estimating equation line Y_c. It is within these limits that 99.7% of all values may be found. In the present case, where $s = 0.294$ ohms as computed, but not shown here

$$3s \text{ Limits} = Y_c \pm 3s = 3.6 + 150.3X \pm 11.36$$
$$= 150.3X \pm {14.99 \atop 7.76} \text{ ohms}$$

Since this is for a sample of 11, any comparison with other samples must make allowance for sample size, by the methods previously used, namely, expansion of s to s' and the application of $s_{\bar{X}}$ equivalents.

Conclusions

The foregoing examples of three types of statistical evaluation have been selected to give an appreciation of the scope of application. In statistical terms, with only a small sample demonstrating such a wide range of application, is it not reasonable to suppose that the full application of statistics will open up still greater possibilities? It is hoped that this paper will have stimulated the reader to apply statistics and to reap the full benefit of the use of this art.

To complete this short exposition a summary of the important points to consider when applying statistical analyses follow:

1. Data can be processed by statistical means, whether on the basis of attributes or measurements. In either case, however, there must be no bias and the influences which cause variations in the measurements or attributes must operate by chance.

2. Inferences formed about a population must be tempered in terms of the sample size.

3. The critical examination of data, as to their fitness for processing, is an exercise of judgment far more important than the mere arithmetical processes which grind out answers. It is in this phase of the work

that the most discriminating abilities are in demand.

In closing, let us confirm our belief that diligent application of the above points, coupled with a willingness to undertake a relatively new approach, will produce better and more easily understood data—with less effort.

Appendix. Glossary and Formulas

Listing follows the sequence of appearance in the test

sample = a randomly selected group of observations representative of a larger group

X = numerical value of one observation, with numerical subscripts denoting the different members of the sample

n = number of observations in a sample

S = sign of summation

$\bar{X}$ = sample average, $\bar{X} = SX/n$

x = deviation of a single observation from the average of a sample, sign disregarded, $x = \bar{X} - X$

s = standard deviation in a sample, $s = \sqrt{\frac{Sx^2}{n}}$

s' = universe standard deviation, where "universe" denotes a group of extremely large size. Universe values are denoted by the superscript apostrophe. The true value is usually unknown and formulas furnish an approximation only,

$$s' = s\sqrt{\frac{n}{n-1}}$$

$s_{\bar{X}}$ = standard deviation of sample averages, generally called the "standard error",

$$s_{\bar{X}} = \frac{s'}{\sqrt{n}}$$

s_s = standard deviation of sample standard deviations, generally called the standard error of sample standard deviations, $s_s = \frac{s'}{\sqrt{2n}}$

$s_{\bar{X}_1-\bar{X}_2}$ = Standard deviation of the difference between sample averages, $s_{\bar{X}_1-\bar{X}_2} = \sqrt{s_{\bar{X}_1}{}^2 - s_{\bar{X}_2}{}^2}$

x/s = standard measure, expressing the deviation of an observation from the average value of a sample in terms of the Standard Deviation in that sample. x need not necessarily be from the sample represented by s,

$$x/s = \frac{\bar{X} - X}{s}$$

n' = degrees of freedom within a sample, $n' = n - 1$

$F_{n'_1n'_2}$ = estimated population standard deviation of one sample as compared with similar parameter of another sample,

$$F_{n'_1n'_2} = \left(\frac{s_1}{s'_2}\right)^2$$

$\bar{X}_P$ = objective average; the mean value about which individual values are expected to group themselves

$\bar{s}$ = average of Sample Standard Deviations,

$$\bar{s} = \frac{Ss}{n}$$

P = probability of the occurrence of an event, expressing it as a decimal,

$$P = \frac{\text{Probability of occurrence}}{\text{Probability of nonoccurrence}}$$

Y_c = computed value of Y, where $Y = (f)X$ and Y_c is an estimate of Y

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Exciter Armature Reaction and Excitation Requirements in a Brushless Rotating-Rectifier Aircraft Alternator

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IN MODERN high-speed high-altitude aircraft, commutation and brush wear become critical problems in the exciters of synchronous a-c generators. To overcome these problems, brushless rotating-rectifier a-c generators have been developed.[1,2] In this type of machine, the commutator and brushes are completely eliminated, along with their associated problems. The design engineer, however, is faced with a new problem. He must design an exciter which is an a-c generator loaded by a rotating rectifier feeding a highly inductive load: the main generator rotating field. The rectifier acts somewhat as a rotating switch, switching the load current from one phase to the other. The nonlinear load then presents design problems: What is the effect of this load on exciter armature reaction, terminal voltage, and power factor? The purpose of this paper is to answer this question. A method of calculation is outlined to convert a given d-c rectifier load to an equivalent a-c load. With the equivalent a-c load known, the generator designer can use conventional formulae to design an optimum exciter and regulator system for a given application.

Discussion

Fig. 1 shows a schematic diagram of the basic circuit for a brushless rotating-rectifier type generator. It is seen that the exciter is basically an a-c generator feeding a highly inductive load by means of a rotating rectifier. Fig. 2 shows the circuit for the basic problem. Several properties of the basic circuit will be considered as follows:

1. Because of the action of the rectifier, the load will only "see" the largest instantaneous terminal voltage of the alternator (reduced by the voltage drop through the rectifier).

2. Only the phases contributing to the largest terminal voltage will carry current.

3. Because of inductance in each phase it is not possible for the load current to shift instantaneously from one phase to another. Thus, there is a transition period, or commutation period, where two phases act in parallel and aid each other in carrying the load current.

4. The inductance of the load will tend to keep the load current constant.

The basic problem, then, is to determine

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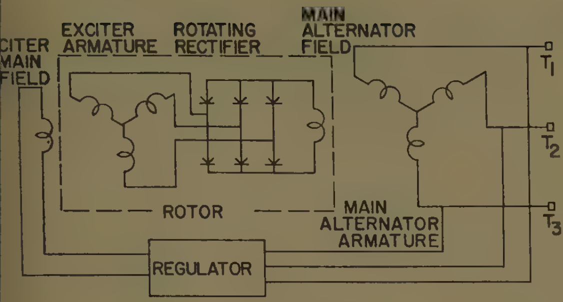


Fig. 1 (left). Basic circuit of brushless generators

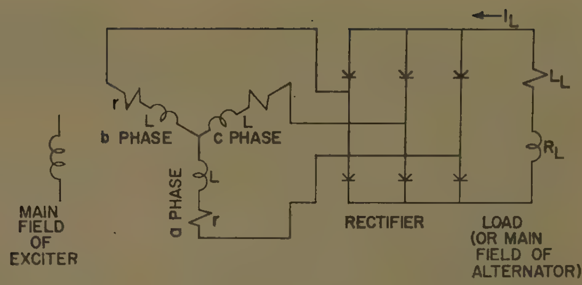


Fig. 2 (right). Circuit of basic problem

e current required by the field, in Fig. to establish a given load current, I_L.

It is necessary to make certain assumptions as follows:

Rectifier cells have infinite resistance their reverse direction.

The forward resistance of a cell is equal the maximum forward voltage drop of the ctifier cell divided by the maximum cell rrent for a given load condition.

The inductance of the rectifier load is so rge that it "swamps" any inclination for e load current to change during load voltge variations such as might be due to comutation.

The per-phase generated voltage is nusoidal with no harmonics.

$K_{dpn}=0$ for $n\geq 5$ (see Appendix V).

n regard to assumption 2, it is realized nat semiconductors tend to have constant rward voltage drop[3] instead of constant rward resistance under various load onditions. However, to facilitate athematical analysis, assumption 2 was sed. Fig. 3 shows the rotating rectifier ad current in a 40-kva aircraft generator nder full load conditions. It is seen that ssumption 3 seems reasonable. Assumpon 4 is true if the effective generator -c exciter) field form is sinusoidal and ne effective armature reaction contains nly the fundamental space and time armonics. Pitch and distribution facrs in the generator tend to make the higher space harmonics in the field form and armature reaction ineffective. It will be shown later (Appendix IV) that because of the rectifier load, 5th and 7th time harmonics are generated by the armature reaction. These time harmonics rotate relative to the stator and thus are dampened by the (a-c exciter) field winding and any damper bars in the exciter pole face. Thus, in the usual case where pitch and distribution factors are operating and damping is effective, assumption 4 seems reasonable.

Looking at Fig. 2, it is first assumed that L, r, and the rectifier resistance are negligible. In this case the load (main rotating field) will see the maximum generated voltage of the exciter. Also, only those two phases contributing to this voltage will carry the load current. Thus, for any given time two phases carry current and one does not, and as a result, every phase must then carry the load current two-thirds of the time. Also, because of assumption 3, the current will be constant and equal to the load current. These ideas are summarized in Fig. 4.

In general, however, the phase current cannot change to the load current value instantaneously because of inductance in the phase. Also, because of the IR (current resistance) drop in the phase, the terminal voltage of the unloaded phase becomes equal to the terminal voltage of the loaded phase sooner than would take place if the IR drop were negligible. At this time, then, the unloaded phase starts to take part of the load current. In effect, the two phases act in parallel with the load current, gradually shifting to the initially unloaded phase until this phase completely takes the current. This period of time, when the two phases act in parallel, will be referred to as "Commutation Period."

Fig. 4 may now be modified as shown in Fig. 5 to show the effect of inductance and resistance. (The dashed curve in Fig. 5 indicates the phase current when L and r were neglected.)

It is now desirable to analyze the effect of commutation. During commutation of c phase with b phase, the two terminal voltages, e_{tc} and e_{tb}', are equal to each other. See list of symbols in Appendix V.

Thus

$$e_{tb}=e_{tc}$$

or

$$e_{gc}-i_c r'-L\frac{di_c}{dt}=e_{gb}-i_b r'-L\frac{di_b}{dt}$$

$$e_{gc}-e_{gb}=i_c r'-i_b r'+L\frac{di_c}{dt}-L\frac{di_b}{dt}$$

From assumption 3

$$i_b=I_L-i_c$$

$$\frac{di_b}{dt}=0-\frac{di_c}{dt}$$

or

$$e_{gc}-e_{gb}=(2i_c-I_L)r'+2L\frac{di_c}{dt}$$

The previous equation is solved in Appendix I for i_c. The solution is expressed as a decimal fraction of the load

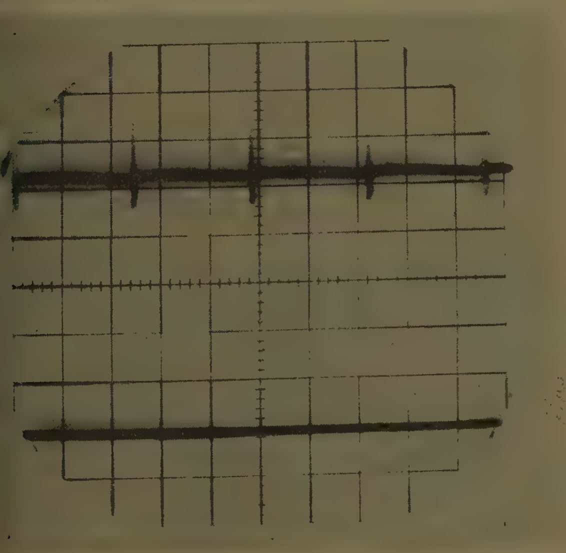


Fig. 3 (left). Rotating rectifier load current in a 40-kva generator

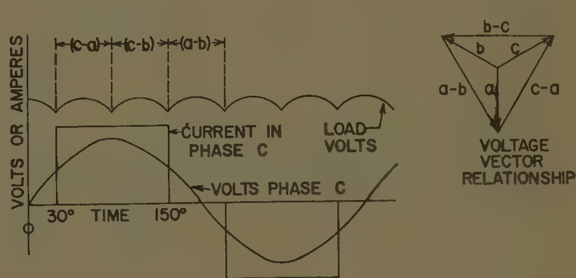


Fig. 4 (right). Voltage and current relationship in C phase assuming L and r are negligible

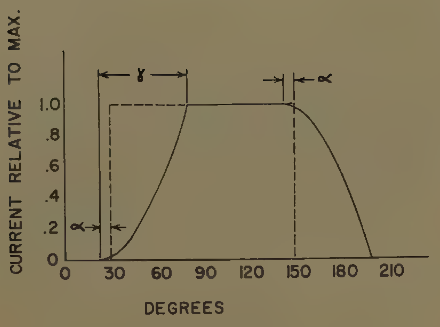


Fig. 5. Calculated exciter armature current for a 40-kva generator where L and r′ are considered

current. This solution applies only during commutation and is

$$\frac{i_c}{I_L}=0.5(1-e^{-\beta/M})+$$
$$K\{e^{-\beta/M}\sin(\theta'-\alpha)+\sin(\beta-\theta'+\alpha)\}$$

When phase c is carrying current but is not commutating, then it is carrying all the load current and $i_c=I_L$. Therefore, the equation for i_c/I_L applies from time $t=0$ until $i_c=I_L$. It is also noted that when i_b commutates with i_c, $i_b+i_c=I_L$.

In the equation for i_c/I_L, two constants, M and K, define the equation as a function of time, β. The firing angle, α (see Fig. 5), and impedance angle, θ', are defined when M and K are known. Fig. 5 shows the calculated current wave shape for an exciter of a 40-kva oil-cooled generator. Since the per-phase current has now been expressed in terms of time, it is next possible to determine armature reaction.

Armature Reaction

The mmf (magnetomotive force) wave for an a-c winding is determined through the use of the Fourier series[4] for an armature current varying sinusoidally with time. In this deviation each phase is handled separately first.

Thus

phase A = constant $\times$

$$I\sin wt\left(K_{dp}\cos\theta-\frac{K_{dp}}{3}\cos 3\theta+\ldots\right)$$

phase B = constant $\times I\sin(wt-120)\times$

$$\left[K_{dp}\cos(\theta-120)-\frac{K_{dp}}{3}\cos 3\theta+\ldots\right]$$

phase C = constant $\times I\sin(wt-240)\times$

$$\left[K_{dp}\cos(\theta-240)-\frac{K_{dp3}}{2}\cos 3\theta+\ldots\right]$$

where

θ = position on the armature in electrical degrees

The three expressions are then added together and the armature mmf obtained as a function of both position on the armature and time. The effect of the rectifier load is to introduce time harmonics into the armature current as shown in Appendix III. By introducing the new expression for the current, it is again possible to obtain the armature mmf as a function of position on the armature and as a function of time. This has been done in Appendix IV. The fundamental mmf (in time) which is stationary with respect to the pole is shown in the appendix to be

$$\frac{\text{ampere-turns}}{\text{pole}}=M_a=0.9\times$$
$$0.707 I_L N_c q a_1' K_{dp}\frac{3}{2}$$

where a_1' is the maximum amplitude for the fundamental per-phase current. The value of a_1' can be found using formulas derived in Appendix III.

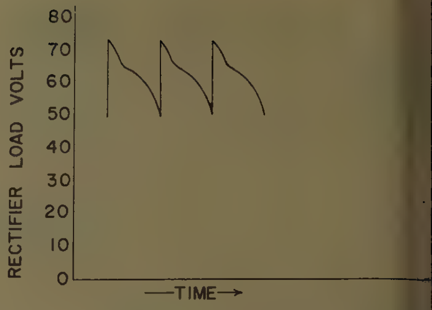


Fig. 7. Calculated rectifier load voltage wave for a 40-kva generator at full load

It is also shown in Appendix IV that there is a 6th harmonic reaction relative to the poles. This harmonic is also discussed in a previous paper.[1] Its effect on exciter field current for a 40-kva generator is shown in Fig. 6.

Voltage Wave Shape

Knowing the current wave shape, it is possible to determine the load-voltage wave shape. This is accomplished by taking the generated voltage in a loaded phase and subtracting the IR drop, $L\,di/dt$, and rectifier cell drop. Assumption of sinusoidal generated voltage simplifies the calculation. In Appendix II the load voltage wave shape is determined. By integrating the expression for load voltage, it is possible to determine an equation to calculate the average load voltage; equation 29 then results.

The rotating-rectifier load-voltage wave shape was calculated at full load for a 40-kva oil-cooled brushless generator using the method shown in Appendix III. The calculated wave shape is shown in Fig. 7. (Time proceeds from left to right to agree with the oscilloscope Fig. 8.) An oscilloscope picture was also taken of

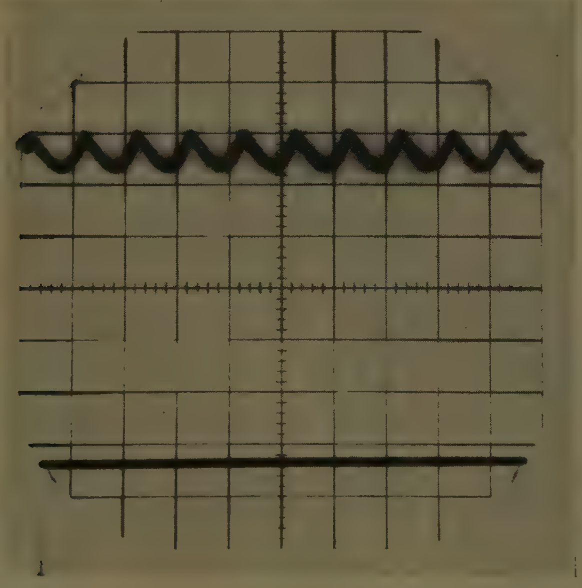


Fig. 6 (left). Exciter field current with battery excitation showing 6th harmonic ripple

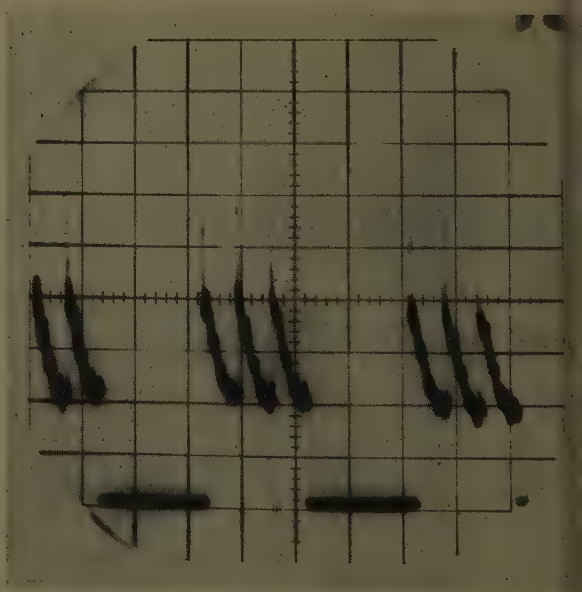


Fig. 8 (right). Volts across a rectifier diode for a 40-kva generator at full load

e rectifier diode voltage in this machine d is shown in Fig. 8. When the diode is nducting, its voltage drop is just the rward voltage drop of the cell. Its con- ction period is 120 degrees $+\gamma$. In g. 8, the conduction period appears to about 180 degrees by observing the ro voltage line. This indicates a com- utation period, γ, of about 60 degrees. is compares with a calculated com- utation period of 44 degrees. In the lculation, the direct-axis subtransient actance, X_d'', was used for X_L because the short time over which commutation kes place. (In the machines tested ere were no damper bars, thus X_d'' ualled X_d'.) When the diode is not nducting, the rectifier load voltage is en. This voltage wave shape can then compared with Fig. 7. The difference the load voltage wave shape in Figs. 7 d 8 is apparently due to assumptions ade in the specific calculation as well as e theory. For example, the tempera- re of the rotating exciter armature re- stance was assumed since determination test was too difficult.

ector Diagram

Although the fundamental armature action mmf has been determined, there e still two things to consider in order to etermine the effect this mmf has on the eld of the machine. One thing is the fect of a nonuniform airgap in the salient- le machine. In determining the mmf action, a uniform air gap was assumed. his effect requires a modification of the uation for armature reaction. The her thing to consider is where the arma- re mmf is relative to the pole. The tter item is determined by a vector agram which will be described shortly. he quadrature and direct axis mmf's are follows: For the direct axis

$$\frac{I}{} = C_d M_a \sin\psi = C_d \times 0.9 \times 0.707 I_L N_c q \frac{3}{2} \times a_1' \times \sin\psi$$

or the quadrature axis

$$\frac{I}{} = C_q M_a \cos\psi = C_q \times 0.9 \times 0.707 I_L N_c q \frac{3}{2} \times a_1' \times \cos\psi$$

where C_d and C_q are constants depending on the pole embrace and are defined under symbols. ψ is the angle between the armature current and voltage, E_d; see Fig. 9.

In the vector diagram the per-phase and rms values of currents and generated voltages will be considered in the con- ventional manner.[4] The armature cur- rent will be used as the reference vector. The rms value of this current is

$$I_1 = a'_1 \times I_L \times 0.707$$

Since the terminal voltage per-phase is not a constant vector in magnitude, it is desirable in this discussion to consider the per-phase generated voltage which is assumed constant in magnitude. It is also possible to determine the angle be- tween this generated voltage and the fundamental armature current. This angle would be 30 degrees if L and r were negligible; however, the angles α and τ_1 must be considered as shown in Figs. 9 and 10, and as determined in Appendixes I and III. In consequence, the angle be- tween the fundamental of armature cur- rent and the generated voltage is

$$\eta = 30 + \alpha + \tau_1 \text{ (for lagging power factor } \eta \text{ is positive)}$$

The next problem is to determine the magnitude of the generated voltage, E_g. To determined E_g, equation 29 of Appen- dix II is used. E_{avg} is known, being the IR drop of the load. However, the angles, α and γ, are not defined until E_g $(=0.707$ $E_m)$ is known. Thus, a trial and error solution is required to determine the generated voltage, E_g. The commuta- tion period, γ, is determined by solving equation 20 for $i_c/I_L = 1$. In order to solve equation 20, it is necessary to assume a value for E_g. This assumption can be made reasonably well by the following equation

$$E_g = \frac{E_m}{\sqrt{2}} \approx \sqrt{(0.45E_{avg} + 0.78 I_L r')^2 + (0.78 I_L X_L)^2}$$

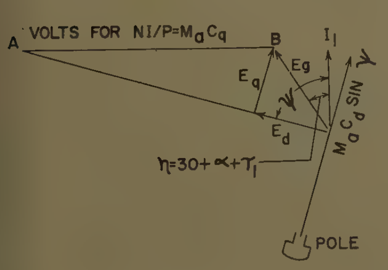
This equation assumes unity power factor and assumes the ratio of average load volts to maximum to be 0.91. If there were no commutation, the ratio would be 0.956. The equation also assumes the ratio of maximum of fundamental current to maximum of per-phase current is 1.1. In the exciters investigated under var- ious loadings, this ratio varied from about 1.06 to 1.12. If there were no com- mutation, the ratio would be 1.1. With the assumption of E_g, α and γ can be determined. If the commutation period is 60 degrees, it is also necessary to find V_1 by a trial and error process. However, in most cases γ is less than 50 degrees in which case $V_1 = 0$. E_g can now be cal- culated using equation 29. The assump- tion of E_g can then be checked.

In order to determine the effect of armature reaction on pole mmf, it is still necessary to determine the angle ψ in Fig. 9. The pole must have sufficient mmf to overcome this reaction and in addition must have sufficient net mmf to generate the voltage E_d. Thus, it is also neces- sary to determine the voltage E_d which can be done by vectors once ψ and E_g are known.

It is seen that if the vector **AB** were known, it would be relatively easy to determine the angle ψ. In Chapter 9 of reference 4, a method of solution is shown whereby **AB** can be considered an artificial voltage in quadrature with I_1 and gen- erated by the artificial mmf$=\mathbf{M}_a$ $\mathbf{C}_q$. With ψ and $\mathbf{E}_g$ now known, the vector diagram can be solved.

Computer Calculations

As is obvious to the reader, the method of calculation presented is impractical for hand calculations. Hand calculations would be practical if more simplifying assumptions could be made. But in order to keep from being limited further by assumptions, it was decided to use an International Business Machines Corpora- tion *704* computer. A program was set up for the computer to take a given d-c rectifier load and find the equivalent a-c load. Thus, using formulas and methods previously explained, the equiv- alent a-c rms per-phase current, and terminal volts were determined as well



Figt. 9 (lefl). Vector diagram

Fig. 10 (right). Relationship of generated volts and armature current

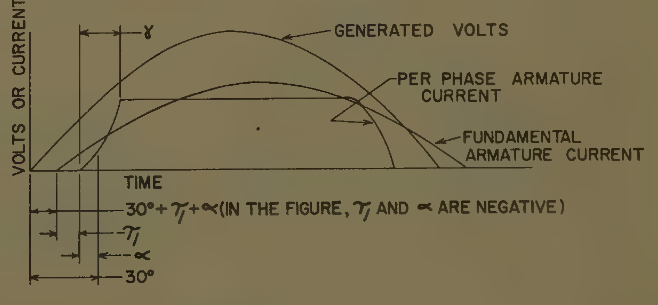


Table I. Excitation Requirements

Main Generator				Rectifier D-C Load			Exciter									
							Degrees			Equivalent A-C Load				Ampere-Turns/Pole		
														Computer		
Kva	W-Type	Load, %	Rpm	Volts, E_{avg}	Amperes	Poles	α	β	a_1'	L-N Volts, V_{LN}	Amperes	P.F. Degrees	Ratio* V_{LN}/E_{avg}	Saturation	Total	Tes Tot
40	6QM40C	0	8,000	17.9	5.2	4	−6.2	43	1.08	8	4	−12.8	0.447		121	11
		100	8,000	62	19	4	−4.2	44	1.10	27.8	14.9	−14.3	0 448	6	470	44
		150	8,000	95	26.1	4	−3.6	41	1.08	42.7	20.1	−13.5	0.449	16	645	66
40	6QM40D	50	8,000	22	15	6	−4.3	39	1.11	9.8	11.8	−13.6	0.445	5	165	14
		100	8,000	44.5	25.8	6	−2.9	36	1.11	19.7	20.4	−14	0.443	4	295	27
		200	8,000	80.2	48.2	6	−2.5	37	1.08	35.8	37.1	−13.2	0.446	11	532	56
		400 Short	8,000	62.2	63.0	6	−3.6	46	1.10	27.9	49.0	−14.9	0.449	7	603	590
31.5	Y6QM31.5	100	7,600	30.5	29.0	8	−4.8	59.3	1.06	14.0	21.9	−14.0	0.459	4	301	28
		150	7,600	50.5	41.8	8	−4.0	56.3	1.067	23.1	31.7	−14.5	0 457	21	462	46
		200	7,600	61.5	55.2	8	−4.0	57.8	1.063	28.1	41.8	−14.4	0.457	188	760	70
		200	8,000	61.5	52	8	−3.9	57.8	1.063	28.2	39.4	−14.4	0.458	110	650	63

* This ratio is the ratio of the equivalent exciter a-c L-N (rms) terminal volts divided by the average d-c rectifier load voltage. If there were no commutatic this ratio would be 0.4275.

as power factor for a given rectifier d-c load current and volts. This equivalent load was determined by first calculating the voltage E_g and angle η as already described. An artificial equivalent a-c terminal voltage and power factor were then calculated so that when IR and IX drops were added to the terminal volts, the voltage E_g would result at an angle η with respect to the fundamental a-c armature current. The equivalent a-c fundamental armature current was determined by using equations 36 and 37. The program was used to determine excitation of three aircraft oil-cooled brushless generators at various loadings. These results are shown in Table I.

It should be pointed out that because of the type of loading, in general the exciters operate unsaturated. In Table I the most saturated operating condition was in the 31.5-kva generator operating at 200% load. Calculations indicated 188 ampere-turns were due to saturation.

Conclusions

The armature reaction has been studied in a three-phase full-wave bridge rectifier loaded alternator. Using formulas derived in this paper it is possible to convert a rectifier d-c load into an equivalent a-c load. The equivalent a-c load is made such that when I_1r' and I_1X_L voltages are added to the "equivalent" a-c exciter terminal volts, the proper generated voltage, E_g, results at the proper angle, η, with respect to the fundamental current, I_1. There is some question as to what reactance should be used for X_L. It appeared in the generators tested that the direct-axis subtransient reactance, X_d'', gave the best results. (In these generators X_d'' equalled X_d' since there were no damper bars.)

In the generators investigated the calculated ratio of equivalent exciter L-N terminal volts divided by the average rectifier load voltage varied between about 0.44 and 0.46. This figure would be 0.427 if there were no commutation. The equivalent exciter rms line current varied betwen 0.755 and 0.791 times the d-c rectifier load current (I_L), whereas the figure would be 0.78 for no commutation. The artificial power factor angle varied between 13 and 15 degrees lagging.

A sixth harmonic armature reaction was shown to exist.

Appendix I. Commutation Current

During the commutation of c phase with b phase in Fig. 2, the terminal voltages are equal. Thus

$$e_{tc}=e_{tb} \tag{1}$$

$$e_{gc}-i_{cr}'-L\frac{di_c}{dt}=e_{gb}-i_{br}'-L\frac{di_b}{dt} \tag{2}$$

By assumption 3 of the third paragraph of this paper

$$i_b=I_L-i_c \tag{3}$$

$$\frac{di_b}{dt}=0-\frac{di_c}{dt} \tag{4}$$

From equation 2

$$e_{gc}-e_{gb}=(2i_c-I_L)r'+2L\frac{di_c}{dt} \tag{5}$$

The boundary values are

$$i_c=0 \text{ at } t=0 \tag{6}$$
e_{gc} is 120° behind e_{gb}

It is now possible to find the firing angle, or that time when current will start flowing in c phase. This firing angle is an angle, in radians, with reference to the time of firing if r' and L were zero.

When $i_c=0$, equation 5 becomes

$$e_{gc}-e_{gb}=-I_Lr'+2L\frac{di_c}{dt}=\sqrt{3}\,E_M\times \sin\,(wt+\alpha)$$

It is desirable to reference time from whe $i_c=0$. Thus, $wt=0$ at $i_c=0$. The angle α the angle which $e_{gc}-e_{gb}$ is at when $i_c=0$, or other words, $\alpha=$the firing angle for $e_{gc}-e_c$ At the time of the firing, e_{gc} is at 30°$+\alpha$ sinc $E_{gc}=E_M\ \sin(wt+30+\alpha)$.

As a result

$$\sqrt{3}E_M\sin(\alpha)=-I_Lr'+2L\frac{di_c}{dt}$$

or

$$\alpha=\sin^{-1}\left(\frac{-I_Lr'+2L\dfrac{di_c}{dt}}{\sqrt{3}E_M}\right)$$

or

$$\alpha=\sin^{-1}\left(\frac{-I_Lr'}{\sqrt{3}E_M}+V_1\right)$$

where V_1 is discussed in Case II of thi appendix.

Case I: $\gamma<60$ degrees

When the commutation takes place less than 60 degrees, the current, i_c, zero for some period of time prior to com mutation. Therefore, $di_c/dt=0$ prior commutation. At the firing of i_c there are no sudden voltages applied in the tw paralleled phases. The two generated vo tages which cause the phases to comm tate change gradually. Thus, there is sudden change in di_c/dt and it is, therefor zero at $t=0$.

Thus, for $\gamma<60$ degrees

$$\alpha=\sin^{-1}\left(\frac{-I_Lr'}{\sqrt{3}E_M}\right)$$

Case II: $\gamma=60$ degrees

Commutation between two specific phas cannot take longer than 60 degrees. Th can be reasoned as follows: When tw phases are commutating, the remainin phase must carry all of the current. Th it must carry the total load current one-thi of the time. The current repeats itse

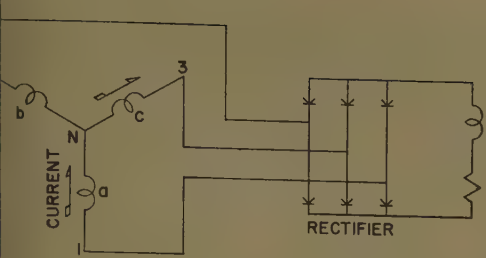


Fig. 11 (left). Direction of assumed current flow

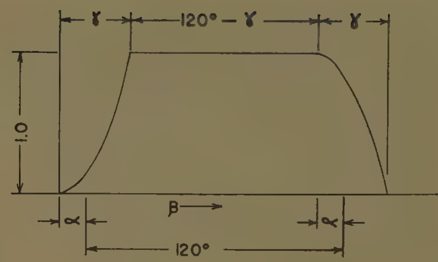


Fig. 12 (right). General current wave shape

magnitude) every 180 degrees. A given
se must then carry all of the load current
at least 60 degrees. This leaves 120
rees in which the phase can com-
tate with the other two phases, or it can
mmutate up to 60 degrees with one of the
er two phases.

n Case I it was shown that $di_c/dt=0$ at
). This is not necessarily true when $\gamma=$
degrees because in this case there can be
udden voltage applied which causes di_c/dt
have a value at $t=0$. This sudden volt-
comes from the $L\ di/dt$ voltage present
t prior to firing.

f there is an initial value of di/dt, then
in equation 7 has a value. V_1 must have
h a value that the firing angle will cause
current to reach its final value in 60
rees. Thus, V_1 must be chosen in such a
y that i_c/I_L in equation 20 equals 1.0 for
60 degrees (if $\gamma=60$ degrees). This cal-
ation may be done by trial and error
uming different values of α.

Having discussed the angle α, the solu-
n for i_c is continued as follows:

om equation 7

$$3E_M \sin(wt+\alpha)=e_{gc}-e_{gb}$$

om equation 5

$$\frac{3E_M \sin(wt+\alpha)+I_Lr'}{2}=i_cr'+L\frac{di_c}{dt} \quad (10)$$

ing

$$(a+b)=\sin a \cos b+\cos a \sin b$$

uation 10 becomes

$$\frac{3E_M}{2}(\sin wt \cos \alpha+\cos wt \sin \alpha)+\frac{I_Lr'}{2}=i_cr'+L\frac{di_c}{dt} \quad (11)$$

Laplace transforms

$$\frac{3E_M}{2}\cos\alpha\left(\frac{\omega}{S^2+\omega^2}\right)+\frac{\sqrt{3}E_M}{2}\times \sin\alpha\left(\frac{S}{S^2+\omega^2}\right)+\frac{I_Lr'}{2}\times\frac{1}{S}=f(i_c)(r+LS) \quad (12)$$

ere S is an operator. Let

$$=\frac{\sqrt{3}E_M}{2}\cos\alpha$$

$$=\frac{\sqrt{3}E_M}{2}\sin\alpha$$

$$=\frac{I_Lr'}{2}$$

en

$$(i_c)=\frac{C_1wS+C_2S^2+C_3(S^2+w^2)}{(S^2+w^2)(S)(r+LS)} \quad (13)$$

From partial equations

$$\frac{A}{S}+\frac{BS+C}{S^2+w^2}+\frac{D}{r+LS}=\frac{C_5S^2+C_6S+C_7}{S(S^2+w^2)(r+LS)} \quad (14)$$

where

$$C_5=C_2+C_3$$

$$C_6=wC_1$$

$$C_7=w^2C_3$$

and solving

$$A=\frac{I_L}{2} \quad (15)$$

$$B=\frac{\sqrt{3}E_M}{2}\left(\frac{r'\sin\alpha-X_L\cos\alpha}{Z^2}\right) \quad (16)$$

$$C=\frac{\sqrt{3}E_M}{2}\omega\left(\frac{r'\cos\alpha+X_L\sin\alpha}{Z^2}\right) \quad (17)$$

$$D=\frac{\sqrt{3}E_M}{2}L\left(\frac{X_L\cos\alpha-r'\sin\alpha}{Z^2}\right)-\frac{I_LL}{2} \quad (18)$$

By conversion, equation 14 becomes

$$i_c=I_L+\frac{\sqrt{3}E_M}{2}\left(\frac{r'\sin\alpha-X_L\cos\alpha}{Z^2}\right)\times \cos wt+\frac{\sqrt{3}E_M}{2}\left(\frac{r'\cos\alpha+X_L\sin\alpha}{Z^2}\right)\times \sin wt+\left[\frac{\sqrt{3}E_M}{2}\left(\frac{X_L\cos\alpha-r'\sin\alpha}{Z^2}\right)-\frac{I_L}{2}\right]e^{-r't/L} \quad (19)$$

Dividing both sides of equation 19 by I_L and substituting $\beta=wt$, $\theta'=$impedance angle $=\cos^{-1}r'/Z$, $M=X_L/r'$, and $K=\sqrt{3}\ E_M/2I_L\ Z$, the following is obtained for the commutation period:

$$\frac{i_c}{I_L}=0.5(1-e^{-\beta/M})+K[(e^{-\beta/M})\sin(\theta'-\alpha)+\sin(\beta-\theta'+\alpha)] \quad (20)$$

where α is determined by equation 8.

Appendix II. Voltage Wave Form

In Appendix I, the phase current was determined for the commutation period. For the rest of the conduction period, the phase current is equal to the load current which is assumed constant in this paper. Since the phase current is thus known, and since the generated voltage is assumed to be a vector of constant magnitude, it is possible to determine the terminal (or load) voltage of the generator. This is done by subtracting, algebraically, the IR and $L\ di/dt$ voltages from the generated voltage. The solution is as follows:

From equations 2, 3, and 4 (during commutation of b phase with c phase)

$$e_{gb}=e_{gc}-2\left(i_cr'+L\frac{di_c}{dt}\right)+I_Lr' \quad (21)$$

By adding $e_{gb}+e_{gc}$ and from equation 21

$$e_{gb}+e_{gc}=2\left(e_{gc}-i_cr'-L\frac{di_c}{dt}\right)+I_Lr'=2e_{tc}+I_Lr' \quad (22)$$

or

$$e_{tc}=\frac{e_{gb}+e_{gc}-I_Lr'}{2} \text{ (during commutation)} \quad (23)$$

The load voltage is found, then: From Fig. 11 and equation 23 and since the current I_L is not changing in phase a at this time

$$e_{ta} \text{ (from } N \text{ to 1)}=e_{ga}-ir'=e_{ga}+I_Lr' \text{ where } I_L \text{ is positive} \quad (24)$$

$$-e_{ta} \text{ (from 1 to } N)=-e_{ga}-I_Lr' \quad (25)$$

$$e_{Lt}=e_{tc}-e_{ta}$$

$$e_{Lt}=\frac{e_{gb}+e_{gc}-I_Lr'}{2}-e_{ga}-I_Lr' \text{ (during commutation)} \quad (26)$$

When commutation is not taking place, the load voltage, e_{Lt}, is determined as follows (current I_L is not changing):

$$e_{tc}=e_{gc}-I_Lr' \quad (27)$$

From equations 25 and 27

$$e_{Lt}=e_{gc}-e_{ga}-2I_Lr' \text{ (no commutation)} \quad (28)$$

In summary then

$$e_{Lt}=\frac{e_{gb}+e_{gc}}{2}-e_{ga}-\frac{3}{2}I_Lr' \text{ (during commutation of } b \text{ and } c \text{ phases)}$$

$$e_{Lt}=e_{gc}-e_{ga}-2I_Lr' \text{ (when there is no commutation)}$$

where

$$e_{gc}=E_M\sin\lambda$$

$$e_{ga}=E_M\sin(\lambda-120)$$

$$e_{gc}=E_M\sin(\lambda+120)$$

Commutation occurs for $\lambda=30+\alpha$ to $\gamma+30+\alpha$

Conduction with no commutation occurs for $\lambda=\gamma+30+\alpha$ to $90+\alpha$

It is sometimes desirable to know the average load voltage in terms of E_g, I_L, r', etc. The average load voltage, E_{avg}, can be determined by integrating equations 26 and 28 and dividing by the time interval. This method was used and the solution is

$$E_{avg}=1.17[\cos\alpha+\cos(\gamma+\alpha)]E_g+0.477I_Lr'(\gamma)-2I_Lr' \tag{29}$$

Appendix III. Harmonics of the Current Wave

A. General Solution

From the assumption of constant load current and from Appendix I, the current, i_c, has been determined. In general, this current is as shown in Fig. 12. The angle β is the variable and is taken from the time when the current is zero. In this appendix the harmonics of this general wave shape will be determined.

By the Fourier Series

$$f(x)=\frac{1}{2}a_0+a_1\sin\beta+a_2\sin 2\beta+\ldots a_n\sin n\beta+b_1\cos\beta+b_2\cos 2\beta+\ldots b_n\cos n\beta \tag{30}$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{+\pi}f(x)\sin n\,d\beta \tag{31}$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos n\beta\,d\beta \tag{32}$$

From Appendix I and assumption 3

$$f(x)\text{ from }(\beta=0\text{ to }\gamma)=0.5(1-e^{-\beta/M})+K[\sin(\theta'-\alpha)\epsilon^{-\beta/M}+\cos(\theta'-\alpha)\times\sin\beta-\sin(\theta'-\alpha)\cos\beta] \tag{33}$$

$$f(x)\ (\text{from }\beta=\gamma\text{ to }120°)=1.0 \tag{34}$$

$$\begin{aligned}f(x)\ (\text{from }\beta=120°\text{ to }120°+\gamma)&=1-0.5(1-\epsilon^{-\beta-120°/M})-K[\sin(\theta'-\alpha)\epsilon^{-\beta-120°/M}+\\&\cos(\theta'-\alpha)\sin(\beta-120°)-\sin(\theta'-\alpha)\cos(\beta-120)]\end{aligned} \tag{35}$$

Applying equation 31 to equations 33, 34, and 35 the solution for a_n is obtained and is

For $n=1$

$$\begin{aligned}a_n=\frac{2}{\pi}\Bigg[&-X_0(\cos\gamma-1)+\frac{X_1\epsilon^{-\gamma/M}}{\frac{1}{M^2}+1}\times\left(-\frac{1}{M}\sin\gamma-\cos\gamma\right)+\frac{X_1}{\frac{1}{M^2}+1}+\\&\frac{X_2\gamma}{2}-\frac{X_2\sin 2\gamma}{4}+\frac{X_3}{2}\sin^2\gamma+X_4\sin\gamma+\frac{X_5\epsilon^{-\gamma/M}}{\frac{1}{M^2}+1}\times\left(-\frac{1}{M}\cos\gamma+\sin\gamma\right)+\frac{X_5}{\left(\frac{1}{M^2}+1\right)M}+\\&\frac{X_6}{2}\sin^2\gamma+\frac{X_7\gamma}{2}+\frac{X_7\sin 2\gamma}{4}+\cos\gamma+0.5\Bigg]\end{aligned}$$

For $n\neq 1$

$$\begin{aligned}a_n=\frac{2}{\pi}\Bigg(&\frac{-X_0}{n}(\cos n\gamma-1)+\frac{X_1\epsilon^{-\gamma/M}}{\frac{1}{M^2}+n^2}\times\left(-\frac{1}{M}\sin n\gamma-n\cos n\gamma\right)+\frac{X_1n}{\frac{1}{M^2}+n^2}+\\&\frac{X_2\sin(1-n)\gamma}{2(1-n)}-\frac{X_2\sin(1+n)\gamma}{2(1+n)}-\frac{X_3}{2}\left[\frac{\cos(n-1)\gamma}{n-1}+\frac{\cos(n+1)\gamma}{n+1}-\frac{1}{n-1}-\frac{1}{n+1}\right]+\frac{X_4}{n}\sin n\gamma+\\&\frac{X_5\epsilon^{-\gamma/M}}{\frac{1}{M^2}+n^2}\left(-\frac{1}{M}\cos n\gamma+n\sin n\gamma\right)+\frac{X_5}{\left(\frac{1}{M^2}+n^2\right)M}-\frac{X_6}{2}\left[\frac{\cos(1-n)\gamma}{1-n}+\frac{\cos(1+n)\gamma}{1+n}-\frac{1}{1-n}-\frac{1}{1+n}\right]+\\&X_7\frac{\sin(1-n)\gamma}{2(1-n)}+X_7\frac{\sin(1+n)\gamma}{2(1+n)}+\frac{\cos n\gamma}{n}-\frac{\cos n120°}{n}\Bigg)\end{aligned} \tag{36}$$

where

$X_0=0.5(1+\cos n\,120°)$

$X_1=K\,[\sin(\theta'-\alpha)](1-\cos n\,120°)-0.5(1-\cos n\,120°)$

$X_2=K\,[\cos(\theta'-\alpha)](1-\cos n\,120°)$

$X_3=-K\,[\sin(\theta'-\alpha)](1-\cos n\,120°)$

$X_4=0.5\sin n\,120°$

$X_5=[0.5-K\sin(\theta'-\alpha)]\sin n\,120°$

$X_6=-K\,[\cos(\theta'-\alpha)]\sin n\,120°$

$X_7=K\,[\sin(\theta'-\alpha)]\sin n\,120°$

Applying equation 32 to equations 33, 34, and 35, the following is obtained:

For $n=1$

$$\begin{aligned}b_n=\frac{2}{\pi}\Bigg(&C_0\sin\gamma+C_1\frac{\epsilon^{-\gamma/M}}{\frac{1}{M^2}+1}\times\left(-\frac{1}{M}\cos\gamma+\sin\gamma\right)+\frac{C_1}{\left(\frac{1}{M^2}+1\right)M}+\\&\frac{C_2}{2}\sin^2\gamma+C_3\left(\frac{\gamma}{2}+\frac{\sin 2\gamma}{4}\right)+C_4(1-\cos\gamma)+\\&\frac{C_5}{\frac{1}{M^2}+1}\left[-\epsilon^{-\gamma/M}\left(\frac{\sin\gamma}{M}+\cos\gamma\right)+1\right]+C_6\left[\frac{\gamma}{2}-\frac{\sin 2\gamma}{4}\right]+\frac{C_7}{2}\sin^2\gamma+\\&\sin 120°-\sin\gamma\end{aligned}$$

For $n\neq 1$

$$\begin{aligned}b_n=\frac{2}{\pi}\Bigg(&\frac{C_0}{n}\sin n\gamma+C_1\frac{\epsilon^{-\gamma/M}}{\frac{1}{M^2}+n^2}\times\left(-\frac{1}{M}\cos n\gamma+n\sin n\gamma\right)+\frac{C_1}{\left(\frac{1}{M^2}+n^2\right)M}-\frac{C_2}{2}\left[\frac{\cos(1-n)\gamma}{1-n}+\frac{\cos(1+n)\gamma}{1+n}-\frac{1}{1-n}-\frac{1}{1+n}\right]+\\&\frac{C_3}{2}\left[\frac{\sin(1-n)\gamma}{1-n}+\frac{\sin(1+n)\gamma}{1+n}\right]+\frac{C_4}{n}[1-\cos n\gamma]+\frac{C_5}{\frac{1}{M^2}+n^2}\times\left[-\epsilon^{-\gamma/M}\left(\frac{\sin n\gamma}{M}+n\cos n\gamma\right)+n\right]+\\&\frac{C_6}{2}\left[\frac{\sin(1-n)\gamma}{1-n}-\frac{\sin(1+n)\gamma}{1+n}\right]+\frac{C_7}{2}\left[\frac{\cos(n-1)\gamma}{n-1}+\frac{\cos(n+1)\gamma}{n+1}-\frac{1}{n-1}-\frac{1}{n+1}\right]+\frac{1}{n}[\sin n\,120°-\sin n\gamma]\Bigg)\end{aligned}$$

where

$C_0=0.5(1+\cos n\,120°)$

$C_1=K\,[\sin(\theta'-\alpha)](1-\cos n\,120°)+0.5(\cos n\,120°-$

$C_2=K\,[\cos(\theta'-\alpha)](1-\cos n\,120°)$

$C_3=-K\,[\sin(\theta'-\alpha)](1-\cos n\,120°)$

$C_4=-0.5\sin n\,120°$

$C_5=(\sin n\,120°)\,[K\sin(\theta'-\alpha)-0.5]$

$C_6=K\,(\sin n\,120°)\cos(\theta'-\alpha)$

$C_7=-K\,(\sin n\,120°)\sin(\theta'-\alpha)$

Appendix IV. Armature Reaction

To determine the ampere-turns of armature reaction, the method used in Chapter

reference 4 is used. In this reference, [th]e armature reaction is shown to be

$$NI/P \text{ for } a \text{ phase} = 0.9 I_{rms} N_c q \times \sin wt \left[K_{dp} \cos \theta - \frac{K_{dp3}}{3} \cos 3\theta + \frac{K_{dp5}}{5} \cos 5\theta - \ldots \right] \quad (38)$$

For b phase the reaction is the same except [i]n wt becomes sin (wt-120) and for c phase [it] becomes sin (wt-240). The equations for [th]e three phases are added together to get [th]e total effect. In the rectifier loaded [a]lternator, the phase current has time har[m]onics as seen in Appendix III. Therefore, [b]y substituting the value of the three phase [c]urrents represented as time harmonics into [t]he armature reaction equations and adding [t]he equations together the total reaction can [b]e obtained.

By checking for 3rd harmonics in a_n of [e]quations 36 and 37 it is found 3rd's and [t]heir multiples cannot exist. The instan[t]aneous current equation is then repre[s]ented as a series of sine terms.

$$i_a = I_L[a_1' \sin (wt+y_1) + a_5' \sin (5wt+y_5) + a_7' \sin (7wt+y_7) \ldots] \quad (39)$$

Equation 38 is now modified by sub[s]tituting 0.707 times equation 39 for "$I_{rms} \sin wt$" in (equation 38). In like man[n]er, phases b and c are modified. The three [r]esulting equations are added together and [t]he result is

[F]or the fundamental

$$f(wt) = a_1' \times 0.9 \times 0.707 I_L N_c q \frac{3}{2} \left[K_{dp1} \times \sin(wt-\theta) + \frac{K_{dp5}}{5} \sin (wt+5\theta) - \frac{K_{dp7}}{7} \sin (wt-7\theta) \ldots \right] \quad (40)$$

(Note: The reference for time $t=0$ has been changed so that y_1 does not appear.)

For the 5th harmonic

$$f(5wt) = a_5' \times 0.9 \times 0.707 I_L N_c q \frac{3}{2} \times \left[K_{dp1} \sin (5wt+\theta) + \frac{K_{dp5}}{5} \sin (5wt - 5\theta) - \frac{K_{dp7}}{7} \sin (5wt+7\theta) \ldots \right] \quad (41)$$

(Note: The reference for time $t=0$ has been changed so that y_5 does not appear.)

For the 7th harmonic

$$f(7wt) = a_7' \times 0.9 \times 0.707 I_L N_c q \frac{3}{2} \times \left[K_{dp1} \sin(7wt-\theta) + \frac{K_{dp5}}{5} \sin (7wt + 5\theta) - \frac{K_{dp7}}{7} \sin (7wt-7\theta) \ldots \right] \quad (42)$$

In order to reduce equations 40, 41, and 42 by approximation, it is desirable to assume that

$$K_{dpn} = 0 \text{ for } n \geq 5 = \text{assumption 5} \quad (43)$$

Equations 40, 41, and 42 can be reduced to equation 44 for harmonics up to the 11th. Armature reaction equals

$$NI/P = 0.9 \times 0.707 I_L N_c q \frac{3}{2} K_{dp1} [a_1' \times \sin (wt-\theta) + a_5' \sin (5wt+\theta) + a_7' \sin (7wt-\theta) \ldots] \quad (44)$$

Realizing that $\sin(wt-\theta)$ and $\sin(7wt-\theta)$ are forward revolving waves, it is seen that the 5th time-harmonic rotates backwards and the 7th time-harmonic rotates forward. These waves are fundamental space harmonics which rotate at their respective harmonic speeds. It is desirable to consider the armature reaction relative to the poles. In this case, then, the terms in equation 44 change by wt as follows:

Armature reaction relative to the poles:

$$NI/P = 0.9 \times 0.707 I_L N_c K_{dp1} q \frac{3}{2} [a_1' \times \sin (-\theta) + a_5' \sin (6wt+\theta) + a_7' \times \sin (6wt-\theta) \ldots] \quad (45)$$

It is interesting to note that the 5th and 7th time-harmonic of armature current have cause a 6th time-harmonic reaction on the poles.

Appendix V. Definition of Symbols

$a_1, a_3, \ldots a_n$ = amplitude of first, third, ... or nth sine harmonic of armature current expressed as a decimal fraction of I_L, dimensionless. The harmonic is relative to the armature. With respect to the poles, the harmonics change by the synchronous speed

a'_n = amplitude of nth harmonic current wave $= \sqrt{a_n^2 + b_n^2}$

$b_1, b_3, \ldots b_n$ = amplitude of first, third, ... or nth cosine harmonic of armature current as a decimal fraction of I_L, dimensionless

C_d = armature mmf factor in the direct axis for salient pole synchronous machines, dimensionless

$$= \frac{\alpha'\pi + \sin \alpha'\pi}{4 \sin \frac{\alpha'\pi}{2}}$$

C_q = armature mmf factor in the quadrature axis for the salient pole synchronous machine, dimensionless

$$= \frac{\alpha'\pi \sin \alpha'\pi + 0.5 \cos \frac{\alpha'\pi}{2}}{4 \sin \frac{\alpha'\pi}{2}}$$

C_1, C_2, C_3, etc. = real constants. See Appendix III for specific values

E_{avg} = average rectifier load voltage, volts. (Y-connected exciter)

E_d = direct axis component of E_g, rms volts

$ERLVM$ = maximum rectifier load voltage, volts

E_g = per-phase generated voltage, rms volts $= 0.707 \times E_m$

E_m = maximum value of e_{ga}, e_{gb}, or e_{gc}, volts $= \sqrt{2}\, E_g$

E_q = quadrature axis component of E_g, rms volts

e_{ga}, e_{gb}, e_{gc} = instantaneous generated voltage of phase a, b, or c in volts

e_{ta}, e_{tb}, e_{tc} = instantaneous terminal voltage of phase a, b, or c in volts including the forward voltage drop of one rectifier cell

e_{Lt} = instantaneous load voltage, volts

f = frequency, cycles per second

I_L = load current in amperes; assumed constant

I_{rms} = rms value of armature current where the current is sinusoidal, rms amperes

I_1 = fundamental armature current, rms amperes

i_a, i_b, i_c = instantaneous current in phase a, b, or c, amperes

Δ_i = change in current i, amperes

K = a constant $= \frac{\sqrt{3}E_m}{2I_L Z}$, dimensionless

$K_{dp1}, K_{dp3}, K_{dpn}$ = distribution factor times pitch factor for the first, third, or nth harmonic, dimensionless

L = self-inductance of the a-c exciter phase due to leakage flux, henrys

L_L = inductance of load, henrys

M = a constant $= \frac{X_L}{r'}$, dimensionless

M_a = fundamental armature reaction, ampere-turns per pole[4]

N_c = conductors per slot

NI_{dmax} = the maximum direct axis component of armature reaction per pole, ampere-turns per pole

NI/P = ampere-turns per pole

n = as a subscript denotes nth harmonic

q = slots per pole per phase

R_L = resistance of load, ohms

r = effective per phase resistance of an alternator phase, ohms

$r' = r$ plus effective forward resistance of one rectifier cell, ohms

S = symbol used in Laplace transforms

t = time in seconds

V_1 = value of $\dfrac{2L \frac{di_c}{dt}}{\sqrt{3}E_m}$ at $t=0$, dimensionless

$w = \omega$

$X_L = \omega L$ = effective phase reactance $= 2\pi fL$ (f = fundamental frequency), ohms

X_1, X_2, X_3, etc. = constants. See Appendix III for specific values

$Y_n = \tan^{-1} \frac{b_n}{a_n}$

Z = impedance $\sqrt{r'^2 + X_L^2}$, ohms

α = firing angle of phase current = zero if r and L are negligible, degrees

$\alpha = \sin^{-1}\left(\frac{-I_L r}{\sqrt{3}E_m} + V_1\right)$, degrees

α' = ratio of pole arc to pole pitch, dimensionless

β = time in radians $= \omega t$, at $t=0$, $i_c = 0$

γ = commutation angle in radians or degrees, that is, the angle through which commutation takes place between two phases such as b and c

θ = position on the armature in electrical degrees or radians

θ' = impedance angle $= \cos^{-1} \frac{r'}{Z} = \cos^{-1} \frac{1}{\sqrt{1+M^2}}$, degrees

η = angle between per phase generated voltage and fundamental per phase current $= 30° + \alpha + \tau_1$ = positive for lagging current

$\lambda=\omega t$ where $e_{gc}=0$ at $t=0$, radians
 $=$angle between the armature current and direct-axis generated voltage, degrees

$\tau_1=\arctan -b_1/a_1$

$\tau_n=$angle between the nth harmonic armature current and the point where the instantaneous armature current equals zero, degrees

$\omega=2\pi f$, radians per second

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Analog Computer Representation of an Aircraft Electric Power Generating System

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IN THE PROCESS of designing and developing an aircraft a-c electric power system, a state of development is reached where some of the components, due to the present state of the art and the steady-state specifications, have their designs essentially fixed. The analysis of an aircraft a-c electric power system presented in this paper illustrates how time, effort, and costs required to complete the development can be substantially reduced if proper system dynamic performance can be assured prior to the construction of the actual hardware.

System Description

The aircraft power system analyzed consisted of:

1. An automatic, magnetic-amplifier type of voltage regulator.
2. A conventional, brushless, synchronous a-c generator.
3. An a-c pilot exciter.

The voltage regulator senses the variation of the main a-c bus voltage from a fixed reference, amplifies the error signal and applies it to the shunt field of a brushless a-c exciter. The a-c output of the exciter is rectified by a 3-phase full-wave bridge rectifier and applied to the fields of the synchronous generator and the pilot exciter. This excitation into the field of the synchronous machine produces the main a-c bus voltage.

The pilot exciter is a 3-phase a-c machine. This machine, rather than the main a-c bus voltage, is used as a voltage supply for the regulator in order that the main generator will receive excitation during main-generator short-circuit conditions.

An additional input signal to the voltage regulator is obtained from the a-c exciter. This signal is used to obtain stable closed-loop control of the main a-c bus voltage and also to improve the transient recovery of this voltage during electric load switching conditions.

When the study was initiated, only calculated performance curves and calculated constants were available for use in analyzing the magnetic amplifiers, exciter, generator, and pilot exciter.

Nomenclature

$k_1, K_1, k_2, K_2=$magnetic amplifier constants
$R_{02}=$second-stage amplifier internal resistance
$s=$Laplace transform operator
$e'=$exciter air-gap voltage
$e_d=$exciter generated voltage
$e_t=$exciter terminal voltage
$K_e=$exciter gain constant
$K_3=$constant relating air-gap flux to pole flux in exciter
$i_f=$exciter shunt field current
$i_t=$exciter line curent
$\phi=$exciter pole flux
$N_{fe}=$turns per pole of a-c exciter shunt field
$R_f=$exciter shunt field resistance
$SRDS=$square root of difference of two squares
$SRSS=$square root of sum of two squares
$x_d=$direct-axis synchronous reactance of a-c exciter
$x_d'=$pilot-axis transient reactance of a-c exciter
$k_p=$pilot exciter gain constant
$C_1, C_2, C_3, C_d, C_t=$synchronous generator constants defined by equations 34 through 36, 39, and 45
$E_d'=$voltage behind direct-axis transient reactance
$E_q'=$voltage behind quadrature-axis transient reactance
$E_{td}=$direct-axis component of generator terminal voltage
$E_{tq}=$quadrature-axis component of generator terminal voltage
$E_t=$generator terminal voltage
$E_p=$pilot exciter terminal voltage
$E_{fd}=$generator field voltage
$I_{fd}=$generator field current
$I_d=$direct-axis component of generator stator current
$I_q=$quadrature-axis component of generator stator current
$I_{fq}=$current flowing in quadrature-axis transient paths
$R_L=$resistive load on synchronous generator
$X_L=$reactive load on synchronous generator
$T_{do}'=$direct-axis open-circuit time constant of generator
$T_{qo}'=$quadrature-axis open-circuit time constant of generator
$X_d=$direct-axis synchronous reactance of generator
$X_d'=$direct-axis transent reacitance of generator
$X_q=$quadrature-axis synchronous reactance of generator
$X_q'=$quadrature-axis transient reactance of generator
$w/w_b=$ratio of operating speed to nominal speed

System Analysis

System analysis requires the writing of mathematical equations governing the performance of the various system parts. For the complex control system under consideration, these mathematical expressions are very complex and are functions of many parameters. Since the control system utilizes nonlinear elements, the system analysis requires the solution of simultaneous, nonlinear integrodifferential equations.

A d-c electronic differential analyzer was in this analysis because it can compute the solution of simultaneous integro-

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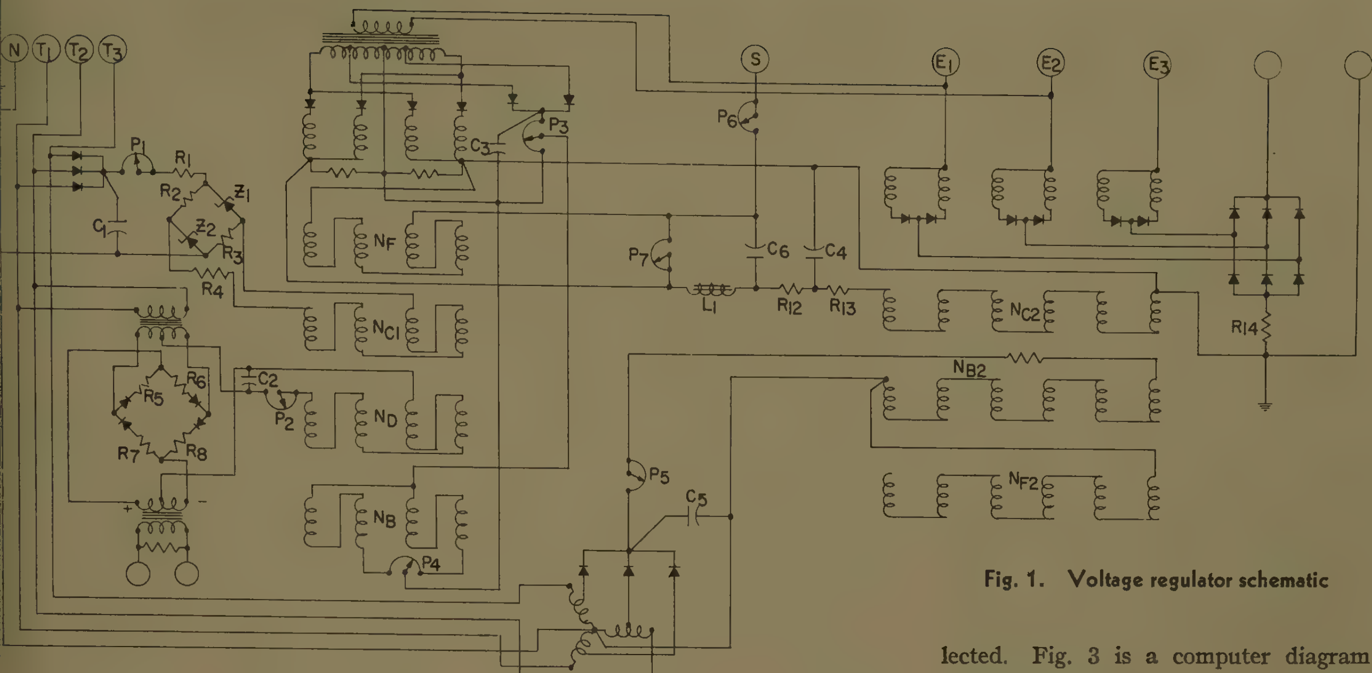

Fig. 1. Voltage regulator schematic

ifferential equations and also simulate onlinear phenomena. In addition, variations in parameters can be accomplished by changing potentiometer settings.

In the development of the following nathematical representations, the numbers in italics refer to computer amplifier CA) numbers in which the equations re solved.

The sensing network shown schematially in Fig. 1 can be reduced to the quivalent circuit shown in Fig. 2.

$$E_C'=I_{C1}R_C'+N_{C1}\ s\phi_{01} \tag{1}$$

$$E_C'=\frac{2(R_1+P_1+R_2)e_r}{2(R_1+P_1)+R_2}-\frac{2E_tR_C}{2(R_1+P_1)+R_2} \tag{2}$$

$$R_C'=\frac{R_2(R_1+P_1)}{2(R_1+P_1)+R_2} \tag{3}$$

$$E_{01}=\frac{k_1K_1\left(\frac{E_c'N_c}{R_c'}-\frac{E_fN_f}{R_f}+\frac{E_dN_d}{P_2+R_d}\right)-E_{01}}{k_1\left(\frac{N_c^2}{R_c'}+\frac{N_f^2}{R_f}+\frac{N_d^2}{P_2+R_d}\right)}$$

(CA *1*, *21*, *3-1*) (4)

here

$$E_d=\frac{\frac{R_5}{2}I_d}{\frac{R_5}{2}Cs+1} \tag{5}$$

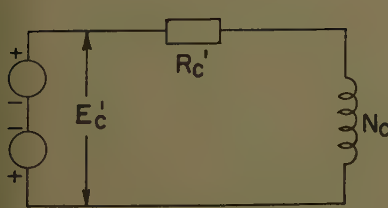

Fig. 2. Equivalent detector circuit

$$I_d=\frac{-N_dse_{01}}{K_1(P_2+R_d)}-\frac{E_d}{P_2+R_d} \tag{6}$$

Much of the present-day literature points to a more complex representation of magnetic amplifiers than the single time constant representation. However, it is felt that for present purposes, the single time constant will be adequate if care is exercised in determining the necessary constants.

Various methods are available for calculating the constant k_1, but past experience with many of these methods left a desire for a closer approximation of its value. This constant was determined by experimentally measuring the frequency response of a similar magnetic amplifier which had the same core design. The ceiling voltage, obtained by linearizing characteristics, is represented in the computer by the limiters connected to amplifier 1. The effect of any changes in bias voltage to the first stage was neglected. Fig. 3 is a computer diagram of the first stage.

The coupling network between the first and second stages is reproduced in Fig. 4. The equations listed subsequently are readily apparent by inspection.

$$I_L=\frac{E_{01}-E_1}{R+sL_1}\ \text{(CA \textit{31})} \tag{7}$$

$$I_1=\frac{E_1-E_2}{R_{12}}\ \text{(CA \textit{36})} \tag{8}$$

$$I_2=\frac{E_2-N_{C2}s\phi_{02}}{R_{13}+R_{C2}}=\frac{E_2-NC_2sE'_{02}}{\frac{K_2}{R_{12}+R_{C2}}}\ \text{(CA \textit{39})} \tag{9}$$

$$I_{C6}=I_L-I_1\ \text{(CA \textit{32})} \tag{10}$$

$$E_{C6}=\frac{I_{C6}}{sC_6}\ \text{(CA \textit{33})} \tag{11}$$

$$E_1=E_F+E_{C6}\ \text{(CA \textit{34})} \tag{12}$$

$$I_{C4}=I_1-I_2\ \text{(CA \textit{37})} \tag{13}$$

$$E_2=\frac{I_{C4}}{C_4s}\ \text{(CA \textit{38})} \tag{14}$$

$$I_6=\frac{E_{XF}-E_F}{P_6}\ \text{(CA \textit{48})} \tag{15}$$

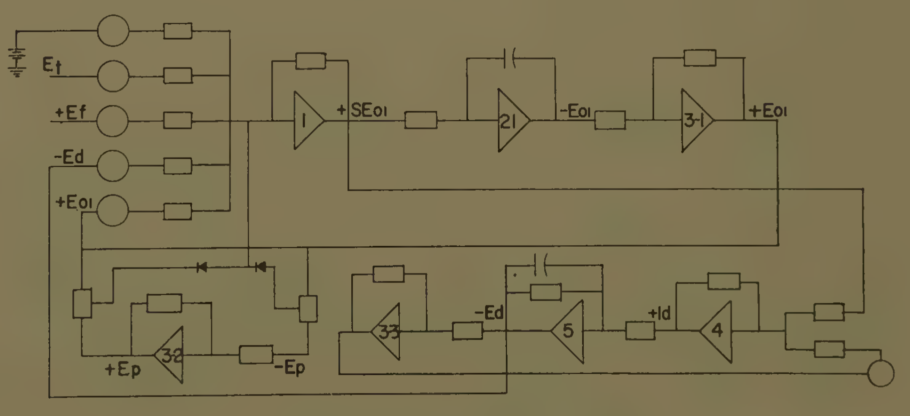

Fig. 3. First-stage computer diagram

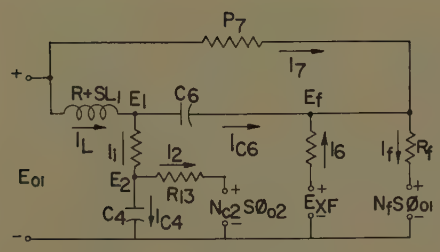

Fig. 4. Coupling network

$$I_7=\frac{E_{01}-E_t}{P_7}\quad(\text{CA } 40) \tag{16}$$

$$I_f=\frac{E_f-Nfs\phi_{01}}{R_f}\quad(\text{CA } 29) \tag{17}$$

$$I_f=I_6+I_7+I_{C6}\quad(\text{CA } 30) \tag{18}$$

Fig. 5 is the computer diagram of the coupling network.

The second-stage transfer characteristics, Fig. 6, are considered to have slopes independent of pilot exciter voltage, but to have ceiling voltages and zero control signal outputs proportional to pilot exciter voltage. This stage is represented by an "internal" voltage, E_{02}'', which is applied to the load in series with an internal resistance. It should be further recognized that the idealized curves can be arrived at by having an artificial voltage E_{02}', Fig. 7, which is zero for zero signal input and has the slope of the real curves but a ceiling voltage proportional to pilot exciter voltage. To this voltage is added a voltage proportional to pilot exciter voltage:

$$E_{02}''=E_{02}'+K_pE_p-A\quad(\text{CA } 26) \tag{19}$$

The equation for E_{02}' is of the form:

$$E_{02}'=\frac{k_2K_2\left[\frac{N_{C2}}{R_{C2}'}E_2-\frac{N_fE_{f2}}{R_{f2}}-\frac{N_BE_{B2}}{R_{B2}'}\right]}{1+sk_2\left[\frac{N_{C2}^2}{R_{C2}'}\ \frac{N_{f2}}{R_{f2}}\ \frac{N_B^2}{R_{B2}}\right]}\quad(\text{CA } 27,\ 6) \tag{20}$$

Fig. 8 is the computer diagram for the second stage.

Limiters are applied to amplifier 27 to limit E_{02}' to a ceiling value to correspond with Fig. 7. A voltage proportional to generator terminal voltage E_t, along with a bias voltage are introduced at amplifier 27 to account for the effect of changes in second-stage bias current.

A voltage proportional to the pilot exciter voltage E_p and a battery voltage are introduced at amplifier 26 along with E_{02}' to generator E_{02}''. A diode is connected to the output of amplifier 26 to limit the output to voltage of one polarity only.

When evaluating k_2 by the same method used to evaluate k_1, the experimental frequency response curve could not be closely approximated by single-time-constant curves. However, a close approximation was arrived at by introducing additional phase shift to account for a transportation lag inherent in this type of magnetic device.

Amplifier 25 and its associated circuit were added to introduce the transportation lag effect of the second stage.

The exciter voltage swings over a wide operating range on the application of load, and for this reason linearized equations for the exciter are not considered to be satisfactory. The exciter was therefore approximated as a round rotor machine with armature current assumed in phase with terminal voltage, and armature transients and resistance neglected. Since the exciter no-load saturation curve does not depart from the air-gap line over the range of operation required, no saturation was considered.

Fig. 9 is a vector diagram of the a-c exciter and the following equations may be written as a representation of the a-c exciter:

$$E_{02}''=i_f(R_{02}+R_f)+8N_{fe}s\phi\times10^{-5} \tag{21}$$

From Fig. 9

$$e_d=K_6i_f=\sqrt{\left(\frac{w_b}{w}\,et\right)^2+(i_tx_d)^2} \tag{22}$$

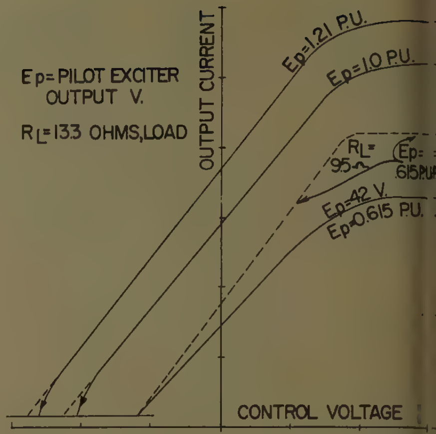

Fig. 6. Second-stage transfer characteristics

$$\frac{w_b}{w}e_t=\sqrt{e^{2\prime}-(i_ex_d')^2} \tag{23}$$

Therefore

$$se'=\frac{E_{02}''-i_f(R_{02}+R_f)}{k_8\times10^{-5}N_{fe}}\quad(\text{CA } 45) \tag{24}$$

The computer diagram for the a-c exciter is shown in Fig. 10.

The mathematical expressions for the synchronous generator were developed from the expressions presented in a previous paper.[1]

$$E_d'=\frac{E_{fd}-I_{fd}}{sT_{do}'} \tag{25}$$

$$E_q'=\frac{-I_{fq}}{sT_{qo}'} \tag{26}$$

$$E_{td}=\frac{w}{w_b}(E_d'-I_dX_d') \tag{27}$$

$$E_{tq}=\frac{w}{w_b}(E_q'-I_qX_q') \tag{28}$$

$$I_{fd}=E_d'+I_d(X_d-X_d') \tag{29}$$

$$I_{fq}=E_q'-I_q(X_q-X_q') \tag{30}$$

For a static load equal to R_L+jX_L the following can be written:

$$I_q-j^Id=\frac{(E_{td}-jE_{tq})}{R_L+jX_L} \tag{31}$$

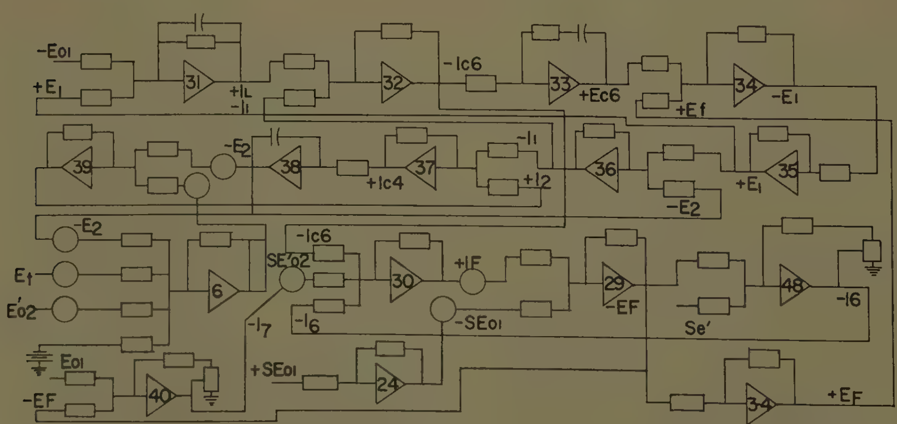

Fig. 5. Coupling network computer diagram

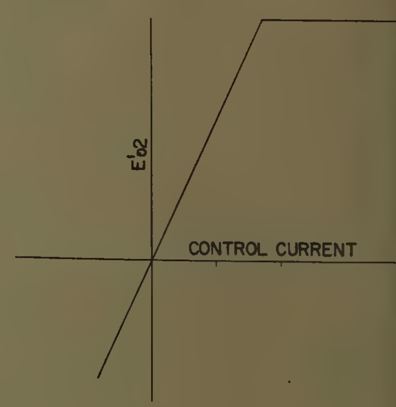

Fig. 7. Artificial voltage transfer characteristics

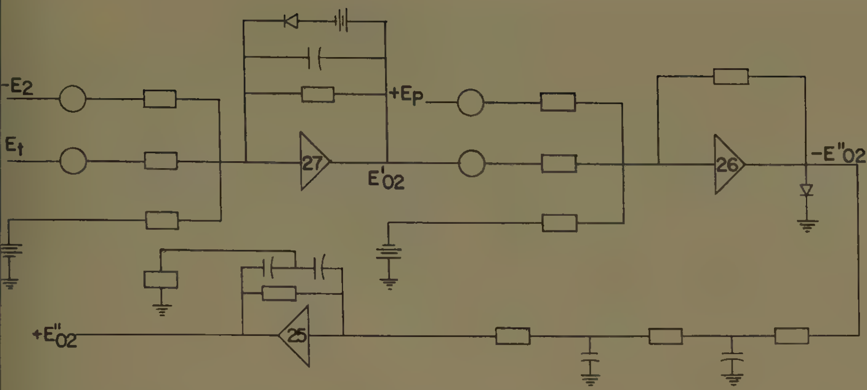

Fig. 8. Second-stage computer diagram

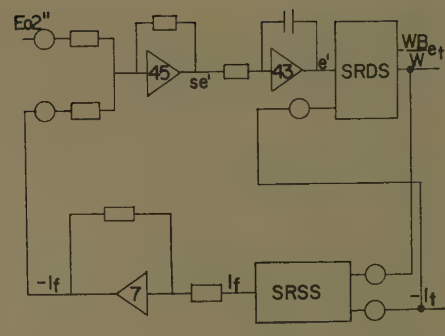

Fig. 10. A-c exciter computer representation

Substitution of equations 27 and 28 into [e]quation 31 and proper algebraic mani[pu]lation results in the following expres[si]ons:

$$= -C_1E_q' + C_2E_d' \quad (32)$$

$$= C_2E_q' + C_3E_d' \quad (33)$$

The constants C_1, C_2, and C_3 are defined [b]y

$$_1 = \frac{\frac{w}{w_b}\left(X_L + \frac{w}{w_b}X_d'\right)}{R_L^2 + \left(X_L + \frac{w}{w_b}X_d'\right)\left(X_L + \frac{w}{w_b}X_q'\right)} \quad (34)$$

$$_2 = \frac{\frac{w}{w_b}R_L}{R_L^2 + \left(X_L + \frac{w}{w_b}X_d'\right)\left(X_L + \frac{w}{w_b}X_q'\right)} \quad (35)$$

$$_3 = \frac{\frac{w}{w_b}\left(X_L + \frac{w}{w_b}X_q'\right)}{R_L^2 + \left(X_L + \frac{w}{w_b}X_d'\right)\left(X_L + \frac{w}{w_b}X_q'\right)} \quad (36)$$

Assuming I_{fq} and T_{q0}' are of a negligible [m]agnitude and saturation effects are [n]eglected, equation 29 may be rewritten [a]s

$$E_q' = I_q(X_q - X_q') \quad (37)$$

$$_q = \frac{C_2E_d'}{1 + C_1(X_q - X_q')} \quad (38)$$

$$I_d = C_3 + \frac{C_2^2(X_q - X_q')}{1 + C_1(X_q - X_q')}E_d' = C_dE_d' \quad (39)$$

$$I_{fd} = 1 + C_d(X_d - X_d')E_d' \quad (40)$$

$$E_{td} = (1 - C_dX_d')\frac{w}{w_b}E_d' \quad (41)$$

$$E_{tq} = \frac{C_2X_q\frac{w}{w_b}E_d'}{1 + C_1(X_q - X_q')} \quad (42)$$

$$E_t = \sqrt{E_{td}^2 + E_{tq}^2} \quad (43)$$

Substituting expression 41 and 42 into 43:

$$E_t = \frac{w}{w_b}C_tE_d' \text{ (CA } 12\text{)} \quad (44)$$

where:

$$C_t = \left[(1 - C_dX_d)^2 + \left(\frac{C_2X_q}{1 + C_1(X_q - X_q')}\right)^2\right]^{1/2} \quad (45)$$

Substituting equation 40 into 25 results in

$$E_d' = \frac{2.34e_t - C_d(X_d - X_d)E_d'}{1 + sT_{do}'} \text{ (CA } 41\text{)} \quad (46)$$

and from equation 39:

$$K_{ft}i_t = [1 + C_d(X_d - X_d')]E_d' \text{ (CA } 13, 47\text{)} \quad (47)$$

Since the generator operates in the saturated region, saturation was added to the analog of E_d'. Those terms of the equations which are functions of the load constants C_d, C_q, and C_t were segregated to permit switching from a no-load to a loaded condition. Potentiometers are provided to permit the exciter and generator voltages to be changed for various operating speeds, but transients in speed are not represented. Fig. 11 is the computer diagram of the generator.

The load characteristics of the pilot exciter do not vary appreciably from the no-load saturation curve. Therefore, the no-load saturation curve was analogued as the transfer characteristic of the pilot exciter for all load conditions.

Therefore

$$E_p = kpi_t \text{ (CA } 49\text{)} \quad (48)$$

Fig. 12 is the computer diagram of the pilot exciter. Saturation was added to amplifier 49 by use of nonlinear circuits.

Results

The analog representation of each of the main components was checked independently and, when combined into a closed-loop system, the system was unstable, oscillating at 16 cycles per second. Stable operation of the simulated system was obtained by removing the inductance

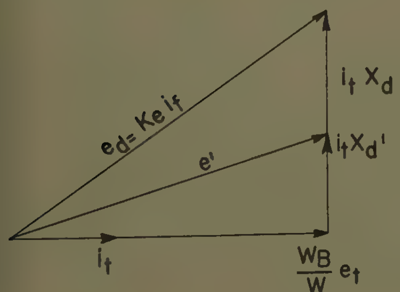

Fig. 9. A-c exciter vector diagram

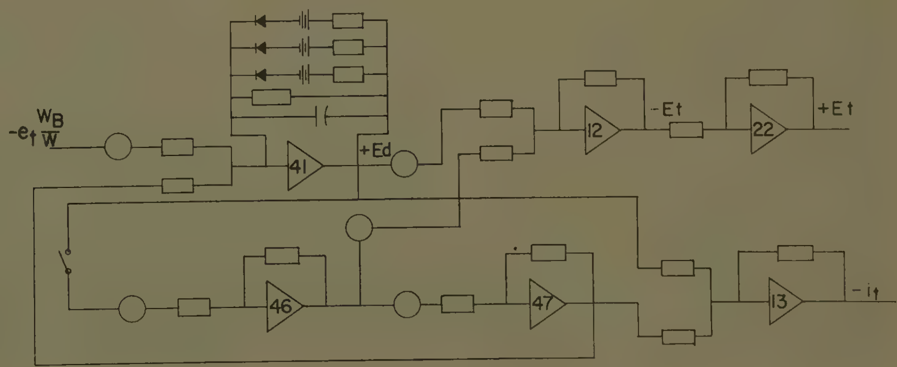

Fig. 11. A-c generator computer representation

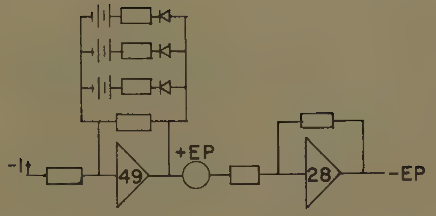

Fig. 12. Pilot exciter computer representation

L_1, Fig. 1, and inserting approximately 300 ohms in series with C_6.

Later, when the first breadboard regulator was connected with a machine into closed-loop operation, the system was unstable, oscillating at 14 cycles per second. Stability was achieved by making the changes indicated by the analog simulation.

The response of the simulated system to load switching indicated that the proposed system would meet the recovery time specification for the conditions investigated. Variations in recovery time for variations in parameter settings were rapidly recorded from the computer. These variations indicated that a maximum tolerance of ±20% in the setting of the stabilizing feedback resistance, P_6, was allowable to assure compliance to the recovery time specification for the wide temperature range of operation. Tests made later on the actual closed-loop system indicated approximately the same tolerance.

The simulated system indicated that a faster system could be obtained by increasing the number of exciter feedback turns. Therefore, the final design was altered to increase the number of turns in the exciter feedback circuit.

Conclusions

Judicious use of system analysis during the preliminary design and development of a-c electric power systems eliminates repeated engineering and design changes resulting from unsatisfactory control performance at later stages in the development. This preliminary step results in a more rapid and less costly development program.

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Development of a Flight Controller for the Delta Space Research Vehicle

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THE National Aeronautics and Space Administration requirements for the Delta Space Research Vehicle imposed conditions of accuracy, flexibility, and an extended period of operation never attained before by United States space vehicles. In order to meet these requirements a new flight controller had to be designed and manufactured. This new design included other salient features; such as: minimum weight and power requirements, a minimum number of relays, and a high degree of reliability. The flexibility in the design had to allow for interchangeability of components or subsystems with a minimum of time and effort due to changes in the flight sequence or trajectory that could occur shortly before the vehicle is launched.

The Delta Space Research Vehicle (Fig. 1) is a multistage vehicle designed to put various payloads in a variety of orbits. The first stage is a modified Thor missile with the guidance system and re-entry vehicle removed. The second stage has a modified and improved Aerojet-General liquid propulsion system with a guidance section and a spin table. A radio guidance system, which provides pitch and yaw steering correction commands to both the first- and second-stage flight controllers, is incorporated in the second stage. These steering correction commands are used to reduce the nominal programmed trajectory errors. The steering correction commands to the first-stage flight controller from the radio guidance system are provided after a programmed flight period of the first stage. After separation of the first and

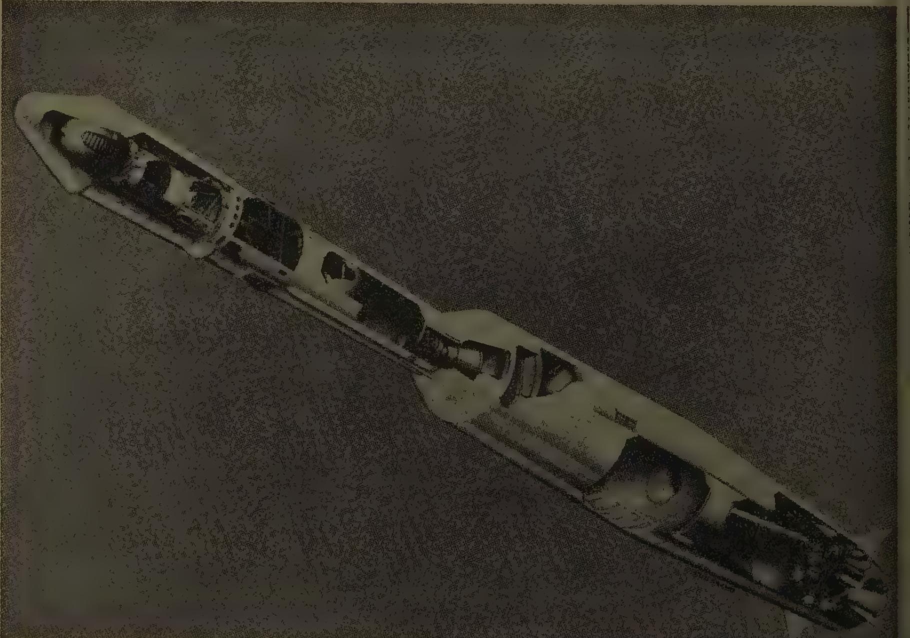

Fig. 1. Thor-Delta Space Research Vehicle

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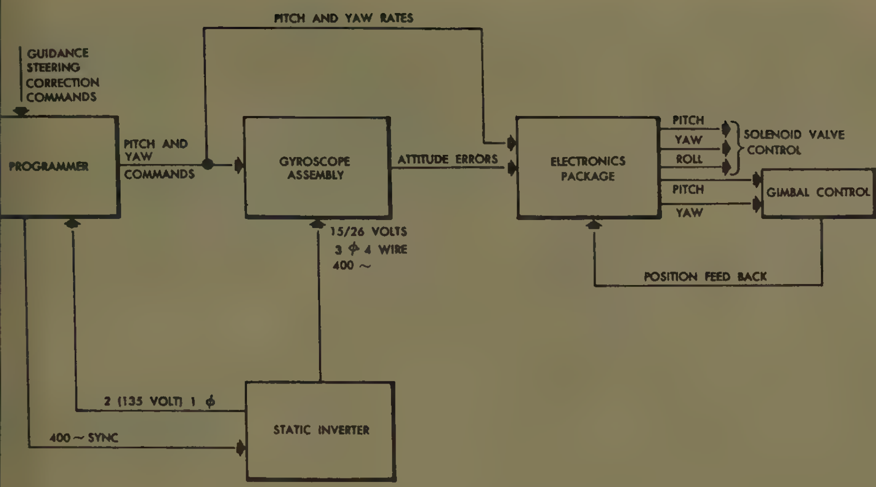

Fig. 2. Flight controller block diagram

cond stage, the guidance steering rrection commands are sent to the cond-stage flight controller until second-age engine cut-off. The radio guidance stem is then left on for a period of time r tracking purposes only. The third age is powered by an Allegheny Bal-stics Laboratory solid propellant motor ith payload attachments.

A unique feature of the Delta vehicle a coast-control phase of second-stage ght. This occurs after the radio guid-ice commanded cut-off of the second-age engine and allows the combined cond and third stage to coast to apogee efore third-stage spin up and injection the third stage and payload into orbit. uring this coast phase the vehicle is abilized to ±0.3 degree in the pitch ad yaw planes and to ±3.0 degrees the roll plane.

The Delta second-stage flight con-oller system consists of a programmer, gyroscope assembly, an electronics ackage, and a static inverter. The pro-rammer provides the time base for the peration of all the vehicle discrete commands with the exception of the radio guidance engine cut-off command. It also provides a time base to the static inverter for frequency control of the output voltage. The torquing current commands from the programmer to the gyroscope assembly are precision controlled in both magnitude and time. The gyroscope assembly provides the 3-axis attitude reference for attitude and stability control of the vehicle. The pitch and yaw gyros are capable of being torqued by the programmer commands for reorientation of the vehicle's attitude. The electronics package provides gimbal control of the engine actuators during the powered phase of flight and provides valve control of the helium reaction jets for attitude control during the coast phase. The static inverter supplies 3-phase voltage to the gyroscope assembly for spin motor power and also for the signal excitation and demodulator reference voltages. Two separate windings of 135 volts each are used to supply single-phase power to the programmer where the voltages are rectified and filtered and used as the power supply for the torquer currents. All four units utilize d-c power for operation and solid-state devices are used throughout. A block diagram of the second-stage flight controller is shown in Fig. 2.

The Miniature Integrating Gyroscopes (MIGS) are used as the attitude references to accomplish the required vehicle stabilization during the coast phase. The a-c attitude error output signals from the MIGS are amplified, demodulated, and fed into switching amplifiers that control the expenditure of helium gas through reaction jets for maintaining the vehicles' attitude within these limits. The switching amplifiers have a preset triggering level which is equivalent to a given gyro gimbal angular displacement. The second stage is also capable of being programmed in pitch and/or yaw during the powered and coast phases of its flight from preset torquing commands to the gyros from the programmer.

The following is a report of the development of the individual units showing some of the major problems encountered and how these problems were solved.

Discussion

Programmer

The length of time for a controlled flight of the second stage dictated the requirements for a programmer that would be capable of at least 1,000 seconds of operation. Electromechanical programmers, with this capability, were either too large, too complex, or lacked sufficient accuracy to be utilized on this

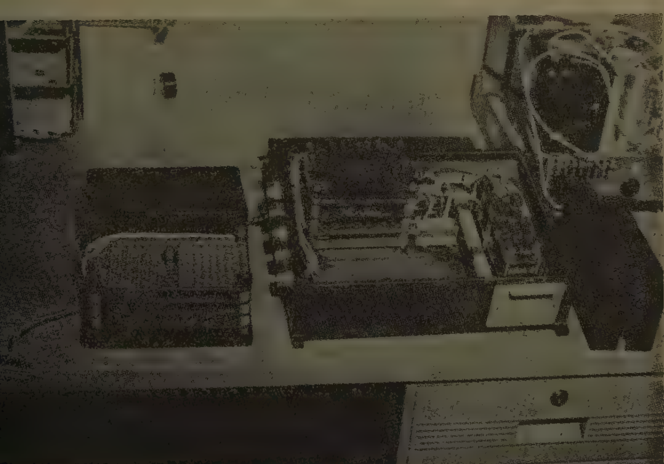

Fig. 3. Programmer design proving model

Fig. 4. Gyroscope heater block temperature study

system. The flexibility of the programmer and the accuracy required (±0.01%) for sequencing of events led to a solid-state programmer. The programmer incorporates a crystal-controlled time base which provides the accuracy required. The frequency of the crystal is 51.2 kc which is counted down to 2 cps (cycles per second) through a series of flip-flop circuits. This countdown to 2 cps provides the 1/2-second time increment capability required for sequencing of events. An accumulator provides the circuitry for counting the time. A diode matrix with screw contact switches provides the selection of times for the sequencing of events on 16 different channels.

The tolerance of ±0.05% for the gyro torquer currents from the programmer dictated that the resistors used for obtaining the currents could not vary because of changes in temperature. A temperature control oven was installed within the programmer for maintaining a constant temperature for the resistors.

The selection of resistors values, through various combinations, to obtain the correct torquer currents over a wide range of torque rates for the pitch channel was based on various series-parallel combinations of ten resistors. Fixed resistors were chosen because of the range (6–60 kilohms) and resolution (±0.05%) required. A potentiometer is not capable of meeting these requirements. The solution of basic equations by an International Business Machines Corporation computer gave over 1,000 combinations of three resistors in series-parallel combinations for the ten resistors. A minimum of 300 combinations were required to cover the range of resistance (60 kilohms) but the grouping of resistance values in the results of the computations by the computer indicated more combinations would be needed.

The final current adjustment is provided by a variable resistor inserted in parallel with a fixed resistor which is in series with these combinations. This gives the linearity required over the torquer rate range. The torquer current circuits are so designed that if the least reliable component (variable resistor) should fail the total resistance in the circuit will not be changed by more than 15% in the worst case.

For a flight, the selection of various combinations of resistors for the proper range and polarity required for the torquer currents is accomplished by connecting short-circuiting plugs to connectors installed on the programmer. The terminal connections for the 10 resistors in the pitch channel and the 6 resistors in the yaw channel are brought out to these connectors. A combination of 5 jumper wires for each series-parallel combination of 3 resistors is required on the short-circuiting plug for a pitch or yaw program. This method of not requiring removal and/or installation of resistors for particular pitch or yaw programs increases the reliability of the system.

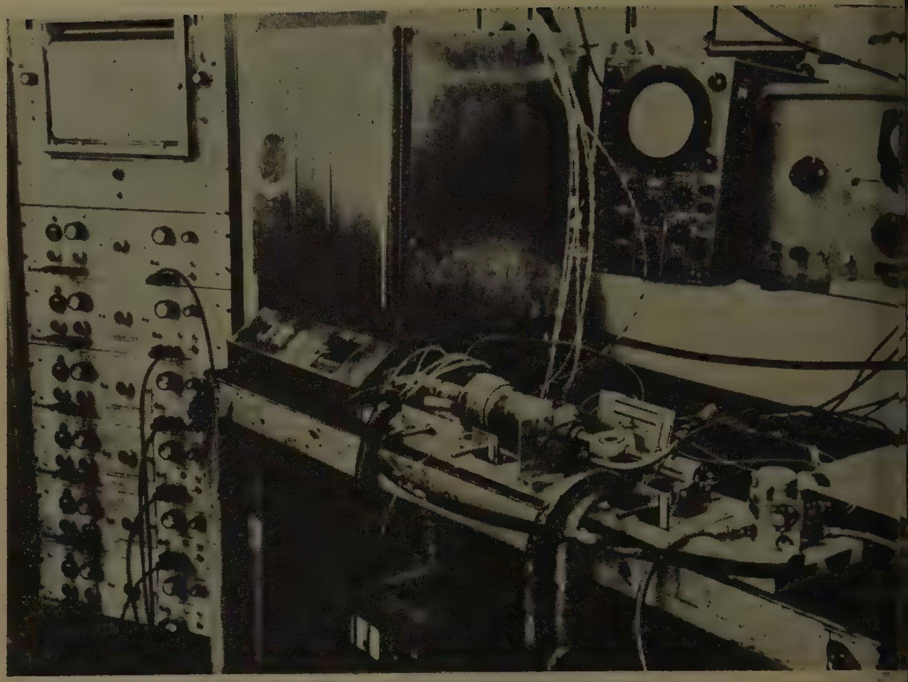

Fig. 5. Servoamplifier environmental test

In the initial design of the programmer an operating time of 1,023 seconds appeared to be sufficient. A further investigation into the time requirements, for the various trajectories which would be encountered, disclosed that a possible addition of a few hundred seconds of operation might be required. It was therefore decided to increase the operating time capability of the programmer. This was easily accomplished since only one additional flip-flop circuit was required to double the existing time capability. The addition of a diode and a screw contact switch in the diode matrix for each function was also required to allow for the added time capability. The programmer is now capable of operating for 2,047.5 seconds, which should meet any requirements for flight for the Delta Space Vehicle.

The time of engine cut-off cannot be predetermined to any degree of accuracy since it is dependent upon the attainment of the velocity vector required for a particular mission. However, the time between achieving the predetermined velocity and the third-stage ignition must be precisely controlled. Therefore, the programmer must have the ability to stop during second-stage burning and be restarted in flight when the predetermined velocity is achieved. In order to accomplish these objectives, a stop-restart capability was incorporated in the programmer as part of the system. The stop-restart circuit is so designed that the programmer will stop counting time when it is commanded to "stop" and when a "restart" command is given it will commence counting from the point at which it stopped. The elapsed time between "stop" and "restart" is not seen by the programmer. With this capability designed into it, the programmer can be commanded to "stop" just prior to the radio guidance command for engine cut-off and then restarted at cut-off. This signals the initiation of the coast phase. The coast phase time varies from mission to mission depending upon trajectory requirements. Fig. 3 is the first complete programmer which was used as the design proving model.

Gyroscope Assembly

The requirement for maintaining attitude stabilization within a quarter of a degree for the pitch and yaw planes for an extended period of time (15 minutes) dictated the use of the MIG gyro. The low drift characteristics (0.1 degree per hour) of the MIG gyro insures that the vehicle will be capable of being reoriented for the final velocity injection of the third stage and payload without introducing an appreciable error.

There are no rate gyros incorporated

the flight controller. Rate signals are ierated electrically by differentiating e position error signals from the gyros. ice the pitch and yaw gyros are being mmanded for a change in vehicle sition the differentiation also produces command rate error signal. This mponent of rate error is undesired in e circuit and is eliminated by subtract- g the gyro command, which is a rate gnal, through an external loop. As the ll gyro is not commanded, no command te error component exists in the position ror signal. Therefore, no external sub- action is required in the roll channel.

Incorporated in the gyroscope assembly ith the three gyros are a-c amplifier- modulators, caging amplifiers, and a mperature control amplifier. The out- it of the a-c amplifier-demodulator for ich channel is a d-c signal of proper plarity which is proportional to the ttitude error of the vehicle. Because of ie low drift rates, and other tightly ontrolled properties of the gyroscope ssembly and its subassemblies, inter- hangeability of gyros, subassemblies, r an entire unit is attained for integration ito the system.

A method for decreasing the number of omponents required and also for dis- ributing the power requirements of the -phase a-c voltage is incorporated in the ignal excitation-demodulator reference ircuit. Normally a design will use the ame phase for the signal excitation nd the demodulator reference voltages. 'o compensate for the phase shift be- ween the signal excitation and the iicrosyn pick-off a phase shifting network s required in order to have proper phase elationship when the signal reaches the lemodulator. An alternative method, which is used in the gyroscope assembly, uses one phase of the 3-phase voltage for the signal excitation voltage and the line-to-line voltage of the other two phases as the demodulator reference voltage. The phase shift between the signal excitation and the microsyn pick-off is approximately 90 degrees and the resultant signal is either in phase or out of phase (for proper polarity) with the demodulator reference.

The gyroscope assembly incorporates features that are both unique to the system and are improvements over known previous design of gyroscope assemblies. Some of the significant features are as follows.

A spin-motor current-monitoring device is incorporated for each gyro which enables an operator to know that the gyros' spin motors start when they are turned on. The gyros are mounted in a heater block for temperature control. The heater block acts as a heat sink, thereby decreasing the "on" time for the heaters. A solid-state switching device is used in a single temperature control amplifier with the heater block.

The MIG gyroscopes which are used in the gyroscope assembly incorporate the following features: a permanent magnet d-c torquer for accuracy and a drift trim which is separate from the command torquer. The signal generator excitation windings are in series which decreases the power requirements over previous parallel designs that would have the same input voltage.

It was first believed that the a-c amplifier-demodulator could be used in a dual purpose, i.e., for both the output and caging circuits. This amplifier was found to have too low a gain for the caging requirements. The amplifier that was designed is a differential d-c amplifier, connected across the torquer winding to provide the caging signal.

The possibility of damaging the gyros before the temperature of viscous fluid within the gyros is raised high enough to allow internal movement existed without a proper turn on sequence. A shear or tension force would be imposed upon the flex leads within the gyros because of the semisolid state of the viscous fluid if any physical movements occurred within the gyros. In order to obtain information regarding the sequence of applying power to both the heaters and the unit, temperature studies were conducted on the heater block with simulated gyros in the block. Through the results of these tests, a proper sequence for applying power and lock-out times were established that prevents damage to the gyros. Fig. 4 shows the bench test setup used for conducting the temperature studies on the gyro heater block.

Electronics Package

The electronics package is used to shape the pitch and yaw error signals from the gyroscope assembly for system stability and amplifies the signals for proper engine gimbaling during the powered phase of flight. An external signal commands the change of control from the servo amplifiers and the valve actuators to switching amplifiers and solenoid valves for control of the expenditure of helium for the coast phase. The roll error is shaped and fed into switching amplifiers for both the powered and coast phases of flight.

The electronics package design is based on solid-state devices in order to minimize weight and power consumption. The major units in the electronics package are servo amplifiers, shaping network, switching amplifiers, and a d-c–d-c converter and regulator. The use of silicon tran-

Fig. 6 (left). D-c–d-c converter regulator

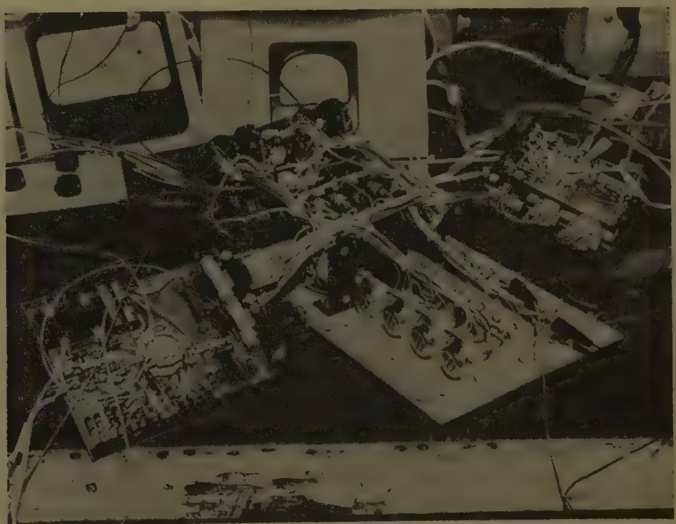

Fig. 7 (right). Static inverter design proving model

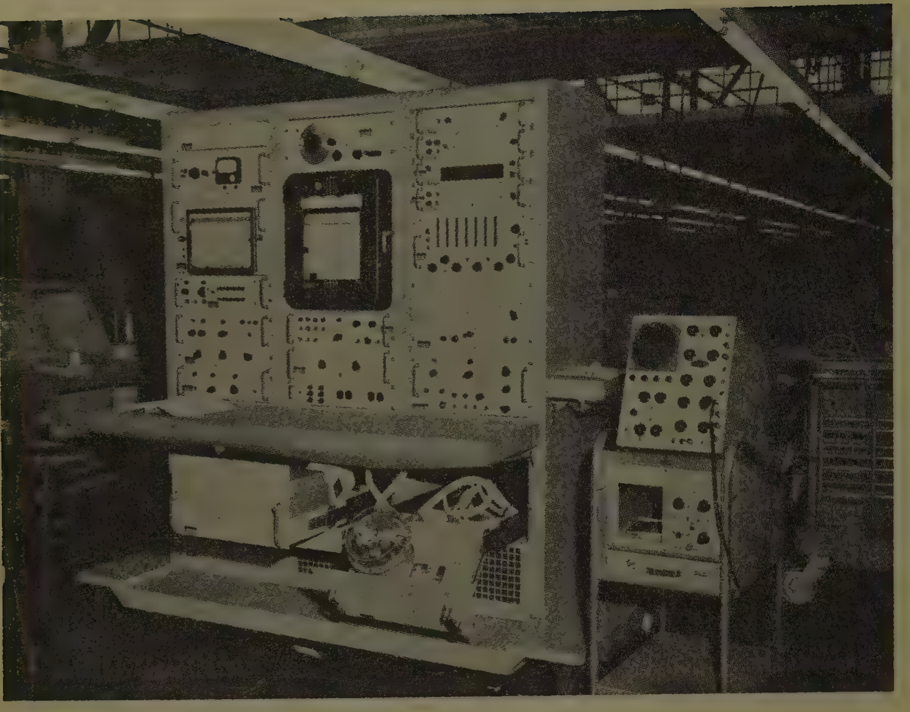

Fig. 8. Flight controller units under test

sistors in this unit insures meeting the anticipated temperature environments of the Delta Space Vehicle with ease.

The flexibility in interchangeability of subassemblies is exemplified in the electronics package. All of the units (servo amplifiers, shaping networks, etc.) are installed on separate printed circuit boards which may be inserted into or removed from the electronics package with ease. A mother board is used for the interconnection between boards and also provides for the signal inputs and outputs. The possibility of inserting the wrong board into a particular position is eliminated by the use of a simple keying method. If a board was inserted in the wrong position by force the pin arrangements are such that no electrical damage can be done to the board or the electronics package.

In order to compensate for any variations in the electromechanical valves an adjustment within the servoamplifier was required. Associated with this another adjustment in the servo amplifier was required for gain control. The two adjustments appeared to be incompatible in the amplifier as the adjustment of one would cause limitations on the capability of the other. The interaction between the two adjustments was worked out with complete compatibility between them. The servoamplifier was breadboarded and environmentally tested with a valve under actual load as shown in Fig. 5.

At the start of the program a commercial d-c–d-c converter and a regulator were considered for supplying the ±20 volts required in the operation of the electronics package. Upon receipt of one set of these units from the vendor for evaluation and testing, it was discovered that the packaging, size, and performance did not meet the Delta requirements.

An investigation was made into the possibility of designing and building a d-c–d-c converter and regulator that would be compatible with the system and packaging design criteria for the electronics package. A circuit was designed and tested, and the results exceeded the requirements for the system operation.

One other significant factor in changing to the new design is that the commerical unit utilized germanium transistors, and the time required to develop a commercial unit utilizing silicon transistors was excessive. From a temperature standpoint, the Delta environmental conditions require the germanium devices be operated at their upper limit, as compared with silicon devices operating at a temperature well within their limits. Fig. 6 is the first production d-c–d-c converter and regulator.

The external signal from the gyro torquer circuit which gives commanded rate error signals was fed into a d-c amplifier in the electronics package. These rate error signals are used to cancel out the commanded rate error signals that appear in the attitude error circuit. The high input impedance required to the d-c amplifier and the inherent property of the amplifier to drift when the input is an open circuit required modifications to this circuit. To eliminate any extraneous error commands to the engines because of the drift of the d-c amplifier, the output is left as an open circuit until the gyros are uncaged. Further investigation into this circuit disclosed that the d-c amplifier could be removed and a divider network could replace it.

The required drop-out time (60 milliseconds maximum) of the solenoid valve and the allowable hysteresis (±1.0%) of the switching amplifier presented a challenge in the design of the switching amplifier. At the outset five different designs were tested and evaluated. Further investigation into the design and test results of these five amplifiers eliminated three. The remaining switching amplifiers differed basically in the approach to the helium solenoid valve switching problem.

One design was a straightforward amplifier with the output power transistors in either the "on" or "off" condition, and the other was a regenerative switch. Tests were conducted with both amplifiers operating with a system under pressure and the results showed both amplifiers performed the same. The two designs incorporated a spike suppression diode and resistor circuit across the solenoid valve. The evaluation of the circuit with and without the diode and resistor installed disclosed considerable difference in drop-out time of the solenoid valve. Without the diode and resistor installed the drop-out time was well within the prescribed limit. A further investigation into the possibility of punch-through occurring in the switching transistors by the voltage spike generated by the inductive reactance of the solenoid pointed out that the diode and resistor could be removed. The switching amplifiers were operated continuosly for more 12,000 times without a diode and resistor installed with no depreciation in the transistor characteristics. The regenerative switch was finally selected based on it being the more reliable circuit of the two.

Static Inverter

Although Douglas Aircraft has had remarkable success in the use of unenclosed rotary inverters for space vehicles the major reasons for utilizing a static inverter in the design of the flight controller were requirements for precision frequency control, continuous duty cycle, and minimum weight. Precision frequency control is required because of the direct effect that it has on the transfer function of the gyroscopes. Three-phase

wer is supplied to the spin motors which synchronous in operation. A change frequency will change the synchronous ed and thus change the transfer funcn of the gyroscopes. The continuous ty cycle requirement for the a-c power urce was dictated by the time duration flight. Rotary devices with the same ntinuous duty rating are heavier and l to meet the frequency control requirents.

The method of cooling the static inrter was of prime concern because of e weight penalty that could be encounted. At first convection cooling was conlered, but this proved to be too heavy d occupied too much space. A thermal alysis was performed for conduction oling that would utilize a slightly larger d thicker base plate. By increasing e area of the base plate 13 square ches and increasing the thickness to 8 inch, conduction cooling of the static verter could be achieved. This reduced e weight of the unit by approximately %. Fig. 7 is the production proving odel of the static inverter.

System Testing

In order to check out the flight controller as a system and also as individual units a special test set was designed and fabricated. This design had to take into account the accuracies required of the flight controller and the tight tolerances on the readings to be taken. Considerable care was taken in the design of the torquer circuit where an error of 0.25 microampere would be greater than the allowable tolerance on the torquer current. The use of a digital voltmeter for voltage readings helped simplify the design of the tester and at the same time helped increase the accuracy in measuring a voltage. This also reduces the operator setup time and chances for operator error. Fig. 8 shows the first flight controller components installed in the test drawers of the test set.

The first unit of each component of the flight controller was functionally checked and a first article test was performed on the system to verify the design intent of the system. The compatibility of all four units operating as a flight controller system was proved with only minor changes required to the units. The d-c–d-c converter in the electronic package was found to be generating spikes of 60 volts peak-to-peak at a frequency of 2 kc. By installing a filter capacitor across the 28-volt input line this spike was reduced to less than 0.5 volt peak-to-peak. There is no detrimental effect upon the system with the spike limited to less than 0.5 volts peak-to-peak.

Conclusions

The design, development, and fabrication of the four components that make up the flight controller progressed smoothly and rapidly. The major problems that were involved regarding interface problems between units and as a system were solved with minor changes to the equipment and minimum time involved. The first article tests that were performed on the system proved that the basic design of the flight controller more than meets all expectations.

A Controlled Rectifier Regulator for Aircraft D-C Generators in 120 C Applications

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TO SELF-EXCITE and regulate a d-c generator, a device is needed which can ary the current supplied to the shunt field the machine so that regulated voltage esults over a wide range of speed and ad. In addition, it must provide some eans of load sharing when one generator nd its regulator are paralleled with one or ore identical systems. In 71 C (degrees entigrade) ambient this can be accomlished by a carbon-pile regulator or by a witching type of regulator using geranium transistors.[1]

For 120 C ambient the need existed for a reliable and accurate static regulator which would exceed the performance of existing 71 C regulators. The development of a regulator to do this was the design goal.

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Selection of Power Element

The first problem was to find a suitable regulating element, one which could carry up to an 8-ampere average current at temperatures in excess of 120 C. Only semiconductors were considered, as the device was required to be static, and only silicon was considered because of the temperature range. This narrowed the choice to either silicon transistors or silicon-controlled rectifiers (SCR). As temperature was indeed a problem, thought was first given to the most efficient use of any semiconductor selected. To minimize power dissipated in the regulating element itself, the switching rather than the linear or proportional mode of operation was selected. In the switching mode, the device quickly passes through the high-dissipation zone where the product of current and voltage (hence power) is quite large. This greatly reduces the self-heating for a given device; a perfect switch with zero switching time would dissipate no power.

Silicon Transistors

A survey of silicon transistors available in early 1959 showed none capable of carrying the 8 amperes, although several 5-ampere models were available. The devices could have been paralleled to handle the field current. However, this would have required the use of transistor drive circuitry, which would have involved additional losses. Their low usable current gain and high saturation resistance also causes inefficient operation, as considerable power is needed to turn and hold the devices ON, and even in the ON state considerable voltage drop and therefore power loss occurs.

Silicon-Controlled Rectifiers

The SCR in its ON state has a forward drop of about one volt, relatively independent of current through it. The forward drop tends to decrease slightly at

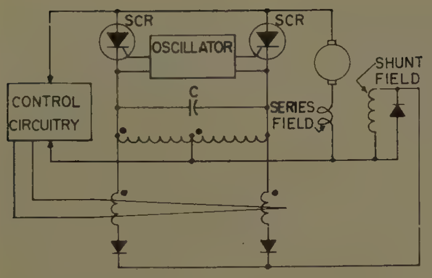
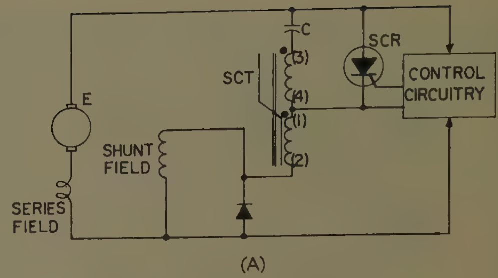


Fig. 1. Parallel-inverter–magnetic-amplifier circuit



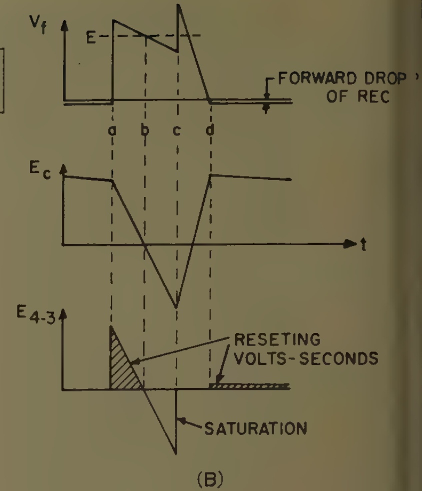


Fig. 3. Magnetic-controlled rectifier
A—Circuit B—Waveforms

higher temperatures. This means that less than one watt per ampere of field current is dissipated in the device. The gating power required by the SCR is negligible as it must be pulsed with only 3 volts, 80 milliamperes, and it will stay on until it is turned off. Special turnoff circuitry must be used to reverse the voltage across the SCR in order to restore its blocking properties at the end of each cycle. A single rectifier is capable of handling the full field current even at the elevated temperature.

Choice of Power Element

The over-all power gain and efficiency of the SCR are much better than those of the silicon transistor. This and the fact that only a single rectifier is needed to handle the maximum field current made it the overwhelming choice. The only disadvantage of the SCR is its thyratron-type turnoff characteristic.

Selection of Power Circuitry

A circuit was needed which could operate from a transiently variable direct-voltage power source and still achieve reliable commutation or turn-off of the SCR. Ideally, it would have minimum complexity, utilizing only one SCR. It would be capable of supplying any value of field current from less than 1 ampere to more than 8 amperes. Several circuits were considered for this application.

Parallel-Inverter–Magnetic-Amplifier Circuit

It was thought that perhaps a parallel inverter[2] could be used to supply a square-wave alternating voltage and its output controlled and rectified by a magnetic amplifier, and then fed to the shunt field. This approach requires two controlled rectifiers and in addition all the power must be handled not only by the rectifiers but by the magnetic amplifier as well; see Fig. 1. For these reasons the approach was quite heavy. Also, no ready means of turning off the inverter during low-voltage transients was seen. This feature would be necessary to prevent commutation failure of the inverter during momentary overloads.

Series Inverter

The series inverter shown in Fig. 2 looked quite promising as it used only one controlled rectifier, and it could easily be turned off during low-voltage transients. Referring to Fig. 2, the current operation is as follows:

When the SCR is gated, current flows through *L*1, charging the capacitor *C*. The capacitor is charged to a voltage higher than *E* because of the resonant overcharge supplied from the stored energy of inductor *L*1. As the current in *L*1 tried to reverse, the *SCR* blocks, and the energy stored in the capacitor *C* is supplied to the shunt field. This circuit was investigated in breadboard tests, and (with one notable exception) was found to operate satisfactorily with reasonably sized components. The maximum average field voltage obtainable was only 75% of that required. As no means for increasing the field voltage was found, the circuit was not considered further.

Magnetic-Controlled-Rectifier Switching Circuit

A third power circuit was investigated. It was the magnetic-controlled-rectifier switching circuit invented by R. E. Morgan of the General Electric Company's general engineering laboratory.[2] Note from Fig. 3(A) that this power circuit consists of only three components, the controlled rectifier SCR, the saturable current transformer (SCT), and the turnoff capacitor *C*. The circuit operation is as follows:

The capacitor *C* is charged to source voltage *E*, as shown. When the SCR is gated, the charged capacitor is switched across the unsaturated SCT so that terminal 4 is plus with respect to terminal 3, and therefore terminal 2 is plus with respect to terminal 1. This voltage on the primary (1-2) then adds to source voltage as can be seen in Fig. 3(B) on the field voltage V_f. The capacitor will supply energy to the field until time *b*, when its voltage goes through zero. At this point there is no voltage across the SCT windings and $V_f \equiv E$. The SCR current has been dividing in the SCT according to its turns ratio, the majority flowing into the field.

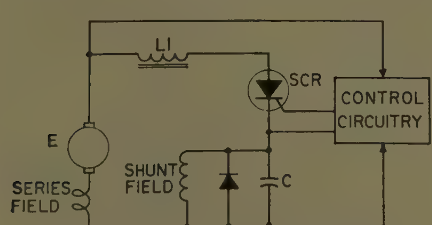


Fig. 2. Series inverter

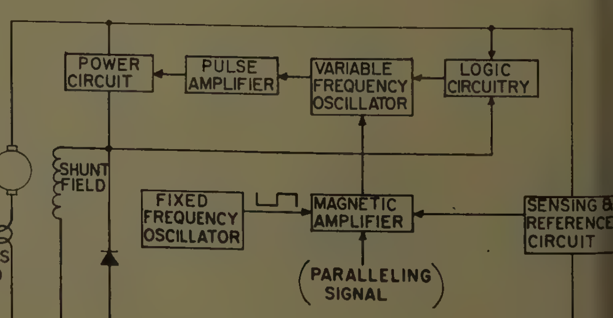


Fig. 4. Block diagram of circuit approach

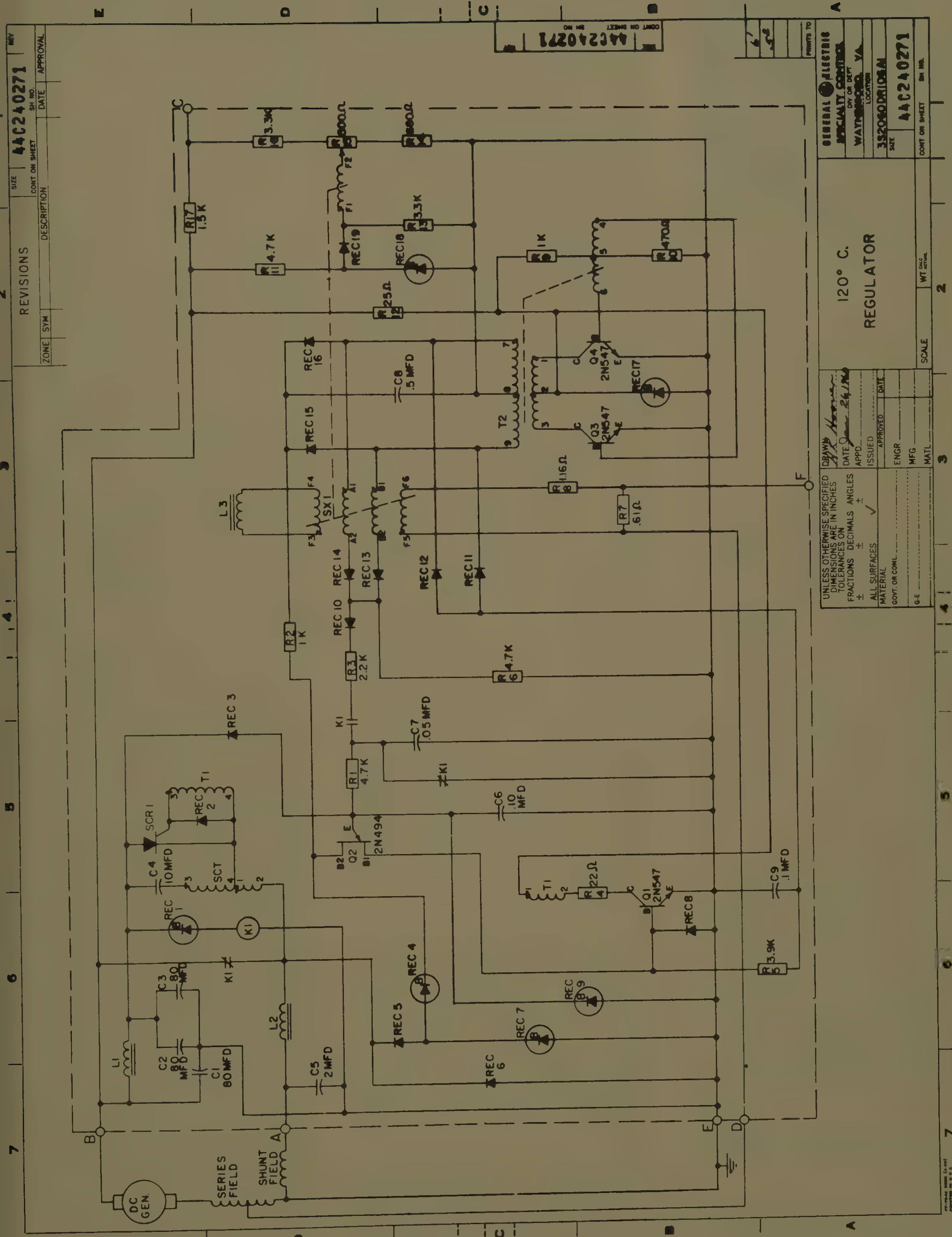


Fig. 5. Detailed circuit diagram

After time b, current continues flowing into C, charging it in the reverse direction. The voltage across the transformer windings (see plot of secondary 4-3 voltage) has reversed. After sufficient volt-seconds (time c), the SCT will saturate sharply, switching the charged capacitor across the SCR with the polarity to turn it off. After commutation, current will continue to flow through C recharging it to E. As the shunt field is highly inductive, the charging occurs in a constant current or linear manner. C will tend to overcharge slightly, owing to the after-saturation inductance of the SCT. When the charge current tries to reverse, the SCT will come out of saturation and the small overvoltage on C will slowly reset the SCT core. Note on the plot of secondary voltage that any area above zero constitutes resetting volt-seconds. If the gate is again triggered before slow reset has been completed, additional resetting will be supplied by C. There, volt-seconds occur in the next cycle during the interval a to b.

At this point, it is important to note that in some manner, sufficient reset must be obtained for each cycle. If capacitor C is not allowed to recharge prior to gating, the SCT may saturate before enough ampere-seconds have been stored. These ampere-seconds are needed to detour the load current around the SCR for at least 20 microseconds during turnoff. Failure could occur on the first cycle of operation as well as at any time "head-to-toe" pulses are being called for. It was also found that an input E of at least 10 volts is needed for reliable commutation at any gating frequency. These limitations necessitate the use of logic circuitry to be discussed later.

This circuit supplies a relatively constant pulse to the field each time that the SCR is gated. The charge time from c to d varies inversely with the field current level, but causes less than a 2-to-1 variation in over-all pulse width.

The circuit was tested and found to supply more than enough average field voltage. Its component simplicity and its efficiency also made it attractive. For these reasons it was selected for use in this application.

Circuit Approach on Block Diagram Level

Once the basic power circuit was selected, an over-all circuit approach had to be developed. The over-all circuit must satisfy the gating requirements of the power circuit, supplying pulses at varying frequency, in order to regulate the output voltage. To obtain equal sharing of the load by paralleled generators, it was also necessary to have control superimposed on the voltage-regulating system, which would raise or lower the field excitation of each generator to obtain load sharing. In conventional systems, this is accomplished by maintaining equal voltage drops across low resistances in series with the armatures of each machine. These resistances could be separate shunts or portions of the generator series fields. The low-voltage low-impedance signals thus obtained are normally applied to the equalizing windings of carbon-pile regulators. These signals are more easily used as current inputs such as are applied to the control coils of magnetic amplifiers. The use of transistors in this low-level circuitry tends to be complex, and subject to thermal drift; it therefore was not used. This dictated the use of a small magnetic amplifier, which in turn requires an alternating voltage source.

Fig. 4 is a block diagram of the circuit approach selected. The over-all circuit operation is as follows: Assume that the output voltage is slightly high. The sensing circuit would compare the output voltage with a Zener diode reference and feed an error signal to the magnetic amplifier, reducing its output voltage. This reduced output voltage would slow up the variable frequency oscillator, which would then supply a lower frequency pulse train through the pulse amplifier to the power circuit. This in turn would reduce the shunt field current and therefore the output voltage. In a similar manner, when a generator is carrying more than its share of the load current, a signal is fed to the magnetic amplifier reducing its excitation to equalize the division of load.

Detailed Circuit Operation

To examine the circuit operation, refer to the circuit diagrams shown in Fig. 5. Terminal C of the regulator is connected directly to the point of regulation, providing remote sensing independent of drop in the generator feeder. *R17* is included to supply a secondary sensing path to avoid ceiling excitation in case the remote-sensing lead should be accidentally opened.

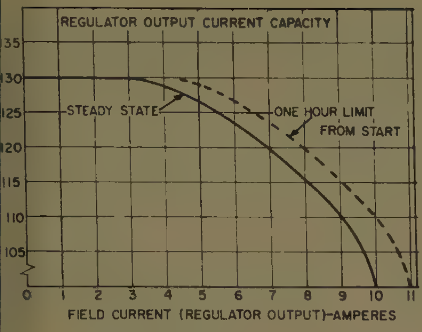
The resistive network *R14*, *R15*, and *R16* forms a divider supplying the proper voltage level for comparison with the Zener voltage reference, *REC18*. The control coil of the magnetic amplifier is placed between the reference and the divider, thereby receiving the error signal. A current limit circuit formed by *REC19* and *R13* prevents the magnetic amplifier from being overdriven into "backside operation" by overvoltage transients. A center-tap-connected magnetically coupled multivibrator is used to supply a square-wave output to the magnetic amplifier. The d-c input voltage to the multivibrator is regulated by a 50-watt Zener diode, *REC17*.

The volt-second characteristic of the core and the input voltage were selected so as to give a constant-frequency 4-kc constant-amplitude square wave on each of the *T2* windings. The voltage on *T2* winding 7-9 is rectified by *REC15* and *16* and supplied to base 2 of the unijunction transistor. In addition it is rectified by *REC11* and *12* to supply negative bias to transistor *Q1*. Third, *T2* supplies the alternating current for the magnetic amplifier *SX1*. As the magnetic amplifier has a capacitive load, an exciting current path is supplied through *R6* to ground. *REC10* decouples from the capacitive load whenever necessary to maintain the exciting current path.

The output of the magnetic amplifier is then fed into a relaxation oscillator formed by the unijunction transistor *Q2*, the capacitor *C6*, and the charging resistor *R1*. The filter *R3-C7* was inserted to supply additional smoothing, but it is not essential for circuit operation. The characteristic of the unijunction transistor is such that with a constant voltage imposed from base 2 to base 1, very little emitter current will flow until the emitter is raised above base 1 potential to approximately 7/10 of the base-to-base voltage. If this point is exceeded, the transistor will "fire," quickly discharging *C6* into the base of transistor *Q1*. Transistor *Q1* is a pulse amplifier, its output being transformer-coupled through *T1* into the gate of the controlled rectifier. The frequency of the relaxation oscillation is determined by the charging time constant *R1-C6* and by the magnetic-amplifier voltage fed into the circuit.

Assume for the moment that the output voltage is high. Current would be flowing from *F2* to *F1* in the magnetic amplifier in such a direction as to turn it off. In this case, the output voltage to the magnetic amplifier would be reduced, and and the relaxation oscillator would run at a lower frequency. This would decrease the average ON time of the controlled rectifier, also reducing the average shunt field current. This would in turn reduce the output voltage until the proper level was again reached. Adjustment of this level can be made by varying the divider resistor *R15*.

The regenerative build-up of generator



g. 6. Regulator output current capacity

oltage from its residal (permanent mag-etism) value to the 28-volt regulated evel is accomplished by relay *K1*, which onnects the field across the armature roviding a minimum resistance path. t was felt that reliable build-up could not e secured with silicon semiconductors ecause of their high forward-voltage lrop. During build-up, when about 15 olts have been reached, current will flow hrough the Zener diode *REC1*, and hrough the operating coil of *K1* relay to ick it up. Both sets of *K1* contacts vill switch, transferring the shunt field urrent to the "free-wheeling" diode *REC6* and also connecting the relaxation oscilla-tor input for normal operation. Note hat prior to *K1* pickup, the capacitor *C4* s essentially short-circuited through the SCT and the choke *L1*. There is suffi-cient time delay between the operating of relay *K1* and the first gating pulse for capacitor *C4* to receive an initial charge in excess of 10 volts. This eliminates com-mutation failures at start-up.

Additional logic imposed on the relaxa-tion oscillator should now be examined. Another limitation of the power circuit is that its operation becomes unreliable with input voltages of less than 10 volts. The voltage levels in the relaxation oscillator were so arranged that the emitter of the unijunction transistor had to charge above the 11 volts in order to fire. The emitter was then connected to the power circuit supply voltage through *REC3*. If for any reason the input voltage should drop below 10 volts, the capacitor *C6* would be immediately discharged, pre-venting further gating until the input voltage rose above 10 volts.

A third precaution necessary in gating the power circuit is that the control rectifier must not be triggered again until it has completed its prior cycle, the turnoff capacitor recharging to the source voltage. To obtain the maximum field current required, it is necessary that we be able to place the field voltage pulses "head to toe," but we must be sure not to gate the control rectifier while it is still conducting, or while current is flowing through the SCT to the shunt field.

This is also the period of time while the field voltage is positive. All that is neces-sary to meet the third power circuit limitation is to prevent the unijunction transistor *Q2* from firing while the field voltage is positive. This bit of logic is accomplished by means of Zener diodes, *REC4*, *7*, and *9*. *REC9* clamps the emitter voltage such that the *Q2* cannot fire when a voltage which is the sum of the breakdowns of *REC4* and *7* appears on base 2 of *Q2*. However, if the base 2 volt-age is reduced to the breakdown of *REC4* only, then *Q2* can fire. Note that the mid-point of the voltage divider formed by *REC4* and *REC7* is tied directly through a diode to the field voltage as seen on the free-wheeler *REC6*. When the field voltage is negative as it is during the free-wheeling interval, the mid-point between *REC4* and *REC7* is at ground potential. This means that a lower volt-age appears on base 2 and the Zener clamp *REC9* is not effective and the relaxation oscillator can function normally.

When the field voltage is positive, *REC5* will block and the base 2 voltage will merely be the sum of the breakdown voltages of *REC4* and *7*. As mentioned earlier, *REC9* will clamp the emitter at a low enough voltage to prevent *Q2* from firing. In this manner head-to-toe field voltage pulses can be obtained, and at the same time failure of the power circuit prevented.

This generator and its regulator can be paralleled with any number of similar sys-tems. The load division is accomplished by the magnetic-amplifier winding *F5-F6*, and the adjustment resistors *R7* and *R8*. Its performance is directly analogous to the load division in a carbon pile regu-lator system or the system used in a 71 C d-c regulator.

To obtain adequate system stability without the sacrifice of regulation ac-curacy, the stabilizing choke *L3* was con-nected across an additional magnetic-amplifier control winding. This pro-vides adequate system phase margin with only minor sacrifice of transient response. The servo analysis of this type of stabiliza-tion is included in reference 1.

A 5-microsecond dip in generator out-put voltage was noticed each time the SCR was gated. The sharp step of cur-rent drawn by the SCR was causing the voltage to appear transiently across the impedance of the generator and not at its terminals. The input filter *C1-L1-C2*, *3* was added to supply the needed field cur-rent transiently and prevent the genera-tor voltage from dipping. The input filter and the field voltage filter *L2-C5* also serve to attenuate radio-interference volt-age generated by the fast switching. It is interesting to note that any load equip-ment using fast-rise switching circuits, i.e., inverters, will chop the generator voltage in the same manner unless filtering is added to draw relatively constant cur-rent from the generator. Radio-inter-ference requirements may demand even greater filtering than that needed to minimize ripple voltage.

Fig. 7. The unit

Packaging and Thermal Design

From the start, it was evident that the SCR and unijunction transistor would have to be operated quite close to their maximum temperature ratings. It ap-peared that some field current derating might be required in 120 C ambient and, in any case, a careful job of thermal design was necessary. A general ap-proach was evolved in which all but the thermally critical components were to be placed in a "hot" section. The critical components were to be thermally isolated from the hot section by means of a foam compound. Metal-to-metal con-tact between the two sections was avoided by using epoxy resin blocks to supply the mechanical connection.

The SCR had to be electrically isolated from the case of the section in which it was mounted, but it had to have low thermal drop to the finned heat sink mounted on the outside of its section. This was accomplished by threading the controlled rectifier into a $2^1/_2$-inch-square plate of 1/4-inch aluminum. The plate was insulated from the case by a 3-mil sheet of mica. This resulted in a less than 3-degree rise of the control rectifier stud above the heat sink at the maximum field current. The hot section containing 75% of the weight was placed on the bottom of the unit. The higher

temperature of this section increased the radiation efficiency to allow better heat rejection. The top unit was quite light and it could therefore withstand the specific shock and vibration environment without the need for heavy mounting.

The thermal isolation allowed a steady-state temperature differential of about 5 C between the top and bottom halves, but its real advantage was in transient isolation.

Referrring to Fig. 6, note that the regulator can supply an 8-ampere field current for one hour at 120 C ambient; its steady-state rating 120 C is 7 amperes.

The completed unit mounts within the envelope of the carbon pile regulator and does not require shock mounts; see Fig. 7. Electric connections are made through a single connector. The over-all weight is about $6^1/_4$ pounds.

Comments on the Development

The rather complex control circuitry was required in order to use the magnetic-controlled rectifier power circuit. The use of a saturating core to obtain reliable commutation justifies the added complexity of these signal-level circuits. The complexity was compounded by the requirement for self-build-up and by the possible presence of low-voltage transients. Care was taken throughout the development of the regulator to meet system requirements which are filled by the carbon-pile type of regulator, but which are not included in present specifications.

Conclusions

The development of this regulator clearly demonstrates that an SCR and its associated circuitry can be used in 120 C ambient to regulate d-c aircraft generators.

Tests have proved that the regulator exceeds the performance required by the 71 C regulator specifications, maintaining a 28.0 ±0.75-volt output over the speed, load, and temperature extremes. A regulator of this type, using all silicon devices, can provide more than adequate performance in advanced aircraft requiring 120 C operation.

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The Effect of Speed Variation on Aircraft Electric Power Systems: Speed Modulation and Voltage Modulation

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HIGHER-PERFORMANCE aircraft and missiles place extreme requirements on electric power systems. These systems cannot be divided into components such as drive generator, or regulator, as has been done in the past; rather, they have to be treated as a whole or a system. This places new emphasis on the division between mechanical drive and electric components. This paper deals with this co-ordination problem and the effect of each component on the others.

Speed modulation of the generator drive shaft will result in frequency and amplitude modulation of the load voltage, which in turn will produce pulsations of generator torque. This pulsating torque is presented to the drive as a load. System synthesis requires a quantitative evaluation of the effect of speed modulation on the load voltage. It is therefore necessary to relate the effect of speed modulation to the transfer function of the electric power system. The open-loop system transfer function is generally used in the study of system performance with respect to voltage regulation and load changes. The objective is therefore to express the effect of speed modulation on system performance quantitatively through use of the open-loop transfer function of the electric power system.

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Open-Loop System Transfer Function

The electric power system is represented by the block diagram of Fig. 1. The diagram contains only one block *G*, which corresponds to the open-loop transfer function of the electric part of the power system at rated speed. *G* also contains terms representing the load connected to the generator. It represents, therefore, the open-loop trransfer function of an electric power system including voltage regulator, exciter, generator, and load. The output variable of this block, v_{L0}, corresponds to the load voltage at rated speed.

As the generator voltage changes proportionally with speed, the load voltage at rated speed is multiplied by the per-unit speed *n*. The voltage across the load v_L is therefore equal to the voltage at base speed v_{L0} multiplied by the per-unit speed *n*. This speed shall be the speed at the drive shaft of the generator or, better, since the drive shaft is not infinitely rigid it shall be the relative speed of rotor and stator in the air gap. This definition eliminates the need for consideration of interaction between the electric system and the drive at this time. It is equivalent to the assumption of a zero-impedance drive with a stiff drive shaft.

In general, electric power systems are not represented by a simple block diagram, as Fig. 1. Normally, several feedback loops exist throughout the system. However, by manipulation it is possible to represent any power system in

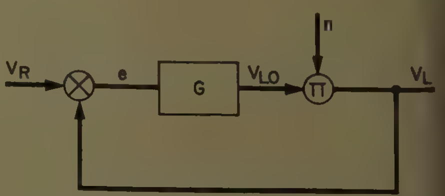
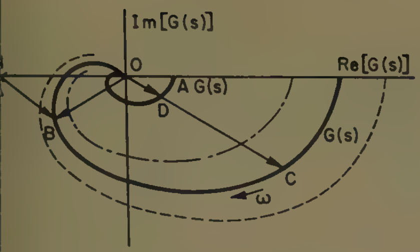


Fig. 1. Block diagram indicating effect of speed on load voltage for electric power system



g. 9. Effect of speed changes on open-op transfer function of electric power system

ie form shown in Fig. 1. The system quation, written with all variables being unctions of time, is

$$(t)=G(p)e(t)\times n(t) \quad (1)$$

see references 1 and 2) in which $G(p)$ the system transfer function expressed n operational form in the time domain. (p) is the ratio of two polynomials in of order i and k, where p stands for the ifferential operator d/dt. To facilitate he analysis, it will first be restricted to inusoidal speed modulation around rated peed $n_0=1$. The instantaneous speed s then given by

$$(t)=n_0+A \cos ut \qquad A \ll n_0 \quad (2)$$

In practice, sinusoidal speed modula-ions will be encountered most fre-quently. Analysis of this special case vill provide sufficient insight into the ffect of speed modulation to allow the nvestigation of system performance vith respect to arbitrary disturbances n speed. The effect of sinusoidal speed nodulation is given by equations 1 and 2:

$$_L(t)=n_0G(p)e(t)+A\ (\cos ut)G(p)e(t) \quad (3)$$

System synthesis requires the system ransfer function to be given in the fre-quency domain; this is accomplished by Laplace transformation of equation 3. The transformation, given in Appendix I, esults in an open-loop system transfer unction, which includes the effect of peed modulation as follows:

$$\frac{V_L(s)}{E(s)}=n_0G(s)+ AG(s)\left[\frac{E(s-ju)}{2E(s)}+\frac{E(s+ju)}{2E(s)}\right] \quad (4)$$

The nature of the open-loop transfer unction given in equation 4 indicates hat speed modulation introduces a non-inearity into the system. It also shows hat the frequency spectrum is translated n both directions by an amount u, as ndicated by the two side-band frequencies $(s-ju)$ and $(s+ju)$. This frequency ranslation is similar to the spectrum of n amplitude-modulated wave with a uppressed carrier.

Discussion of Open-Loop Transfer Function

The first term in the open-loop transfer function of equaton 4 corresponds to the system transfer function at a constant base speed n_0. The second term accounts for the effect of a sinusoidal speed modulation with an amplitude A. For reasons of discussion it shall be assumed that the electric power system has an open-loop transfer function $G(s)$, measured at constant base speed, as shown in Fig. 2. The curve of Fig. 2 represents the Nyquist diagram of the electric power system in the s-plane with frequency as parameter. Most electric power systems will have transfer functions of the same general shape as the one shown here. The following discussion, however, is not restricted by the shape of the system transfer function.

The closed-loop frequency response V_L/V_R can be found from the open-loop transfer function by well-known techniques. To facilitate the analysis of the expression for the open-loop transfer function (equation 4) a second curve has been drawn in Fig. 2, equal to $A\ G(s)$. A is a constant and corresponds to the amplitude of the sinusoidal speed modulation. Since rated speed n_0 equals 1 per unit, the magnitude of A is small with respect to 1. A is a real number and the transfer function $A\ G(s)$ is therefore obtained by shrinking the open-loop transfer function $G(s)$. The frequency scale remains unchanged. Three special cases are next considered:

Case 1

In the first case, it is assumed that the speed has been increased from n_0 to n_0+A. The speed is constant and no sinusoidal modulation exists: i.e., $u=0$. As a results, the content of the parentheses in equation 4 becomes equal to 1 and equation 5 reduces to

$$\frac{V_L(s)}{E(s)}=n_0G(s)+AG(s) \quad (5)$$

The open-loop transfer function for this speed is obtained by adding the vector **OD** to the vector **OC** at each frequency, as shown by the dotted line. An increase in speed can be represented by blowing up the curve $G(s)$ representing the open-loop transfer function at base speed.

Case 2

For the second case, assume a steady-state decrease in speed to $n=n-A$. The open-loop transfer function for this speed is found as before, by subtracting the vector **OD** from the vector **OC**, resulting in a transfer function corresponding to the dash-dotted line of Fig. 2. This curve is obtained by shrinking the open-loop transfer function $G(s)$.

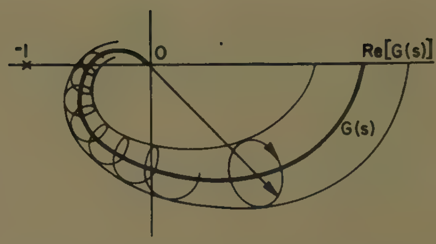


Fig. 3. Vector locus of open-loop transfer function of electric power system with sinusoidal speed modulation

The results obtained here for a steady-state change in speed are obvious. The voltage gain of the generator is directly proportional to speed and the range of the open-loop transfer function for steady-state changes in speed as shown in Fig. 2 could have been derived by inspection. However, in the case of sinusoidal speed modulation $u\neq 0$, the process is not as simple.

Case 3

In the third case the open-loop transfer function will lie within the band established by the maximum and minimum value of speed $n=n_0\pm A$. In addition, there will be an effect from the side-band frequencies $\omega\pm u$ which let the end of the vector **OC** describe a circular path; see Fig. 3. This circular path will be bounded by the upper and lower limits obtained from the steady-state characteristics for $n=n_0\pm A$, and by boundaries in the direction of frequency established by the sum and difference frequencies $\omega\pm u$.

System Stability

The above discussion indicates that speed modulation will definitely affect the stability of the power system and may very well change the stability margin of a system. As the magnitude of the speed modulation increases, the open-loop transfer function shown as a solid line in Fig. 2 changes to a band in the complex plane as indicated in Fig. 3. As the frequency of speed modulation increases, the vector locus of the open-loop transfer function moves on a spiral along the frequency scale.

The proximity of the open-loop transfer function to the point (-1) indicates the relative stability of a closed-loop control system. Fig. 3 indicates that the distance to the point (-1) has definitely been changed by the speed modulation and it is possible that the vector locus

will encircle the point (−1) if the magnitude of speed modulation is sufficient. This can cause the system to become unstable.

Equations for Speed Modulation and Voltage Modulation

The effect of speed modulation on load voltage can be found by evalution of the closed-loop transfer function from Fig. 3. However, the process is very tedious since the closed-loop transfer function must be determined point by point. It is therefore desirable to derive this functional relationship analytically.

The equations for the system of Fig. 1 can be written by inspection

$$v_L(t)=v_B\frac{n(t)G(p)}{1+n(t)G(p)} \tag{6}$$

where $G(p)$ is the transfer function expressed in operational form, and p represents d/dt.

The equation for the load voltage at rated speed $n=n_0=1$, again written in the time domain, is given by

$$v_{L0}(t)=v_B\frac{G(p)}{1+G(p)} \tag{7}$$

The magnitude of voltage modulation is given by the ratio of equations 6 and 7.

$$\frac{v_L(t)}{v_{L0}(t)}=\frac{n(t)[1+G(p)]}{1+n(t)G(p)} \tag{8}$$

The effect of speed variation on the load voltage can be represented by the total differential of load voltage with respect to speed. (It shall be assumed that the open-loop transfer function $G(p)$ is independent of speed.)

$$\frac{d(v_L/v_{L0})}{dn}\approx\frac{1+G(p)}{[1+n(t)G(p)]^2}\approx\frac{1}{1+n(t)G(p)} \tag{9}$$

The approximation of equation 9 is justified if variations in speed are restricted to near unity:

Again, consider sinusoidal speed modulation around rated speed n_0 as defined by equation 2. Transformation of equation of equation 9 from the time domain into the frequency domain results in a transfer function of the electric power system relating voltage modulation; see Appendix II. This transfer function is (with $v_L/v_{L0}=v$)

$$\frac{V(s)}{N(s)}=\frac{1}{1+n_0G(s)+1/2AG(s)\times\left(\frac{V(s-ju)}{V(s)}+\frac{V(s+ju)}{V(s)}\right)} \tag{10}$$

The nonlinear character of the system may be seen by inspection of the right-hand side of equation 10, which contains the variable $V(s)$. In general we are interested in the amplitude ratio of voltage and speed modulation for a speed modulation at constant frequency. Here $s=ju$ and the transfer function of equation 10 reduces to

$$\frac{V(u)}{N(u)}=\frac{1}{1+n_0G(s)+1/2AG(s)\times\left(\frac{V(o)}{V(u)}+\frac{V(2u)}{V(u)}\right)} \tag{11}$$

Equation 11 reveals that the nonlinearity in the denominator is determined by the two side-band frequencies. This term corresponds to the voltage ratio at zero frequency to the voltage at frequency u plus a term representing the ratio of voltage at twice the frequency and the base frequency u. If the system is operating in a frequency modulation range where the terminal voltage is constant and independent of frequency, i.e., where the closed-loop frequency response is flat over a given frequency range, the term in brackets reduces to 2.

With this assumption, and since $n_0=1$, and $A<<n_0$, the transfer function of equation 11 reduces to the following simple term:

$$\frac{V(s)}{N(s)}=\frac{1}{1+G(s)} \tag{12}$$

Equation 12 represents the ratio of amplitude to frequency modulation which can be expected in an electric power system.

Voltage-Speed Transfer Function

Equation 12 indicates that the ratio of voltage to speed modulation is determined by the open-loop transfer function of the electric power system. Referring to Fig. 2, the voltage-speed transfer function is given by the reciprocal of the vector **AB**. As u increases, **AB** describes the following locus.

At low frequencies the vector is large. Its amplitude decreases, with increasing frequency, reaches a value of 1, decreases to values smaller than 1, and reaches a minimum at a point where the open-loop transfer function is closest to the point (−1). As the frequency increases further the vector **AB** increases in magnitude and finally reaches a value equal to 1 at infinite frequencies. The ratio of voltage to speed modulation according to equation 12 is given by the reciprocal of **AB**. The magnitude of voltage modulation caused by speed modulation is very small at low frequencies, and increases as the frequency increases approaching one at high frequencies. Fig. 4 is a sketch of the voltage to speed transfer function corresponding to an open-loop transfer function as shown in Fig. 2.

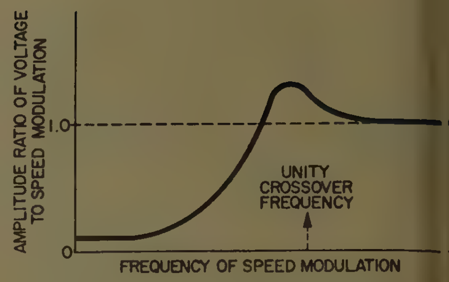
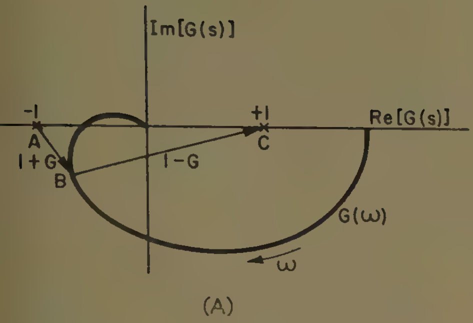


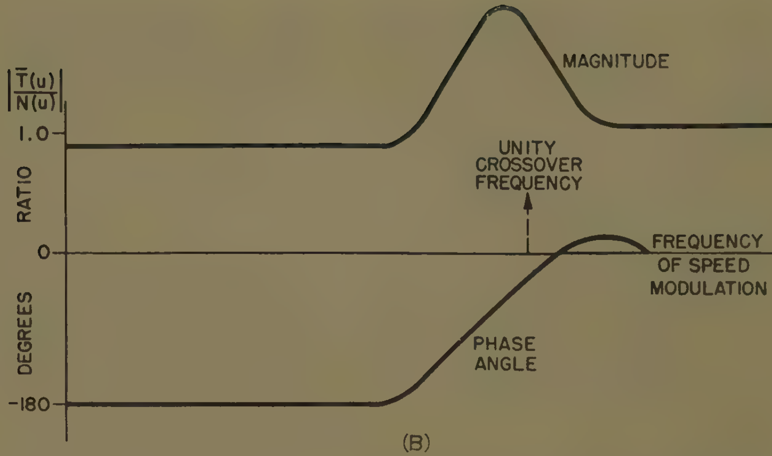
Fig. 4. Amplitude ratio of voltage and speed modulation as function of speed modulation frequency

Results of Fig. 4 can be interpreted easily. The regulator is capable of providing corrective signals to the generator at very low frequencies so that the effect of speed changes does not affect the terminal voltage. As the frequency increases, the voltage regulator becomes less and less capable of correcting for the changes in speed. At high frequencies the voltage regulator has lost completely its capability to correct for speed modulation on the system. The magnitude of speed modulation manifests itself directly at the load voltage of the generator with a 1-to-1 ratio.

At some intermediate frequencies, the possibility exists of amplification of the magnitude of speed modulation which manifests itself as load voltage modulation. The amount of amplification can be estimated from the open-loop transfer function as indicated in Fig. 2. The amplification will be the higher, the closer the open-loop transfer function comes to the point (−1). However the proximity of the open-loop transfer function with respect to the point (−1) is also an indication of the stability margin. The closer the open-loop transfer function approaches the point (−1), the more oscillatory the system will be. In addition, the oscillatory character of the system in the vicinity of the point (−1) explains the magnification of speed modulation in a frequency band around the unity crossover frequency, as shown in Fig. 4.

In practice, specifications on transient response time after load disturbances, require the system to have damping factors between 0.2 and 0.7. These will result in amplification factors for voltage modulation approximately between 1 and 3. If system requirements specify voltage modulation of less than 1%, then the speed modulation introduced from the prime mover must be smaller than 0.3% at frequencies around the closed-loop resonant frequency. Additional amplification is possible through mechanical tor-





;. 5. Transfer function of electric power sys-
tem. A—Open-loop. B—Torque-speed

onal resonance of the drive shaft. Am-ification factors between 2 and 5 have en measured under different conditions an aircraft power system developed cently.

quations for Torque Modulation

Consider again the simple electric ower system of Fig. 1. Speed varia-ons will result in load voltage modula-on, which in turn will cause pulsation of e generator torque. The objective of is analysis is to determine the functional lationship between torque pulsation nd speed variation.

The torque demanded by the generator given by the output power divided by e instantaneous speed. If we restrict e analysis to resistive loads on the stem, the following equation holds for e torque:

$$=\frac{1}{n}\frac{v_L^2}{R_L}=\frac{1}{n}v_Li_L=v_{L0}i_L \qquad (13)$$

The subscript zero refers to quantities base speed. Equation 13 for torque ssumes that no additional losses occur the generator or other components of e electric power system. In other ords, the efficiency of the electric power stem is assumed to be 100%. This ssumption, although crude, will produce 1 insight into the transfer function of orque versus speed. The torque shall e expressed in relationship to the torque base speed $n_0=1$:

$$_0=\frac{v_{L0}^2}{R_L}=v_{L0}i_{L0} \qquad (14)$$

The load voltage at rated speed and oeeds different from rated in terms of e transfer function $G(p)$ and expressed the time domain was given by equations and 7.

Squaring and inserting into equations 3 and 14 gives the ratio of torque at any oeed to the torque at rated speed as a nction of time:

$$\frac{T(t)}{T_0(t)}=\frac{v_L(t)^2}{n(t)v_{L0}(t)^2}=n(t)\frac{[1+G(p)]^2}{[1+n(t)G(p)]^2} \qquad (15)$$

To obtain the effect of speed on torque, equation 15 must be differentiated with respect to speed n. (The open-loop transfer function $G(p)$ is assumed to be independent of speed.)

$$\frac{d(T/T_0)}{dn}=[1+G(p)]^2\frac{1-n(t)G(p)}{[1+n(t)G(p)]^3}\approx\frac{1-n(t)G(p)}{1+n(t)G(p)} \qquad (16)$$

Since speed modulations are of relatively small magnitude, the instantaneous per-unit speed n will be near unity and the above approximation is justified.

Again, consider sinusoidal speed modulation around rated speed as defined by equation 2. Transformation of equation 16 from the time domain into the frequency domain results in a transfer function of the electric power system relating variations in generator air-gap torque to speed modulation; see Appendix III. This transfer function (defining the torque ratio $T/T_0=\bar{T}$) is given by

$$\frac{\bar{T}(s)}{N(s)}=\frac{1-G(s)}{1+G(s)} \qquad (17)$$

This equation indicates that torque modulation caused by sinusoidal speed modulation is represented by a non-minimum phase function; the properties of this function will be investigated next.

Torque-Speed Transfer Function

If an electric power system is driven by its prime mover at a nonuniform rate, the torque demanded by the generator from the prime mover will not be constant. The torque will be a function of the instantaneous speed of the prime mover and also a function of the frequency spectrum of the prime mover's speed of rotation. This relationship expressed in the frequency domain is given by equation 17.

The following may be said by inspection: $G(s)$ is equal to the open-loop transfer function of generator, exciter, and voltage regulator. Because of other system requirements, $G(s)$ will be large at low frequencies and will approach zero if the frequency approaches infinity. The ratio of torque to speed modulation will therefore almost equal 1 at low frequencies and will equal 1 at high frequencies, according to equation 17. The phase angle at low frequencies approaches −180 degrees, and zero degrees at high frequencies.

The behavior of the transfer function in the intermediate frequency range can best be visualized by use of the system open-loop transfer function $G(s)$ shown in Fig. 5(A). Equation 17 can be interpreted in terms of this graph. For a specific frequency the numerator term is given by the vector **BC** connecting the open-loop transfer function locus with the point (+1). The denominator term is given by **AB** connecting the point (−1). with the open-loop transfer function locus. The torque to speed ratio at any frequency u is given by the ratio of these two vectors.

$$\frac{\bar{T}(u)}{N(u)}=\frac{\mathbf{BC}}{\mathbf{AB}} \qquad (18)$$

When the frequency of speed modulation approaches the unity crossover frequency of the subsystem, **AB** becomes smaller than unity and the ratio of torque to speed according to equation 17 becomes larger than 1. A sketch of the complete transfer function for the torque to speed ratio is shown in Fig. 5(B). Fig. 5(B) indicates that the torque demanded by the generator from the prime mover can increase considerably in a frequency range around the unity crossover frequency of the electric power system. Two examples will show the application of this method.

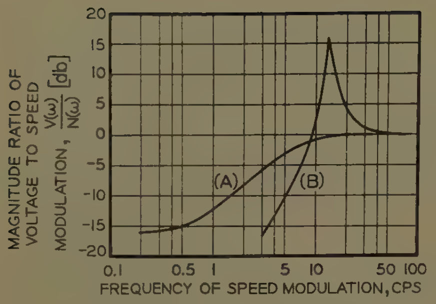


Fig. 6. Voltage and speed modulation of two electric power systems

Example 1

The open-loop transfer function of the electric power system is assumed to correspond to a first-order system. The generator is represented by a single time constant equal to a field time constant of $T=0.16$ second. The voltage regulator is assumed to be ideal and the steady-state open-loop gain of the regulator-generator combination shall equal 5. With these assumptions, the open-loop transfer function is given by

$$G(s)=\frac{K}{1+sT} \tag{19}$$

Inserting equation 19 into equation 12 will yield the transfer function for voltage modulation as a function of speed modulation. This transfer function is given by

$$\frac{V(s)}{N(s)}=\frac{1}{1+G(s)}=\frac{1}{1+K}\,\frac{1+sT}{1+sT/(1+K)}=\frac{1}{6}\,\frac{1+0.16s}{1+0.027s} \tag{20}$$

Equation 20 indicates that the amplitude of voltage modulation for a given amplitude of speed modulation at low frequencies is attenuated by the factor $1/(1+K)=1/6$. The final value theorem of Laplace transform shows that this ratio approaches 1 for high frequencies.

Inserting equation 19 into equation 17 will produce the transfer function for torque modulation as function of speed modulation. This transfer function is given by

$$\frac{T(s)}{N(s)}=\frac{1-G(s)}{1+G(s)}=\frac{1-K}{1+K}\,\frac{1+sT/(1-K)}{1+sT/(1+K)}=-\frac{4}{6}\,\frac{1-0.04S}{1+0.027S} \tag{21}$$

An electric power subsystem having a transfer function according to equation 19 has been simulated on an analog computer according to the block diagram shown in Fig. 1. Speed modulation was simulated by a sinusoidal disturbance into the multiplier on the analog computer. This multiplier was located on the computer setup as indicated in Fig. 1. Torque was calculated by means of a multiplier in the analog computer according to equation 13. The simulation was restricted to resistive load. Computer results of voltage and speed modulation are shown in Fig. 6, curve A, which represents the amplitude ratio of voltage to speed modulation as function of the frequency of speed modulation. The transfer function as determined on the computer shows excellent agreement with the predicted performance according to equation 20. The initial attenuation at low frequencies is equal to 1/6 or −15.6 db (decibels). Voltage modulation increases with a slope of 20 db per decade according to the numerator time constant of 0.16 second, corresponding to a corner frequency of 1 cps (cycles per second).

As the frequency of speed modulation increases, the curve starts to flatten out at a corner frequency of 6 cps. The agreement between actual system performance as determined on the analog computer and the theoretical analysis according to equation 12 is surprisingly good despite the inherent nonlinearity of the system equations. The computer analysis has been repeated at different magnitudes of speed modulation corresponding to 1.0%, 2.0%, and 4.0% of rated speed, with practically identical results.

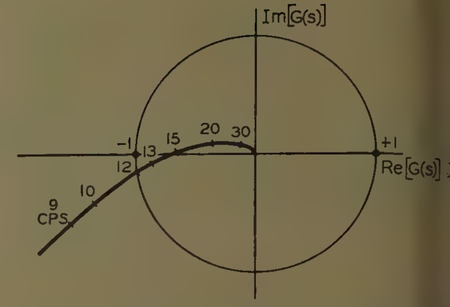


Fig. 8. Open-loop transfer function of electric power system, example 2

Computer results of the torque-speed transfer function are shown in Fig. 7. The transfer function as measured on the computer shows excellent agreement with the predicted performance according to equations 17 and 21 despite the inherent nonlinearity of the system equations.

Example 2

The second example investigates the effect of speed on a system which is represented by four time constants, corresponding to corner frequencies of 1, 8.5, 100, and 100 cps, with an open-loop gain of 28. The system transfer function is given by

$$G(s)=\frac{28}{1+0.16S}\,\frac{1}{1+0.019S}\,\frac{1}{(1+0.0016S)^2} \tag{22}$$

The system has been selected so that it exhibits a highly oscillatory character. The damping factor is 0.074 of critical. The vicinity of the point (−1) is of special interest to the speed-voltage and speed-torque transfer function; see Fig. 8.

The system was simulated on an analog computer and to it was applied sinusoidal speed modulation of different frequencies. The ratio of the magnitude of voltage modulation to speed modulation was then

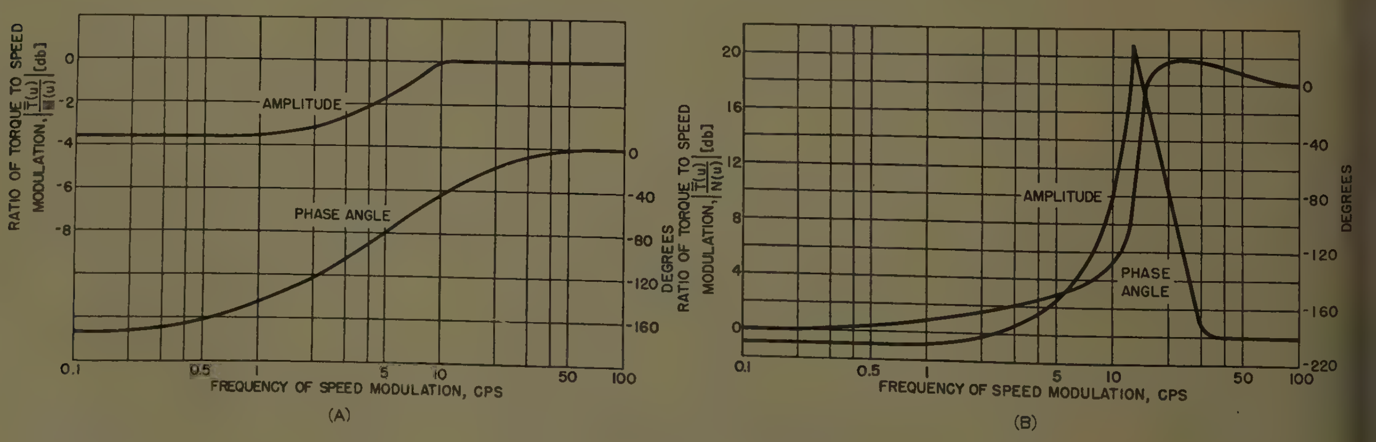


Fig. 7. Torque-speed transfer function. A—Example 1. B—Example 2

etermined. According to equation 17 nd Fig. 8, a peak was to be expected at a requency of 12.5 cps with a magnitude qual to the reciprocal of the distance from he point (−1) to the open-loop transfer unction. This distance is equal to 0.135, which corresponds to an overshoot of .4 or 17 db.

Results of this simulation on the analog computer are shown in Fig. 6, curve 3. There is indeed a peak of the transfer function at approximately 13 cps with a magnitude of 16 db in good agreement with the analysis.

Torque was calculated by means of a multiplier on the analog computer according to equation 13. Determination of the torque-speed transfer function followed the same procedure as in example .

From Fig. 8 and equation 17 it can be predicted that there will be a peak in torque which, according to equation 18, is equal to 1.865/0.135=13.8, or 23 db. This peak should occur at a frequency of 12.5 cps.

Fig. 7(B) presents results of the analog computer simulation. As predicted, there is a peak in torque of 21 db at a frequency of 13 cps. The phase angle starts at −180 degrees at low frequencies and advances to zero degrees at high frequencies. Positive phase angles occur at frequencies slightly above the unity crossover frequency. These phase angles are positive for a range of frequencies where the open-loop transfer function of the electric power system stays in the second quadrant of the complex plane.

Conclusions

Equations and transfer functions have been derived for voltage and torque modulation of electric power systems produced by speed modulation of the prime mover. These equations related the voltage-speed and torque-speed transfer function to the open-loop transfer function of the electric power system. Analytically derived transfer functions have been verified by analog computer simulation.

The voltage-speed transfer function indicates that between amplitude and frequency modulation a definite relationships exists which can reach ratios as high as 5 in certain ranges of frequencies.

The torque-speed transfer function indicates the presence of torque peaks at certain modulation frequencies which are presented as load to the prime mover.

A torque-speed transfer function has been derived for the electric power system. A similar, but inverse, transfer function exists for the drive: the speed-torque transfer function. Both the torque-speed (electric power system) and speed-torque (drive) transfer functions have to be compatible so that one component does not excite the other. Based on this definition, system stability can be determined, performance evaluated and changes incorporated to improve system performance.

The value of these new transfer functions for system analysis and performance evaluation lies in the fact that voltage and torque modulation as produced by drive speed modulation are expressed in terms of the open-loop transfer function of the electric power system. This same transfer function is used in stability analysis and voltage regulation. Therefore, no new information on the power system is required to evaluate the effect of speed variation on the different system components.

List of Symbols

A = amplitude of speed modulation
E, e = error signal
G = transfer function
H, Q = polynomials, general
N, n = speed
p = differential operator d/dt
s = Laplace operator
$T, \bar{T}$ = torque
T = time constant
u = frequency of modulation
V, v = voltage
ω = frequency

Subscripts

L = load
0 = value at rated speed

Appendix I. Open-Loop Transfer Function

Equation 3 has given the system open-loop transfer function:

$$v_L(t)=n_0G(p)e(t)+AG(p)e(t)\cos ut \qquad (3)$$

Before performing the Laplace transformation, $G(p)$ must be examined. $G(p)$ is the open transfer function of the power system in operational form. It is, generally, the ratio of two polynomials $H(p)$ and $Q(p)$ in p.

$$G(p)=\frac{\sum_0^i a_ip^i}{\sum_0^k b_kp^k}=\frac{H(p)}{Q(p)}$$

Therefore

$$Q(p)V_L(t)=n_0H(p)e(t)+AH(p)e(t)\cos ut$$

$H(p)$ and $Q(p)$ are operators and polynomials in p; n_0 is a constant. The cosine function can be replaced by an exponential.

$$\cos ut=1/2(\epsilon^{jut}+\epsilon^{-jut})$$

With these assumptions, and assuming initial conditions equal to zero, Laplace transformation of equation 3 is possible, and results in

$$Q(s)V_L(s)=n_0H(s)E(s)+1/2AH(s)[E(s-ju)+E(s-ju)]$$

from which follows

$$V_L(s)=n_0G(s)E(s)+1/2AG(s)[E(s-ju)+E(s+ju)]$$

Dividing by $E(s)$ results in equation 4.

Appendix II. Closed-Loop Transfer Function

The effect of speed variation on the load voltage was given by the total differential of equation 9. The analysis shall be restricted to sinusoidal speed modulation around rated speed, as defined by equation 2.

When equation 2 is inserted into equation 9, and the voltage ratio $v_S/v_{S0}=v$ is defined, the system equation, written in the time domain, becomes

$$dv(t)+n_0G(p)dv(t)+A\cos(ut)G(p)dv(t)=dn(t) \qquad (23)$$

The differentials of voltage and speed can be interpreted as perturbations around an operating point or as small signal quantities. This allows us to drop the differential notation in equation 23. Then, $v(t)$ and $n(t)$ are redefined as small signal quantities replacing $dv(t)$ and $dn(t)$ in equation 23. Replacing the cosine function by an exponential, equation 23 becomes

$$v(t)+n_0G(p)v(t)+1/2AG(p)v(t)\epsilon^{jut}+1/2AG(p)v(t)\epsilon^{-jut}=n(t)$$

Before performing the Laplace transformation, $G(p)$ must be examined. $G(p)$ is the open-loop transfer function of the power system and therefore generally the ratio of two polynomials $H(p)$ and $Q(p)$ in p.

$$G(p)=\frac{\sum_0^i a_ip^i}{\sum_0^k b_kp^k}=\frac{H(p)}{Q(p)}$$

Therefore

$$Q(p)v(t)+n_0H(p)v(t)+1/2AH(p)v(t)\epsilon^{jut}+1/2AH(p)v(t)\epsilon^{-jut}=Q(p)n(t)$$

Since $Q(p)$ and $H(p)$ are operators and polynomials in p, and since n_0 is a constant, Laplace transformation is possible (assuming initial conditions equal to zero).

$$Q(s)V(s)+n_0H(s)V(s)+1/2AH(s)V(s+ju)+1/2AH(s)V(s-ju)=Q(s)N(s)$$

Collecting terms results in the voltage-speed transfer function of the system as given by equation 10.

Appendix III. Torque-Speed Transfer Function

The effect of speed variations on the air-gap torque of the generator was given

by equation 16. The analysis shall be restricted to sinusoidal speed modulation around rated speed n_0 as defined by equation 2. Inserting equation 2 into equation 16 and defining the torque ratio $T/T_0=\bar{T}$ gives the system equation, written in the time domain:

$$d\bar{T}(t)+n_0G(p)d\bar{T}(t)+A\cos(ut)G(p)d\bar{T}(t)$$
$$=dn(t)-n_0G(p)dn(t)-A\cos(ut)G(p)dn(t) \quad (24)$$

The differentials of torque and speed in equation 24 can be interpreted as perturbations around an operating point or as small signal quantities. The differential notation can then be dropped and $\bar{T}(t)$ and $n(t)$ are redefined as small signal quantities, replacing $d\bar{T}(t)$ and $dn(t)$ in equation 24. Replacing the cosine function by the corresponding exponential and inserting into equation 24 gives

$$T(t)+n_0G(p)\bar{T}(t)+1/2AG(p)\bar{T}(t)\epsilon^{jut}+$$
$$1/2AG(p)\bar{T}(t)\epsilon^{-jut}=n(t)-n_0G(p)n(t)-$$
$$1/2AG(p)n(t)\epsilon^{jut}-1/2AG(p)n(t)\epsilon^{-jut}$$

Before performing the Laplace transformation, $G(p)$ must be examined as in Appendix II. Therefore

$$Q(p)\bar{T}(t)+n_0H(p)\bar{T}(t)+1/2AH(p)\bar{T}(t)\epsilon^{jut}$$
$$+1/2AH(p)\bar{T}(t)\epsilon^{-jut}=Q(p)n(t)-$$
$$n_0H(p)n(t)-1/2AH(p)n(t)\epsilon^{jut}-$$
$$1/2AH(p)n(t)\epsilon^{-jut}$$

Since $Q(p)$ and $H(p)$ are operators and polynomials in p, and since n_0 is a constant, Laplace transformation is possible (assuming initial conditions equal to zero):

$$Q(s)\bar{T}(s)+n_0H(s)\bar{T}(s)+1/2AH(s)\bar{T}(s+ju)$$
$$+1/2AH(s)\bar{T}(s-ju)=Q(s)N(s)-$$
$$n_0H(s)N(s)-1/2AH(s)N(s+ju)-$$
$$1/2AH(s)N(s-ju)$$

Collecting terms yields the torque-speed transfer function of the electric power system, which is then given by

$$\frac{\bar{T}(s)}{N(s)}=\frac{1-n_0G(s)-1/2AG(s)\times\left[\dfrac{N(s-ju)}{N(s)}+\dfrac{N(s+ju)}{N(s)}\right]}{1+n_0G(s)+1/2AG(s)\times\left[\dfrac{\bar{T}(s-ju)}{\bar{T}(s)}+\dfrac{\bar{T}(s+ju)}{\bar{T}(s)}\right]} \quad (25)$$

The nonlinear character of the torque-speed transfer function is obvious by inspection of equation 25. The two variables, torque and speed, appear on both sides of the equation. Generally the system design engineer is interested in the amplitude and phase angle of torque modulation for speed modulation of constant frequencies, because torque modulation will be excited by distinct frequencies u. In this case, $s=ju$, and the transfer function of equation 25 reduces to

$$\frac{1-n_0G(s)-1/2AG(s)\left[\dfrac{N(o)}{N(u)}+\dfrac{N(2u)}{N(u)}\right]}{1+n_0G(s)+1/2AG(s)\left[\dfrac{\bar{T}(o)}{\bar{T}(u)}+\dfrac{\bar{T}(2u)}{\bar{T}(u)}\right]} \quad (26)$$

The nonlinearity in equation 26 is given by the two side-band frequencies which appear in the speed and torque frequency spectrum. Their spectra are similar to the spectrum of an amplitude-modulated carrier with suppressed carrier frequency. The terms in brackets correspond to the speed or torque ratio of zero frequency to their value at the frequency u and another term representing the ratio of speed or torque at twice the exciting frequency u to their value at the exciting frequency. If we restrict the analysis to small perturbations around unity, the terms in the brackets become approximately 2. With this assumption, the transfer function of equation 26 reduces to the following simple transfer function representing the ratio of torque to speed modulation:

$$\frac{\bar{T}(s)}{N(s)}\approx\frac{1-G(s)}{1+G(s)} \quad (17)$$

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A High-Speed Voltage-Regulating and Static-Excitation System for A-C Aircraft Generators

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WITHIN the past few years there has been developed a new type of voltage-regulating and static-excitation system for a 40-kva (kilovolt-ampere) a-c aircraft generator. The system is unique in its manner of excitation control, and exhibits rather remarkable performance. This paper describes the system and its principle of operation, and presents performance data.

General Description

Excitation System

The excitation system, shown schematically in Fig. 1, is a static type. All of the excitation power required by the generator is derived from the output of the machine itself by means of a 3-phase magnetic-core device known as a current-potential transformer (CPT). Each phase of this device contains three separate a-c windings: a potential winding, a current winding, and an output winding. The current winding is connected in such a manner that the line current of the given phase of the machine flows through it. The potential winding is connected in series with a reactor from generator line to ground. The output winding is connected to the output windings of the other two phases to form a closed delta, which supplies the power that is ultimately rectified and applied to the generator as field excitation.

Field excitation is controlled by three magnetic amplifiers, or "volt-second gates," which are connected between the output winding of the CPT and the generator field. The degree to which the amplifiers are turned on, i.e., the phase angle at which they are made to fire, together with the magnitude of their a-c supply voltage, which is the voltage appearing at the terminals of the output winding of the CPT, governs the average voltage applied to the generator field. Control current for the magnetic amplifiers is supplied by the voltage regulator.

This type of static exciter is similar to others described in the literature, in that output power is a function of both generator terminal voltage and generator line current.[1,2] It differs from these in that

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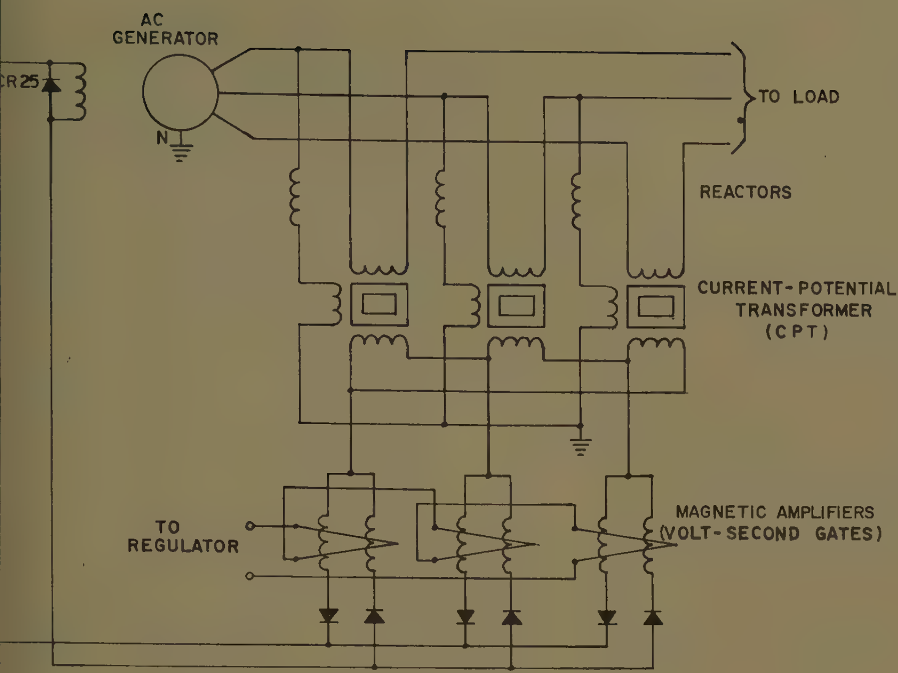

Fig. 1. Simplified schematic diagram of static exciter

output is controlled not by the degree of saturation of the core of the CPT, but by phase control of separate magnetic amplifiers in series with the CPT output.

Voltage Regulator

The regulator, Fig. 2, consists of a sensing circuit which produces a direct voltage proportional to the average value of the three line voltages, a transistor (*Q1*) preamplifier stage, and a magnetic amplifier (*SX1*) output stage. The output stage supplies control power for the three magnetic amplifiers in the static exciter. The voltage reference for the regulator consists of two breakdown diodes (*CR1*,*2*) in series. Direct current for the transistor preamplifier is supplied by a small transformer-rectifier which is energized from a permanent-magnet generator (PMG) mounted on the generator shaft. The PMG also supplies power to the magnetic amplifier (regulator output stage).

A more complete schematic diagram for the entire system is shown in Fig. 3.

Operation

Excitation System

The steady-state operation of the static excitation system can be understood qualitatively by referring to phasor diagrams for the generator and CPT. Consider first the generator. Fig. 4 represents the phasor relationships of voltage, current, and mmf's (magnetomotive forces) of the machine based on the Potier analysis, which is more useful for this purpose than the 2-reaction analysis.[3,4]

In this diagram, which represents the generator on a per-phase basis, phasor MMF_{RO} represents the mmf required to produce the flux which generates no-load terminal voltage. It leads this voltage by 90 degrees. For convenience this voltage is represented on the diagram by phasor E_T, which also represents the terminal voltage existing when the generator is supplying load current. For a well-regulated machine, terminal voltage under load does not in fact depart substantially from its no-load value.

When the machine is supplying some load current I, which lags terminal voltage by a phase angle θ, there occurs within the machine an equivalent voltage drop which is presented as synchronous reactance drop, IX_S, leading the current by 90 degrees. This voltage represents equivalent voltage drops due to armature leakage reactance and armature reaction, and when added to generator terminal voltage, results in a theoretical internal generator voltage represented by the phasor E_0. The mmf producing this voltage is MMF_F, which leads E_0 by 90 degrees. This mmf, which is produced by the generator field, must equal the phasor sum of MMF_{RO} and $-MMF_A$, the latter being that due to armature current and represented as a negative phasor because it has a demagnetizing effect.

The usual simplifying assumptions used in this type of analysis have been made; saturation and armature resistance are neglected. Furthermore, it has been assumed that mmf's and fluxes have sinusoidal air-gap distributions.

Consider now the phasor diagram for the current-potential transformer, Fig. 5. Take E_T, generator terminal voltage, as the reference phasor. At no load on the generator there is no current flowing in the current windings of the CPT, and these may be ignored. (Ignore also for the moment the volt-second gates. Assume that they are fully turned on, i.e., essentially short-circuited.) Field excitation supplied the generator from the CPT output windings is delivered by transformation from the potential windings, which are connected to the terminals of the generator through the reactors. Since generator field resistance referred to the CPT potential windings is small compared with the impedance of the reactors, which themselves are assumed to have no resistance, the current in the potential windings will lag E_T by essentially 90 degrees. This current establishes in the CPT cores an mmf represented on the phasor diagram as MMF_{RE}.

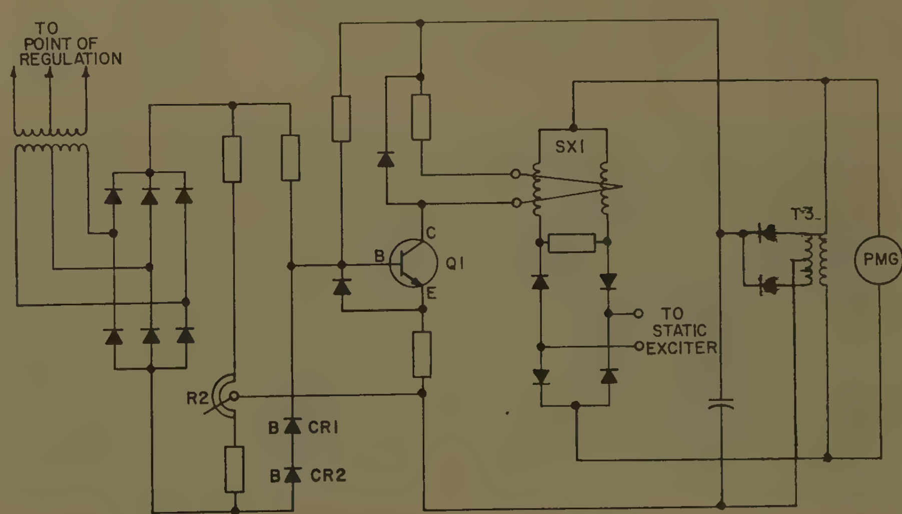

Fig. 2. Simplified schematic diagram of voltage regulator

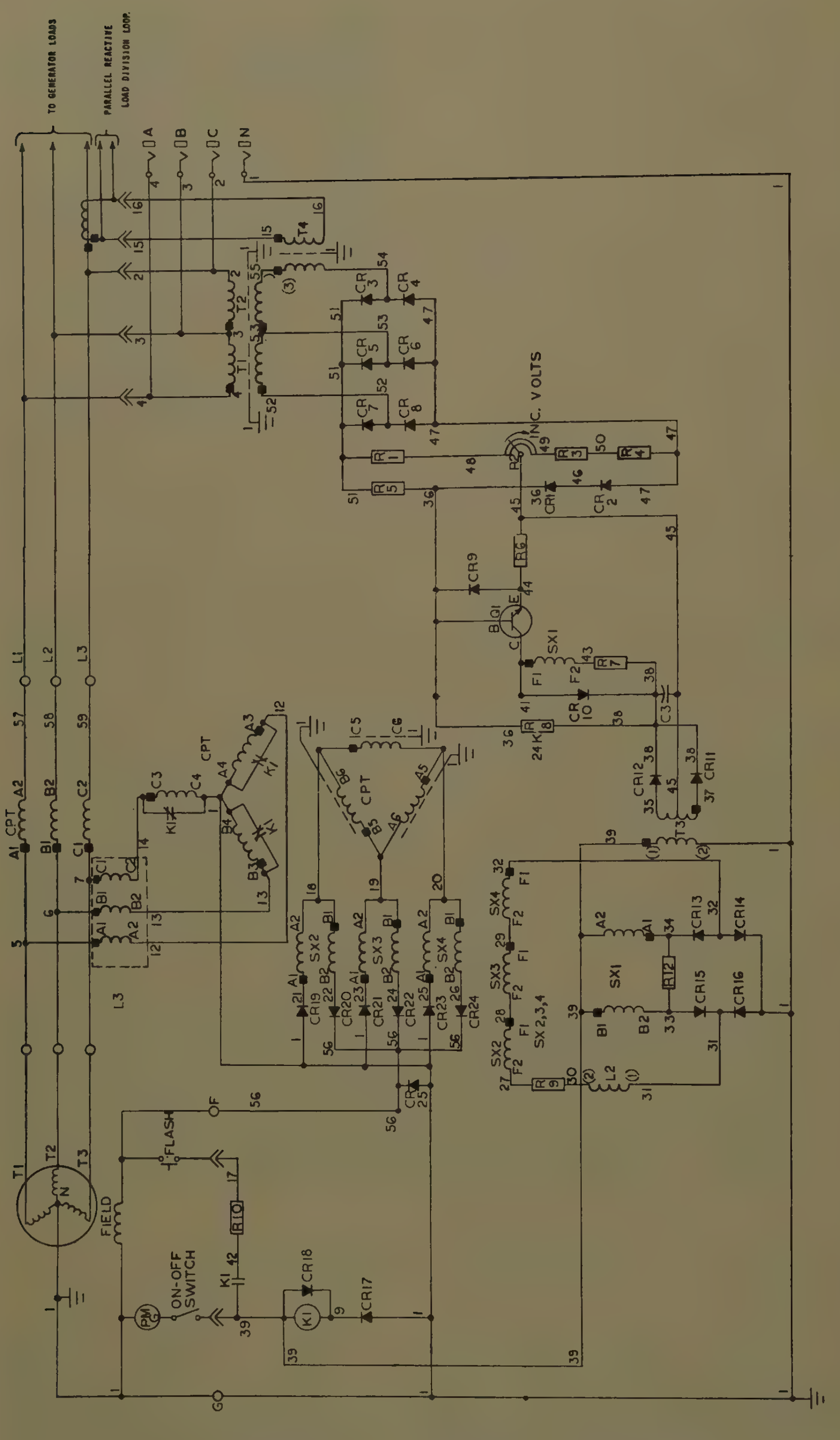

Fig. 3. Schematic diagram of entire regulating and excitation system

When the generator is loaded, the flow of current through the current windings of the CPT produces a second mmf which is in phase with the current. Calling the current I at an angle θ to the terminal voltage, this mmf is represented by the phasor MMF_{AB}. The sum of these mmf's is MMF_{FB}, which, to satisfy the ampere-turns law, must be exactly opposed by an mmf created by current flowing in the output windings. The phasor representing this mmf is termed $-MMF_{FB}$ and is shown in Fig. 5 as the phasor sum of $-MMF_{RB}$ and $-MMF_{AB}$. Again, the usual simplifying assumptions have been made; leakage reactances, magnetizing reactance, and winding resistances of the CPT are all assumed to be negligible; saturation is neglected; all voltages, currents, and mmf's are assumed to be sinusoidal.

Note the similarity between Figs. 4 and 5. In the generator, to maintain constant terminal voltage, it is necessary to supply an mmf (MMF_F) which is the phasor sum of an mmf (MMF_{RO}) which is proportional to and in quadrature with no load terminal voltage, and an mmf (MMF_A) which is proportional to and in phase with armature current. In the CPT there is produced a current which is the result of the phasor sum of an mmf (MMF_{RB}) which is proportional to and in quadrature with no-load terminal voltage, and an mmf (MMF_{AB}) which is proportional to and in phase with armature current. Thus the CPT, in combination with the reactors in the potential circuit, inherently produces an output of the proper characteristics to provide generator excitation which automatically compensates for inherent generator regulation; the output of the CPT automatically increases under the conditions which require greater generator excitation.

In the actual case it is possible to design the CPT and reactors to be under-compensating, flat-compensating, or over-compensating. In the particular design described in this paper, the system is made somewhat overcompensating, i.e., more excitation voltage is available at the output winding of the CPT for any condition of generator load than is required to sustain that load condition when that voltage, rectified, is applied to the field.

Control of the output voltage to bring it to the level required by the generator is accomplished by means of the magnetic-amplifier volt-second gates. By controlling the saturation of these by means of the output of the voltage regulator, it is possible to change the voltage drop occurring across them from a very low value to a relatively high value. Thus, regardless of the output voltage of the CPT, it is possible to exercise complete control over the excitation applied to the generator. Furthermore, this control is very rapid; hence, excitation can be adjusted swiftly. This accounts for the high rate of response of the system.

Note that voltage can be increased rapidly and can be reduced to zero rapidly. The actual rate of decay of field current when the voltage is removed by action of the volt-second gates is governed by the time constant of the generator field itself, inasmuch as current decays through the discharge rectifier *CR25*.

Voltage Regulator

In the voltage regulator rectifiers *CR3-8* (Fig. 3) rectify the generator alter-

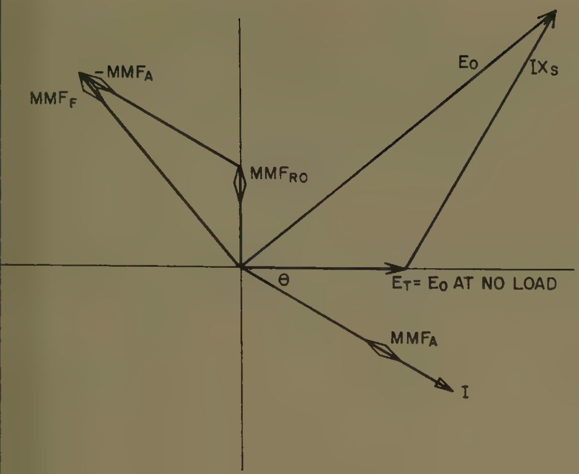

Fig. 4 (left). Phasor diagram of generator

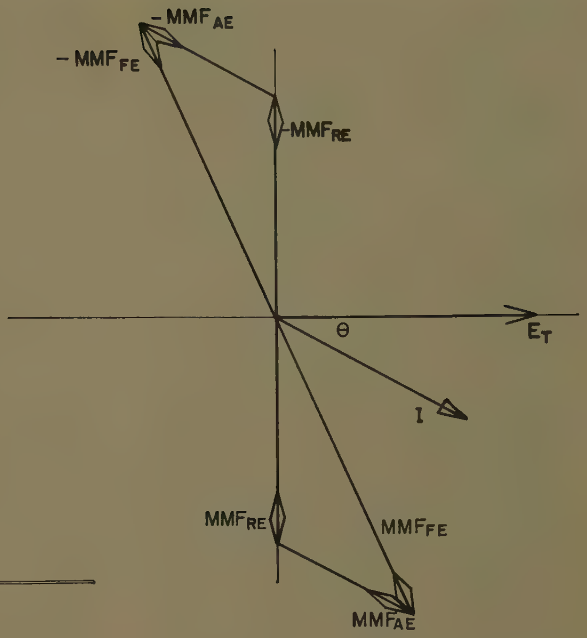

Fig. 5 (right). Phasor diagram of CPT and reactors

Table I. Performance Comparisons

	Rated Load Applied		Rated Load Removed		Three-Phase Short Circuit Removed			
	Minimum Voltage	Recovery Time	Maximum Voltage	Recovery Time	Maximum Voltage	Recovery Time	Volt-Seconds Over 125 Volts	Voltage Regulation, Per Cent
ntegral d-c exciter and a-c generator, 15-kva system	90	0.06	143	0.09	200	0.15	4.0	±2.5
tandard static exciter and a-c generator, 20-kva system	95	0.060	132	0.09	160	0.10	1.8	±2.2
igh-speed voltage regulator and excitation system, 40-kva system	110	0.02	123	0.06	150	0.09	1.2	±1

oltages are measured from line to neutral. All systems originally set to 115 volts. Data taken from est reports. Recovery time measured in seconds. Voltage regulation includes variations due to both oad and ambient conditions.

nating voltage and produce a direct voltage proportional to it. A portion of this voltage, points 47 to 45, is compared with the reference voltage existing across breakdown diodes *CR1* and *CR2*. The error voltage, points 36 to 45, is amplified by transistor *Q1* and magnetic amplifier *SX1*. The output of this device controls saturating current to the volt-second gate magnetic amplifiers *SX2*, *3*, and *4*.

The system is energized after the generator comes up to speed by closing the ON-OFF switch and depressing the FLASH button. This causes relay *K1* to be energized, completing the circuit from the PMG to the field. Permanent-magnet generator voltage, which is generated at a frequency of 1,600 cps (cycles per second), is then applied to the field, which is shunted with the discharge rectifier *CR25*. This results in a net direct voltage and causes initial excitation of the generator.

To shut down the system, the ON-OFF switch is opened, de-energizing relay *K*1. This short-circuits the potential windings of the CPT, effectively short-circuiting the output of this device and thus depriving the system of its means of excitation.

Performance

The high-speed performance of the system is illustrated by the oscillographic recordings shown in Fig. 6(A) which shows application and removal of rated load at 400 cycles, and Fig. 6(B), which shows application and removal of a 3-phase short circuit. Note the rapid rise and decay of field voltage when the transient is initiated.

Note that in the case of the full-load transient, recovery is faster upon application of load than upon release of load. This results from the fact that the control system can exercise greater influence on the application of excitation than upon

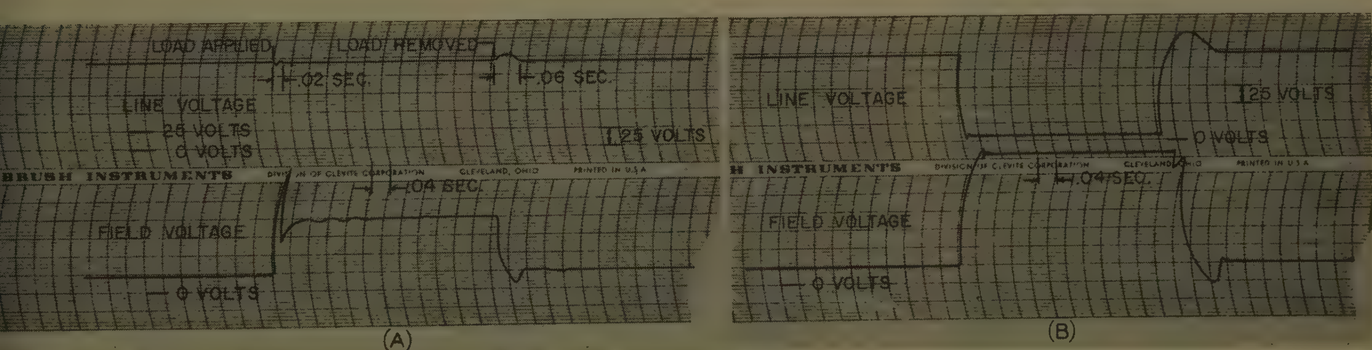

Fig. 6. Transient response

A—On application and removal of 40-kva 0.75-power-factor load, 400 cps
B—On application and removal of 3-phase short circuit

its removal. Note, however, that the transient is over in 60 milliseconds, even on removal of load.

Table I compares the performance of this generating system with other types of aircraft systems. Note that the system herein described is substantially better in recovery time, in undershoot or overshoot of transient voltage, and in volt-second envelope of the transient over a 125-volt base upon removal of short circuit.

Fig. 7 compares the performance of this generating system with the transient response requirements of specification *MIL-G-6099A*.[6]

All data apply to a generating system consisting of the voltage-regulating and static-excitation system described here and the 40-kva 0.75-power-factor 400-cps 24,000-rpm generator described in reference 5. Reactances and time constants of this machine are of the same order of magnitude as those of typical a-c aircraft generators. The most significant difference is in the value of direct-axis transient reactance, X_d', which is lower than that normally encountered.

A—Rated load removal, specification limit
B—Rated load removal, test result
C—Rated load application, specification limit
D—Rated load application, test result
E—Maximum transient, specification limit
F—Removal of 3-phase fault, test result

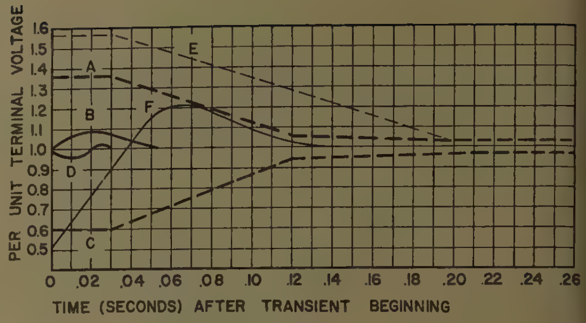

Fig. 7. Comparison of actual system performance and MIL-G-6099A specification limits

Conclusions

The field of static excitation systems for a-c aircraft generators has been broadened by the development of a system utilizing a current-potential transformer and magnetic-amplifier volt-second gates. This type of system, together with a matched high-speed voltage regulator, permits unusually rapid control of generator excitation voltage and provides superior generating-system performance.

Nomenclature

CPT = current-potential transformer
E_T = generator terminal voltage, line to neutral
I = generator line current
E_O = theoretical internal generator voltage
MMF_A = mmf in generator resulting from the flow of armature current
MMF_F = mmf in generator produced by field excitation
MMF_{AE} = mmf in CPT established by flow of line current in CPT current windings
MMF_{FE} = In CPT, phasor sum of MMF_{AE} MMF_{RE}: equal and opposite to mmf established by flow of current in output windings
MMF_{RE} = mmf in CPT established by current flow in potential windings at no load
MMF_{RO} = mmf in generator due to no-load field excitation
$SX1$ = magnetic-amplifier output stage in regulator
$SX2,3,4$ = volt-second gates

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Step-Up Frequency Changer

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WITH THE SCIENTIFIC advancement of the aircraft and missile fields, a need arises for power at high frequencies. High frequency power is needed in commercial and industrial fields as well; for example, in fluorescent lighting and ultrasonic cleaning. Presently available semiconductor devices can be used in the construction of completely static frequency conversion equipment, capable of power outputs of many kilowatts at frequencies as high as 25 kc.

In the past, static frequency conversion equipment has been mainly of the dual-conversion type, in which the input alternating voltage is first converted to direct voltage by rectification. The resulting direct current is then converted to high-frequency alternating current by means of class *B* amplifier or switching-type circuits. The amplifying or switching devices used may be either vacuum tubes or gas-filled tubes, or more recently transistors or controlled rectifiers.

The method of frequency changing described in this paper is a single-conversion type, wherein 3-phase low-frequency power is directly converted to single-phase high-frequency power. If three of these circuits are used, a 3-phase high-frequency output can be produced. Similar frequency changers can be built for any single-phase or polyphase input voltage. Controlled rectifiers are used as the switching elements, since they are capable of handling greater power than power transistors. The circuit described possesses inherently high power capabilities, high efficiency, excellent frequency stability, and good voltage regulation.

The title, "Step-Up Frequency Changer," is used to denote power conversion equipment in which the frequency of the power output is higher than the frequency of the input power. A companion paper[1] describes a similar circuit operated

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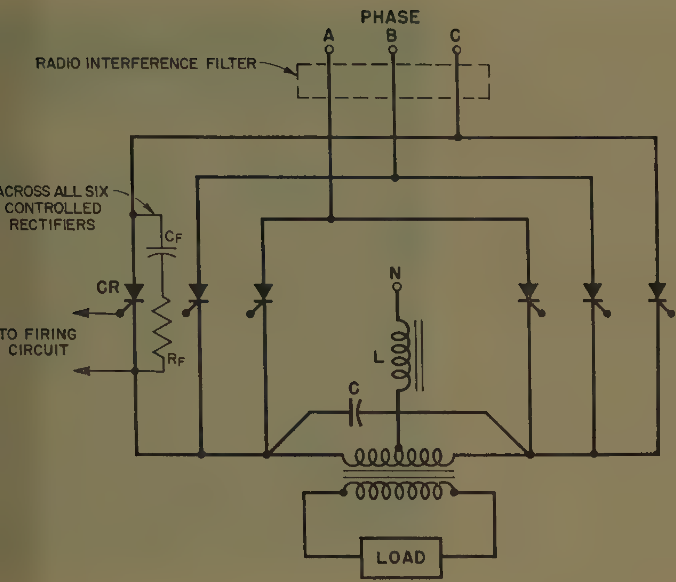


Fig. 1. Circuit diagram of a controlled rectifier step-up frequency changer

is a stepdown frequency changer, in which the frequency of the output is lower than that of the input.

The frequency changer is described assuming, in most cases, a 3-phase 400-cycle input power source converting to a single-phase 3,200-cycle output. The 3-phase input power is assumed to be obtained from a 4-wire system. However, these are not limiting conditions necessary to the operation of the frequency changer. The input frequency need not be stable; it may drift, or can even be frequency wild, without affecting the accuracy of the output frequency.

Frequency Changer Description and Operation

Power Transformer

It is possible to design power switching circuits for a step-up frequency changer in such a way that neither an input nor an output power transformer is needed. However, with such an arrangement, there is a definite relationship between input and output voltages. In most applications, either an input or an output power transformer, or both, are needed to give the voltages required by the application. The circuit in this paper is based on the use of an output power transformer but requires no input power transformer. With this arrangement, the high-frequency output voltage can be any value. With presently available controlled rectifiers, direct connection to a 3-phase 120/208-volt power supply is possible. For higher input supply voltages rectifiers can be connected in series.

Power Circuit

Smart and Weaver[2] have recently described a step-up frequency changer circuit which utilizes a multianode mercury-arc tube as the switching device. The mercury pool of the tube forms a common cathode, requiring that the high-frequency output transformer have three separate center-tapped primary windings.

The circuit described in this paper, Fig. 1, is similar in concept to the Smart and Weaver circuit. Since the controlled rectifiers, used as the switching devices, do not require a common cathode connection, the output transformer needs only a single center-tapped primary winding. This results in a saving in size and weight of the output transformer.

Basic operation of the frequency changer is similar to that of the parallel static inverter described by Wagner[3,4] except that the inverter battery power source is replaced by an alternating voltage source. During the 120 electrical degrees that the line-to-neutral voltage of phase A of the supply to the frequency changer is higher than that of the other two phases, the two controlled rectifiers connected to phase A of the supply alternately conduct current. The rate at which these two controlled rectifiers switch is determined by the timing of the firing pulses that are sent to their gates. The commutation or transfer of the load current from one rectifier to the other is accomplished by the commutating capacitor. The manner in which the high-frequency output voltage and its waveshape is produced is similar to that of the parallel inverter.

During the next 120 electrical degrees, the line-to-neutral voltage of phase B of supply is the highest, and the two controlled rectifiers connected to this phase take over the switching of the load current. The controlled rectifiers connected to phases A and C are idle during this period.

The transfer of the switching of the load current from one pair of controlled rectifiers to the next pair, takes place in a cyclical manner, which is very similar to the transfer of the direct current from one rectifier to the next in a 3-phase half-wave rectifier circuit.

The output frequency is dependent on the frequency of the controlled rectifier firing which exactly follows some reference, such as an oscillator. The unfiltered output voltage waveshape is shown in Fig. 2, and its shape is dependent on the power factor of the load and on the circuit time constants, primarily that of the commutating capacitor. The output contains a 3-phase ripple or modulation, but is smoothed by the choke L in the neutral or center-tapped lead.

Commutation

During the operation of the frequency changer, when one rectifier is fired, the capacitor charges according to a time constant determined by its capacitance, the inductance of the neutral choke, and the impedance of the load. A transformer voltage is then produced since the capacitor is in parallel with the transformer. When the second rectifier of the pair, which is connected to the other half of the primary, is fired, the capacitor discharges around the near short-circuit path. The discharge current passes forward through the controlled rectifier that has just been turned on, and inversely through the outgoing rectifier turning it off. The capacitor now charges in the opposite direction until the first rectifier is again fired. This commutation process continues with the controlled rectifiers

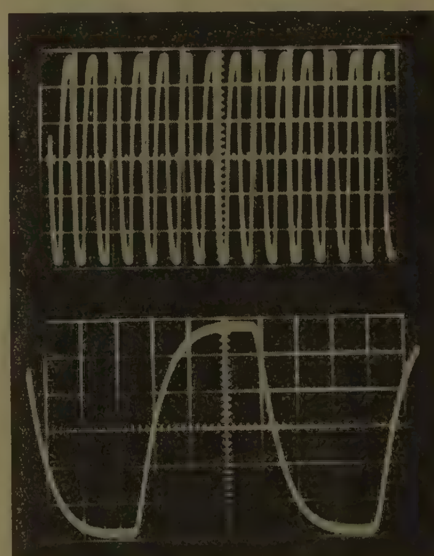


Fig. 2. Output voltage at a frequency of 3,200 cps

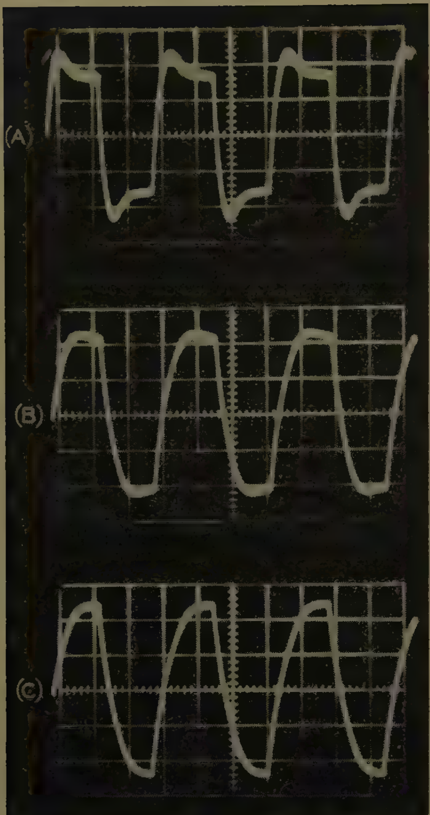


Fig. 3. Output voltage wave at 3,200 cps for three different leading load power factors

A—0.96 power factor
B—0.82
C—0.72 (load includes the commutating capacitor)

producing the high-frequency voltage in the output transformer.

In order for this circuit to commutate, the total load must be capacitive or represent a leading pf (power factor); the commutating capacitor being considered part of the load. If an inductance is added to the load, the capacitor value may have to be changed to keep the total pf still leading. The circuit can operate with a maximum pf of approximately 0.96, and a minimum pf of zero. In other words, the circuit could operate no load with only the commutating capacitor. However, if the value of capacitance is large (approaching zero pf), excessively high voltages will appear across the power transformer and across the controlled rectifiers. These voltages are much in excess of the applied supply voltage, and could easily exceed the voltage ratings of the switching components.

Fig. 3 shows three different unfiltered output voltages at a frequency of 3,200 cycles with different load pf. The neutral choke is constant for all three waves. Fig. 3(A) represents a total pf of 0.96 leading, wave (B), 0.82; and wave (C), 0.72. The change in pf for these waves was accomplished by changing only the value of commutating capacitance. Identical waveshapes result by changing either the load or the neutral choke value instead of the capacitor. These waves actually show the output voltage resulting from a change in the circuit time constant.

When the circuit time constant is long with respect to the operating frequency, the efficiency of the circuit is very dependent on the pf of the load. If the pf becomes too leading, a reduction in efficiency is noted because of high circulating currents. In order to operate over a large load range, it is necessary to change the value of the commutating capacitor to keep a leading pf, but yet not excessively leading. If it is possible in a given situation, the best method to accomplish the change in capacitor would be to connect a capacitor of proper valne across each load that the frequency changer will handle. In this way, the capacitor would be added to or removed from the circuit with the applied load. This would allow reliable operation with high efficiency from no load to full load.

The circuit of Fig. 1 shows the commutating capacitor across the primary winding for clarity of discussion; however, if a primary capacitor is used, it becomes necessary to use a small choke in series with the capacitor to lengthen the discharge time. Without this small choke, the capacitor causes a very high discharge current and places a rapid rise of voltage on the controlled rectifier; the combination of which could eventually be destructive to the rectifier. If this commutating capacitor is placed across the secondary winding instead of the primary, the series inductance is already built in (leakage reactance of the windings), and therefore, requires no other series choke.

Firing Circuit

The firing of the controlled rectifiers can be accomplished in many different ways. A very efficient method utilizes a low-power square-wave oscillator, whose output is differentiated by a series capacitor to produce pulses in a firing transformer. The output windings of the firing transformer are tied to the gates of the controlled rectifiers through a diode. This diode has the two-fold purpose of keeping the negative pulse from appearing across the gate, and also prevents anode to cathode leakage current from flowing out of the gate, through the firing transformer winding, and to the cathode after the unit has turned on. Since both groups of three controlled rectifiers each are so arranged as to have common cathodes, only two output windings are necessary

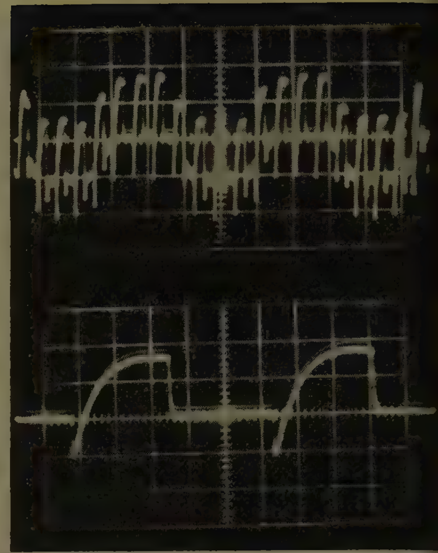


Fig. 4. Voltage across one controlled rectifier at a frequency of 3,200 cps

to fire the six rectifiers. The three rectifiers on one-half of the power transformer primary simultaneously receive the high-frequency (3,200-cycle) positive firing pulse, while the other three rectifiers connected to the other half of the primary are triggered by the firing transformer negative pulse. However, even though three units receive firing pulses, only the unit having the highest anode to cathode voltage will actually turn on. Therefore, sequential firing of the rectifiers is not necessary, and allows the firing circuit to be very simple in construction.

A second method of firing the controlled rectifier uses a 3,200-cycle sine-wave oscillator with a choke and saturable transformer in series. The choke is used to give a current source in order to obtain better pulses from the firing transformer.

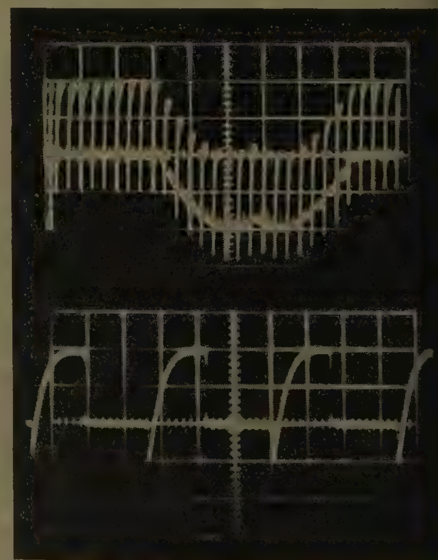


Fig. 5. Controlled rectifier voltage at 10 kc showing the voltage envelope

he secondary circuit would be the same the previously mentioned circuit.

esign Considerations

ONTROLLER RECTIFIER VOLTAGE

Probably, one of the more important esign considerations is the selection of the roperly rated controlled rectifier. The oltage, as seen by these units during rcuit operation, cannot be simply explained in an exact equation because there e so many variables and circuit parameters to consider. To begin with, the rimary winding of the power transformer btains its voltage from what might be onsidered an effective battery supply, r an average input voltage whose value or a 3-phase system is

$$_{DC}=\left(\frac{3}{\pi}\sqrt{\frac{3}{2}}\right)E_{LN}=1.17E_{LN}$$

here E_{LN} is the rms line-to-neutral voltge of the input supply. This voltage is npressed across one-half of the primary hen one of the rectifiers turns on, therere, the total primary sees $2E_{DC}$. Hower, this value is never exactly true in eality since this requires that the comutating capacitor value be zero. Since his value is not attained in a working cirit, the voltage across the transformer s always greater than $2E_{DC}$ by an amount ependent on the choke and load power actor. Calculation of the transformer oltage is possible using the material preented by Wagner.[3,4] This voltage is the orward blocking voltage a nonconducting ectifier must support, since it and its nate are in a series loop with the entire ransformer primary.

The reverse blocking voltage a conrolled rectifier must withstand is again ifficult to state in an exact simple equaion. In general, there are two major oltages to contend with in the inverse locking condition as shown in Figs. 4 and . Fig. 4 shows a controlled rectifier voltge at an operating frequency of 3,200 ycles from a 400-cycle input. Fig. 5 is he same voltage at a frequency of 10 kc, iving a clearer picture of the envelope f the voltage peaks.

The first of the two reverse voltages is resent during each of its nonconducting igh-frequency half-cycles. Assuming ne rectifier to be conducting, its comnutating mate receives a firing pulse and urns on. The capacitor begins its disharge and turns off the previously onducting unit. At the instant of turnff, this inverse voltage appears on the utgoing rectifier, and also appears on the ther two units of its group since they are n parallel with each other. In magnitude, the inverse voltage would exactly equal the transformer voltage at the instant of switching, if the rectifier would turn off at the exact instant of turn-on of the other unit. Since it actually takes a finite time to turn off a rectifier, the reverse voltage only approaches the forward blocking voltage (transformer voltage) as a limiting value, plus a switching transient which will be discussed later. This voltage is essentially equal to the forward blocking value only if the neutral choke L is infinite or at least large with respect to the switching frequency, and if the leakage inductance of the power transformer is zero. Since all transformers have leakage, this voltage will be less than the maximum stated, and will be smaller for greater values of leakage inductance, and reduces still further for higher current operating levels. Transformer leakage effects are discussed further in the next topic.

The second inverse voltage occurs only during its noncommutating period or for 240 electrical degrees of the input, and is equal to the peak voltage of the line-to-line input. This voltage is clearly shown in Fig. 5, and is the dual-like lobe appearing below the zero voltage line. The first reverse voltage rides along on this lobe, and the total reverse voltage becomes the sum of the two.

In general, then, it can be stated that

$E_F =$ Peak forward voltage $= 2(1.17)E_{LN}$
$= 2E_{DC}$ (minimum value)
$E_R =$ Peak inverse voltage $= 2E_F + E_{LN}\sqrt{6}$
(maximum value)

TRANSFORMER LEAKAGE EFFECTS

During the commutation and conduction of a pair of controlled rectifiers, there exists a time of overlapping currents where the incoming controlled rectifier has just turned on, and before the outgoing rectifier has completely turned off. This overlapping period is almost entirely dependent upon the leakage reactance of the transformer and the current passing through it. The greater the leakage reactance, the longer the overlapping time. This results because the current passing through the power transformer and its leakage reactance cannot change or stop immediately. This same current is passing forward through the rectifier to be turned off, and must decay in a given time, not instantaneously as would be desired. When the other rectifier is fired, the commutating capacitor must first cause the decay of current present in the outgoing rectifier, and then maintain the anode negative with respect to the cathode for a sufficient time for the unit to recover to its blocking state. In the meantime, the

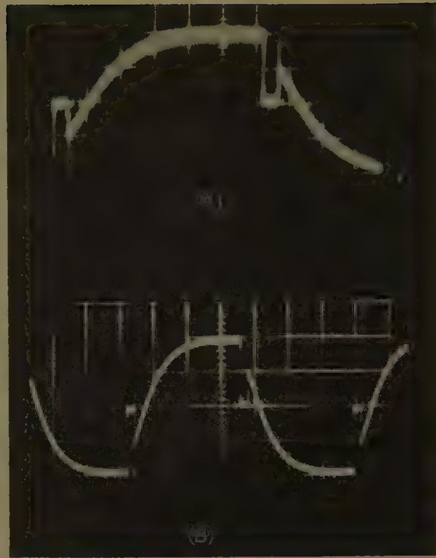


Fig. 6. Transformer primary voltage showing the effect of transformer leakage reactance on current overlap of controlled rectifiers

incoming rectifier has turned on and immediately begins conducting. During this time of overlap, both units conduct, but cause zero voltage in the primary of the transformer. It can, therefore, be understood that this time required, from the turn-on of the incoming rectifier to the actual turn-off of the outgoing, is proportional to the leakage reactance and the current level passing through it. This effect can be seen in Fig. 6, which shows the primary voltage of the power transformer.

As the overlap or no transformer voltage time increases, two effects occur with regard to the controlled rectifiers. It reduces the high-frequency inverse voltage of E_R since the capacitor is losing its voltage, and it also shortens T, the time available for recovery of the outgoing rectifier to its forward blocking state, which is in the neighborhood of 5 to 20 microseconds. This is generally referred to as the "turn-off" time of a controlled rectifier. Fig. 7 shows these effects. From this figure, it can also be seen that the maximum operating frequency is reduced by the same effects, since this frequency is determined by the time available for overlap T_O, plus the required inverse voltage time before the forward voltage can be applied. The controlled rectifier voltage waveshape in the left of Fig. 7 exists in a circuit where the neutral choke is large with respect to the switching frequency, or where the choke is small, but a large commutating capacitor is used. Note that the voltage rise, whose shape is determined by the circuit time constant, does not reach its peak value because of the turn-on of the other rectifier. In this case, the

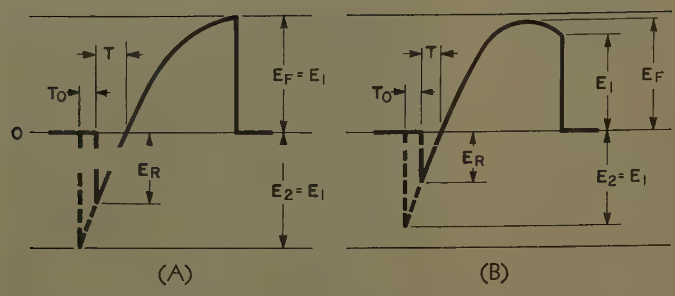
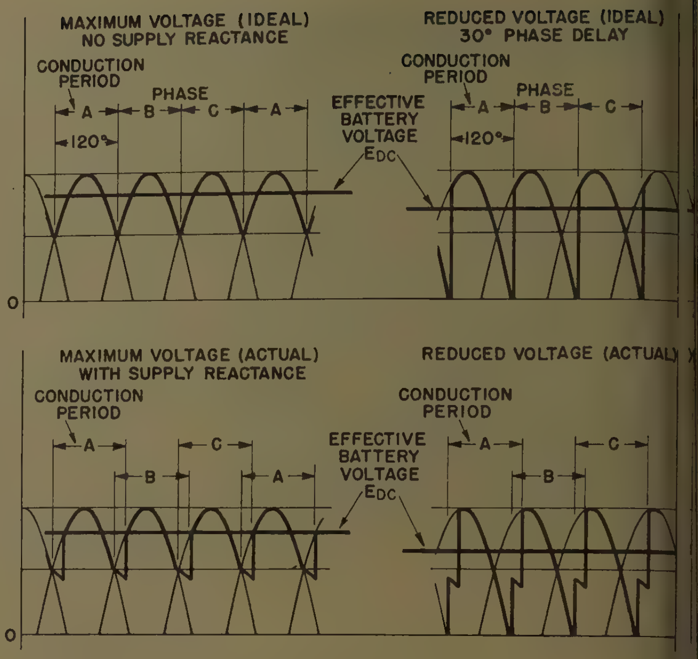


Fig. 7 (above). Diagram of controlled rectifier voltage showing current overlap effects on inverse voltage magnitude

(A) has a longer circuit time constant than that of (B)

Fig. 8 (right). A method of voltage regulation by reducing the value of the effectve battery voltage, accomplished by phase angle delay of the input voltages



reverse voltage E_2 would exactly equal the forward voltage E_F if no leakage reactance existed, as shown by the dashed line. But because of leakage, overlap exists and the actual reverse voltage E_R appears on the waveform. The voltage wave in the right of Fig. 7 exists when the neutral choke and commutating capacitor are smaller. Note that the positive voltage rise reached the peak value because of a shorter time constant, and then decreased before the other rectifier turned on. The instantaneous voltage at the time of switching E_1 is less than E_F. The magnitude of E_1 is now the maximum reverse voltage E_2, assuming no leakage reactance exists, and is shown again by the dashed line. The overlap time shows the amount of voltage discharged from the capacitor before the rectifier becomes blocking.

Switching Transients

The switching transients mentioned earlier occur at the tail of the inverse voltage peak, and actually give rise to a greater inverse voltage than mentioned. However, by proper design of a resistance-capacitance network across each of the rectifiers, the transients can be limited to a very small value. These voltage transients are generally in the megacycle region, and therefore, a relatively small value of capacitance can be used as a filter. The resistance used in series with this capacitor is adjusted to a value yielding the minimum voltage transient across the rectifiers. This transient is actually a damped, very-high-frequency oscillation caused mostly by the transformer capacitance and inductive effects at the time of switching, and appear across the controlled rectifier. Using one R-C (resistance-capacitance) network of proper design across the entire primary winding, eliminates most of the transient, but since the rectifier possesses junction and lead reactance, its effects are added to the oscillation set up by the transformer. Thus, more effective reduction of the transient is accomplished by placing an R-C network across each rectifier. Fig. 6(A) shows this unfiltered transient appearing on the primary of the transformer at the instant of turn-off of one rectifier, and hence, appears as an inverse voltage on the controlled rectifier.

Frequency Limitations

Operating output frequencies for this type of circuit can be as high as 25 kc at relatively high power levels, but for this frequency, controlled rectifiers selected for short turn-off times must be used. As an example, using controlled rectifiers with a 7-microsecond turn-off time, the maximum frequency with which this circuit could operate is approximately 35 kc at light loads, and with negligible transformer leakage reactance. The operation of the frequency changer at a high-frequency level, using a transformer having considerable leakage reactance, yields very inefficient operation because of the long overlapping current duration. The overlapping time is then quite appreciable with respect to the operating frequency period. Since there is no output voltage during the time of this overlap, the current flowing in the circuit at this time results in a power loss. When using this same transformer with the given leakage inductance at lower frequencies, the overlap time is reduced, and the overlapping time ratio with respect to the frequency period is smaller, thus improved efficiency results.

Transformer Design

As would be expected from the previous discussions, the transformer design plays an extremely important role at these high frequency levels. Leakage reactance must be kept as small as possible by designing the transformer to have a small number of primary turns and using bifilar wound windings on the transformer core. At these high frequencies, then, using a small number of transformer turns forces the designer to pay the penalty of using a large transformer core to support the voltage in spite of the frequency level, especially since a low operating flux density must be used to maintain an efficient circuit.

For operation at 3,200 cycles, which we are primarily concerned with, good transformer design is still important, but is not nearly as critical. Much better utilization of the transformer iron can be had by operating at higher flux densities, and still maintain a low power loss in the transformer. Leakage reactance is less at lower frequencies allowing more turns to be added to the windings. However, it is still desirable to use the bifilar method of winding the transformer in order to minimize leakage.

Circuit Features

In an ordinary Wagner-type parallel inverter operating from a direct voltage supply, it is impossible to turn off the inverter without interrupting the power circuit. With the step-up frequency changer, since the supply voltage is alternating, the power can be turned on and off merely by applying and removing the firing signals. Hence, this frequency changer is suitable for pulsed or keyed operation.

Reliable operation is achieved at any

tput power level, provided the commutating capacitor is sufficient in magnitude to sustain commutation. Should is total power factor not be sufficiently leading (0.96 or lower), commutation will fail. In the case of the parallel inverter, this will cause a short-circuit current to exist through the switching components. The power circuit must then be interrupted by a circuit breaker in an attempt to save the circuit; this becomes more difficult at high power levels. But with the frequency changer, completely static sensing circuitry prevents short circuit currents, and thus, prevents damage to any component of the circuit or the load without breaking the input power. Should this condition of insufficient leading power factor be rectified, the control circuitry can be arranged to cause the frequency changer to restart on its own and produce output power. In the same manner, a short-circuit fault in the load or output of the changer results in no ill effect to the load nor the circuitry. Upon removal of the short, power again flows. A description of control and protection is not presented since it depends on the particular application and is outside the scope of this paper.

Voltage Regulation

Voltage regulation of this type of frequency changer is accomplished by control of the firing circuit signals, and is performed at a very low power level. An increase in voltage appearing at the output of the transformer is detected. The firing signals to the incoming input phase are then held off while the outgoing phase is permitted to commutate for a longer duration than its natural time before shifting to the next pair of rectifiers in the next phase. The effective battery voltage is reduced by this action causing a decrease in output. This continues with each input phase. Actually, each pair of controlled rectifiers still conducts for approximtely 120 degrees, but the conduction angles have been shifted depending on the amount of regulation required. See Fig. 8.

Under ideal maximum voltage conditions, the effective battery voltage E_{DC} is equal to 1.17 E_{LN} or 0.827 of the peak value. In actuality, since the power supply has internal inductance, this value is somewhat reduced depending on the value of supply reactance and upon the current drawn, as depicted in the lower portion of Fig. 8. A regulated voltage causing a delay of 30 degrees in operation changes the average effective battery voltage to a smaller value equal to 0.827 cos 30 or 0.717 of the peak input voltage. This value is again ideal and actually is slightly less than this.

From the diagram, it can be seen that the ripple content or modulation in the output voltage will be greater because of regulation, but will be held to a small value by the neutral choke.

Conclusions

The step-up frequency changer described in this paper, utilizing controlled rectifiers in a single conversion process, directly converts 3-phase low-frequency into single-phase high-frequency power. Although a 4-wire 3-phase power supply was assumed for discussion, the frequency changer can be adapted to use other input supply configurations, such as a 3-wire 3-phase supply, or other single- or polyphase supplies. The high-frequency output may be made polyphase by conventional techniques.

1. The maximum output frequency is limited by the controlled rectifiers and circuit parameters, and presently, may be as high as 25 kc for relatively heavy loads, and 35 kc for light loads.

2. The output frequency being entirely independent of the input frequency, allows the circuit to follow exactly any reference signal frequency, provided the switching rate does not cause insufficient time for turn-off of the controlled rectifiers. Frequency accurateness and stability are completely dependent on the reference oscillator or generator, thus, extremely close tolerances are obtainable. With a crystal oscillator, frequency stability of the frequency changer can be 0.001%.

3. Since the frequency changer operates directly from an alternating supply, voltage regulation is efficiently accomplished by phase-angle control of the input voltages without the aid of additional power components.

4. Simplicity of the power circuit and control circuitry result in a highly reliable power conversion unit. The use of controlled rectifiers as semiconductor switches enhance total reliability, since semiconductors themselves are inherently reliable devices when used in the proper manner.

Nomenclature

C = commutating capacitor
C_F = transient filter capacitor
CR = controlled rectifier
E_F = peak forward blocking voltage of the controlled rectifier
E_R = peak inverse voltage of the controlled rectifier
E_1 = forward blocking voltage at the instant of firing a controlled rectifier
E_2 = maximum peak inverse voltage assuming no leakage reactance in the power transformer
L = neutral choke
R_F = transient filter resistor
T = time available for the recovery (turn-off) of the controlled rectifier
T_0 = time duration of the controlled rectifier or transformer current overlap

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